

SCIENZA DELLE COSTRUZIONI – CORSO B

prof. arch. G. Guerzoni A.A. 2017 – 2018

Prova parziale di strutture. 27-11-2017

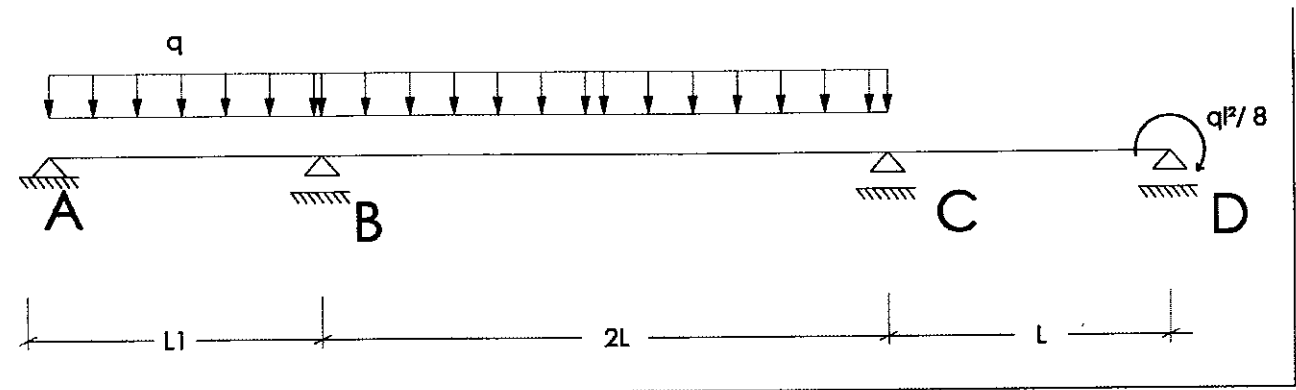
Cognome: _____

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I° Esercizio:

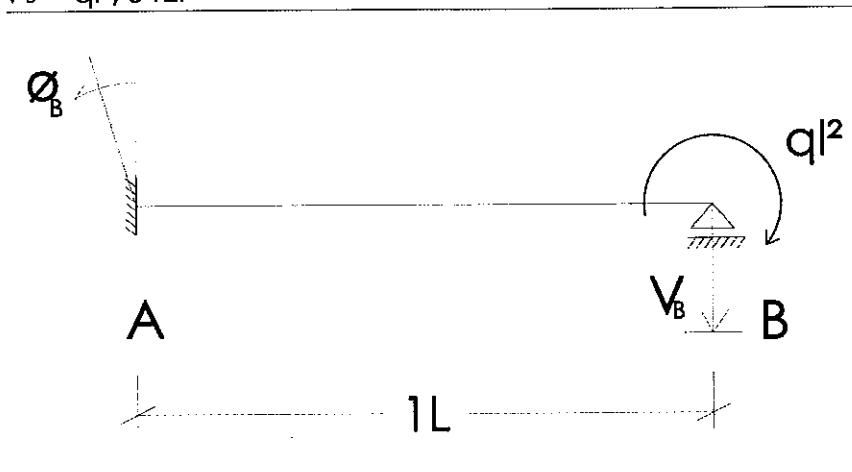
Nella Seguente struttura determinare reazioni vincolari e tracciare i diagrammi quotati e in scala delle caratteristiche della sollecitazione N , T e M mediante equazione dei tre momenti (trave continua) e verificare i risultati con il metodo degli spostamenti.



II° Esercizio:

Nella seguente trave determinare l'incognita iperstatica attraverso il Metodo delle Forze (Linea elastica) e verificare i risultati con la composizione cinematica degli spostamenti.

$$\varphi_A = ql^3/32EI \quad V_B = ql^4/64EI$$



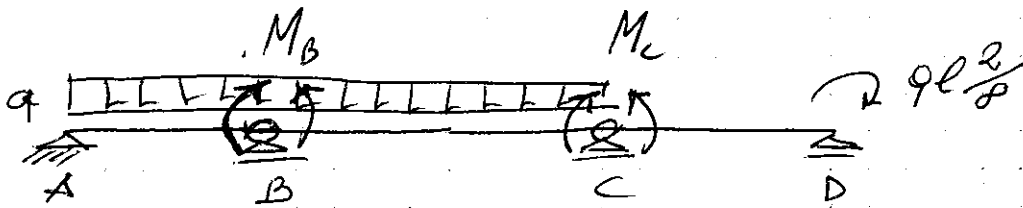
III° Esercizio Facoltativo:

Risolvere la precedente struttura con il Principio dei lavori virtuali

PER SUPERARE POSITIVAMENTE LA PROVA E' NECESSARIO TRACCIARE I DIAGRAMMI DELLE CARATTERISTICHE DELLA SOLLECITAZIONE CORRETTI DI ALMENO UNA DELLE DUE STRUTTURE.

ESERCIZIO 1

①

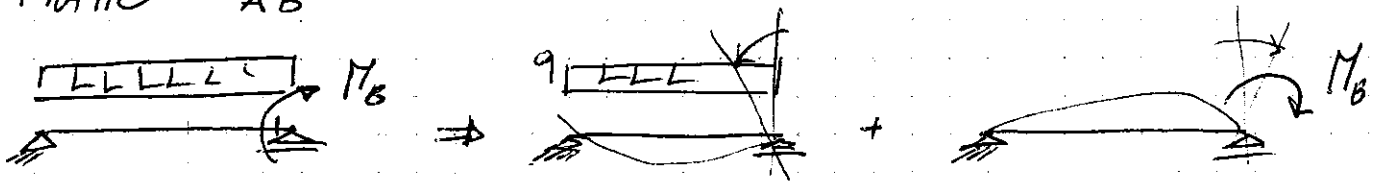


SI INSERISCONO 2 CERNIERE IN B E C
LIBERANDO I DUE MOMENTI M_B E M_C

SI IMPONE CHE $\phi_B^{sr} = \phi_B^{dx}$ e $\phi_C^{sr} = \phi_C^{dx}$

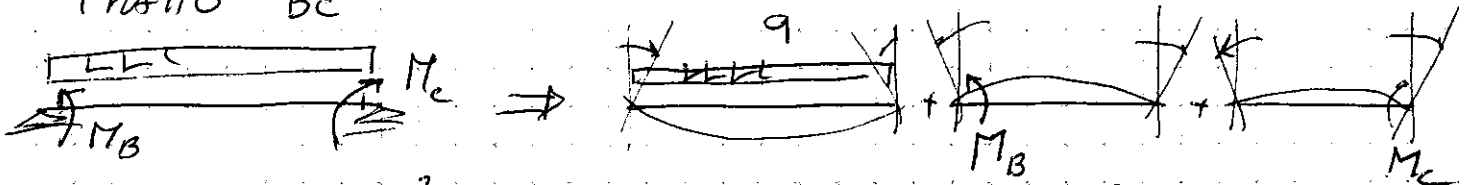
ESAMINIAMO TRATTO X TRATTO

TRATTO AB



$$\phi_B^{sr} = \frac{ql^3}{24EI} - \frac{M_B l}{3EI}$$

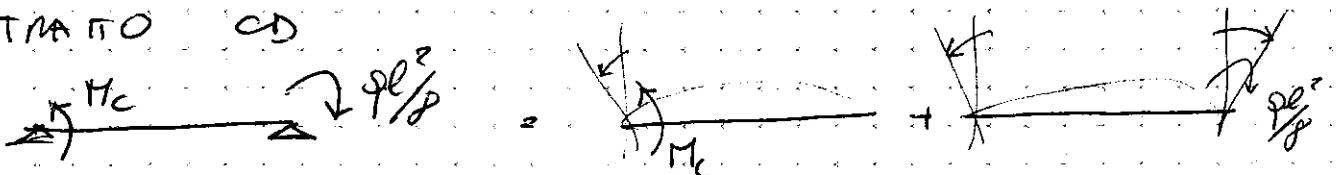
TRATTO BC



$$\phi_B^{dx} = -\frac{q(2l)^3}{24EI} + \frac{M_B \cdot 2l}{3EI} + \frac{M_C \cdot 2l}{6EI} = -\frac{ql^3}{3EI} + \frac{2M_B l}{3EI} + \frac{M_C l}{3EI}$$

$$\phi_C^{sr} = +\frac{q(2l)^3}{24EI} - \frac{M_B \cdot 2l}{6EI} - \frac{M_C \cdot 2l}{3EI} = +\frac{ql^3}{3EI} - \frac{M_B l}{3EI} - \frac{2M_C l}{3EI}$$

TRATTO CD

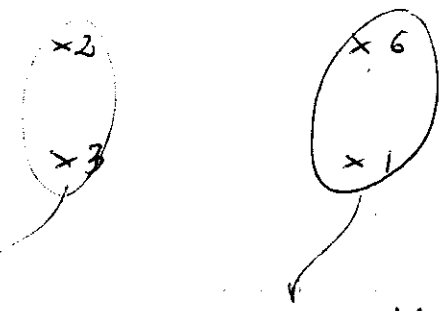


$$\phi_C^{dx} = \frac{M_C l}{3EI} + \frac{\frac{ql^2}{8} l}{6EI}$$

②

$$\left\{ \begin{aligned} \phi_B^M &= \phi_B^{\Delta x} & \frac{ql^3}{24EI} - \frac{M_B l}{3EI} &= -\frac{ql^3}{3EI} + \frac{2M_B l}{3EI} + \frac{M_C l}{3EI} \\ \phi_C^M &= \phi_C^{\Delta x} & \frac{ql^3}{3EI} - \frac{M_B l}{3EI} - \frac{2M_C l}{3EI} &= \frac{M_C l}{3EI} + \frac{ql^3}{48EI} \end{aligned} \right.$$

$$\left\{ \begin{aligned} -8M_B - 16M_B - 8M_C &= -8ql^2 - ql^2 \\ -16M_B - 32M_C - 16M_C &= ql^2 - 16ql^2 \\ 24M_B + 8M_C &= 9ql^2 \\ 16M_B + 48M_C &= 15ql^2 \end{aligned} \right.$$



$$\begin{aligned} - & \left\{ \begin{aligned} 48M_B + 16M_C &= 18ql^2 \\ 48M_B + 144M_C &= 45ql^2 \end{aligned} \right. \end{aligned}$$

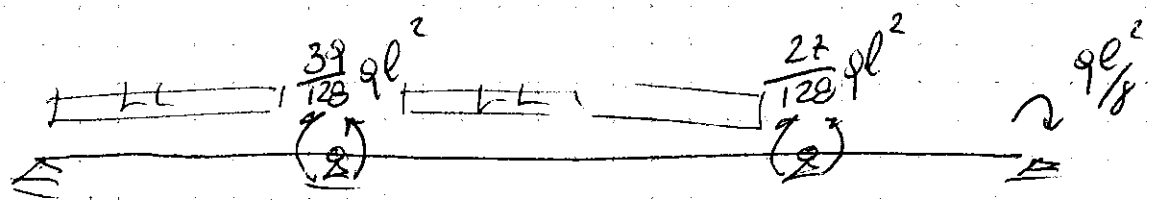
$$\begin{aligned} + & \left\{ \begin{aligned} 144M_B + 48M_C &= 54ql^2 \\ 16M_B + 48M_C &= 15ql^2 \end{aligned} \right. \end{aligned}$$

\checkmark $128M_C = 27ql^2$
TOLGO LA PRIMA EQ. ALLA TERZA

$$\underline{128M_B} \quad \checkmark \quad = 39ql^2$$

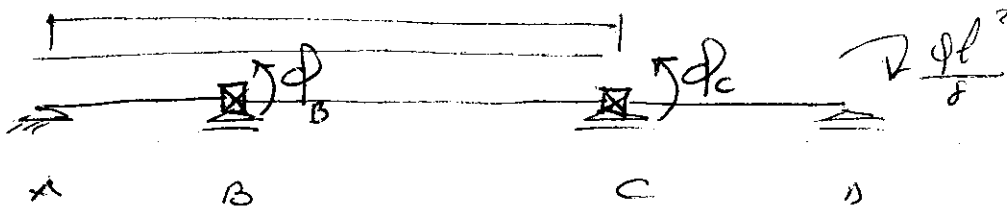
$$M_C = \frac{27}{128} ql^2$$

$$M_B = \frac{39}{128} ql^2$$



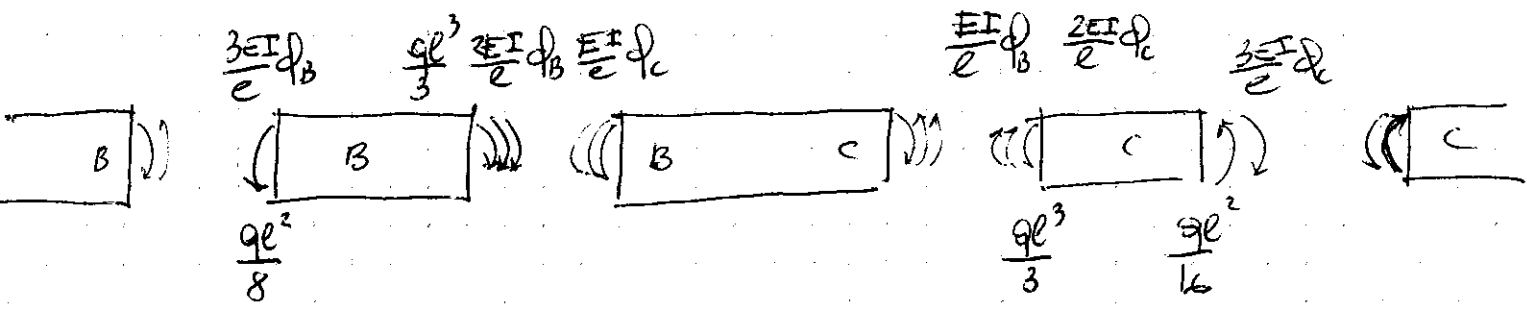
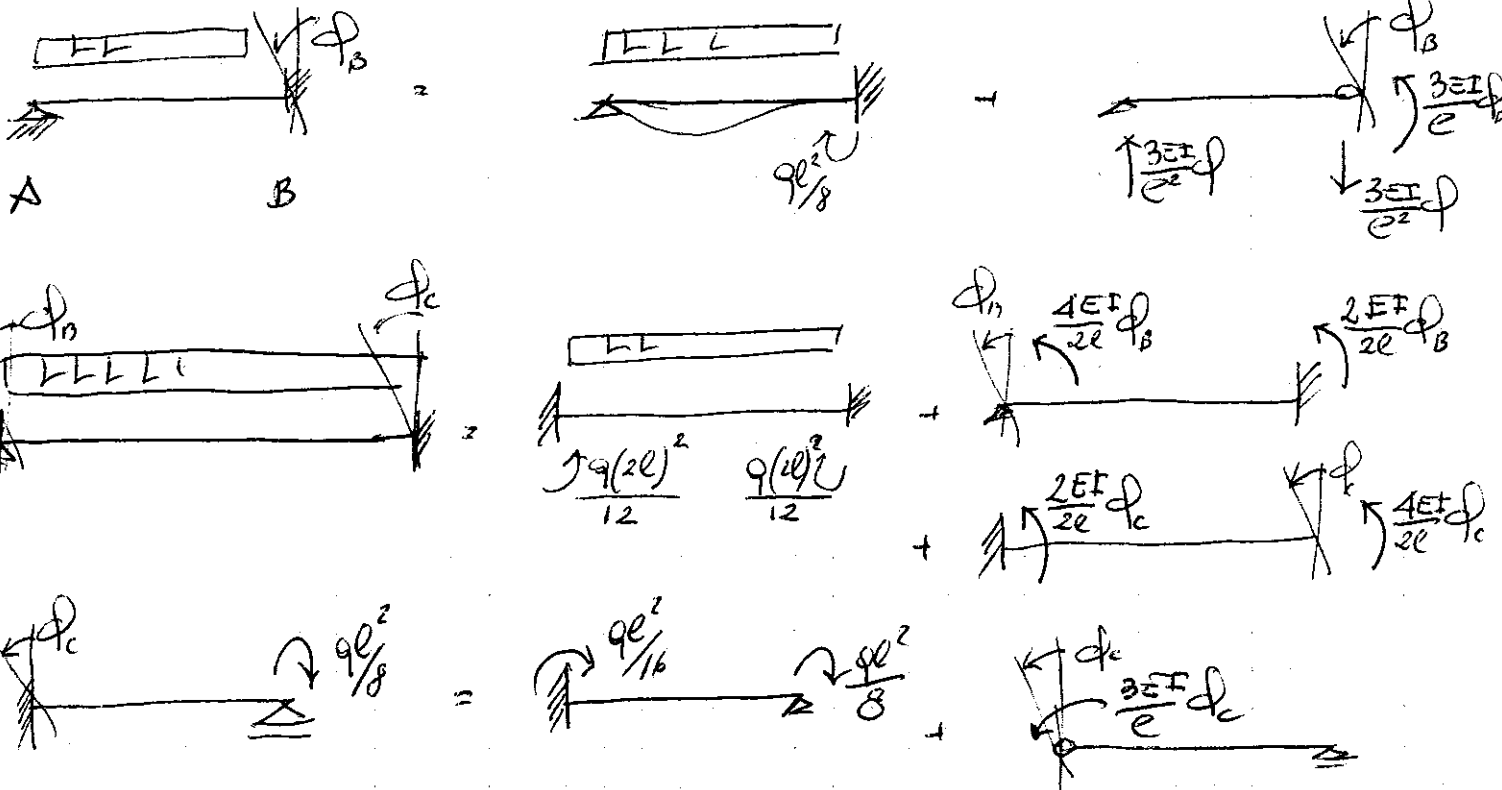
VERIFICA CON METODO DEGLI SPOSTAMENTI

(3)



I NODI INTERNI SONO IL NODO B E IL NODO C
 LE UNICHE COMPONENTI DI SPOSTAMENTO NON NULLE SONO LE
 ROTAZIONI ϕ_B e ϕ_C A CUI CORRISPONDONO LE
 EQUAZIONI RISOLVENTI $\sum M_B = 0$ $\sum M_C = 0$

ANALIZZIAMO TRATTO A TRATTO



4

$$\begin{cases} \sum M_0 = 0 & \frac{3EI}{e} \phi_B + \frac{2EI}{e} \phi_B + \frac{EI}{e} \phi_C = \frac{ql^2}{8} - \frac{ql^2}{3} \\ \sum M_c = 0 & \frac{EI}{e} \phi_B + \frac{2EI}{e} \phi_C + \frac{3EI}{e} \phi_C = \frac{ql^2}{3} + \frac{ql^2}{16} \end{cases}$$

$$\begin{cases} 5\phi_B + \phi_C = -\frac{5ql^3}{24EI} \\ \phi_B + 5\phi_C = \frac{19ql^3}{48EI} \end{cases}$$

$$\begin{cases} 25\phi_B + 5\phi_C = -\frac{25ql^3}{24EI} \\ \phi_B + 5\phi_C = \frac{19ql^3}{48EI} \end{cases}$$

$$\begin{cases} 5\phi_B + \phi_C = -\frac{5ql^3}{24EI} \\ 5\phi_B + 25\phi_C = \frac{95ql^3}{48EI} \end{cases}$$

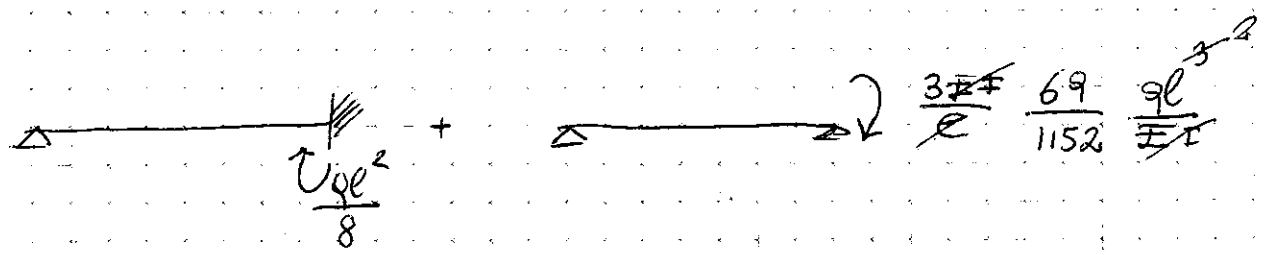
$$24\phi_C = \frac{105ql^3}{48EI}$$

$$24\phi_B = -\frac{69ql^3}{48EI}$$

$$\phi_B = -\frac{69ql^3}{1152EI}$$

$$\phi_C = \frac{105ql^3}{1152EI}$$

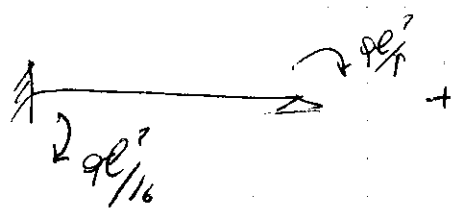
TRATTO AB



$$M_B = \frac{ql^2}{8} + \frac{207}{1152} ql^2 = \frac{144 + 207}{1152} ql^2 = \frac{351}{1152} ql^2$$

$$= \frac{39}{128} ql^2$$

M_c

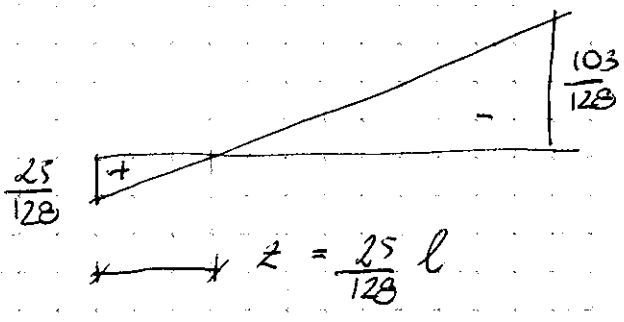
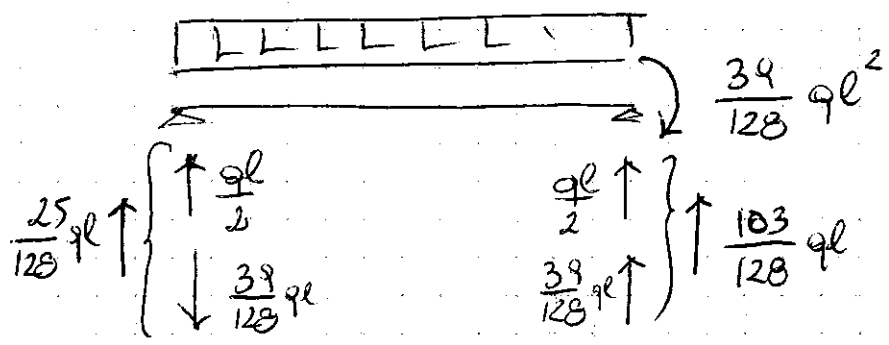


$$\frac{3 \cancel{27}}{2} \frac{105}{1152} \frac{ql^3}{\cancel{27}} = \frac{315}{1152} ql^2$$

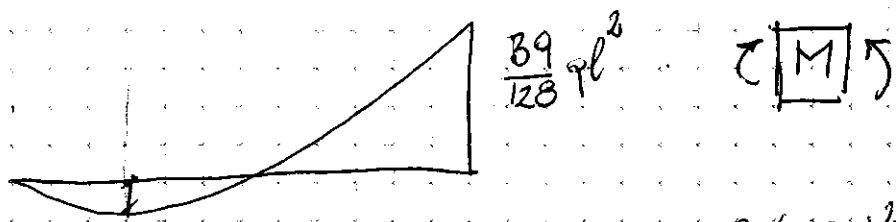
$$M_c = \frac{315}{1152} ql^2 - \frac{ql^2}{16} = \frac{315 - 72}{1152} ql^2 = \frac{243}{1152} ql^2 = \frac{81}{384} ql^2 = \frac{27}{128} ql^2$$

DIAGRAMMI

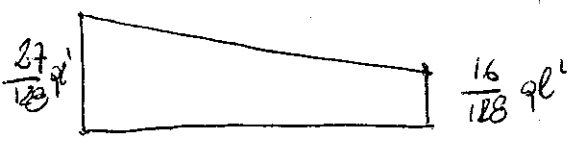
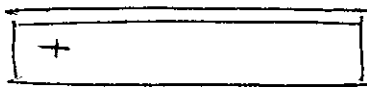
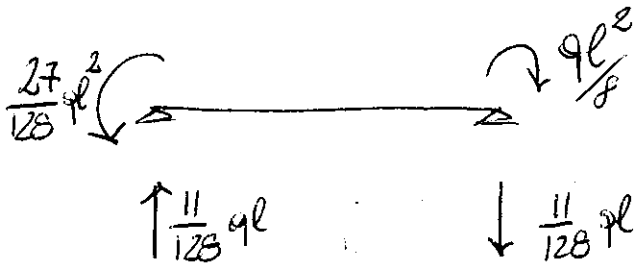
TRATTO AB



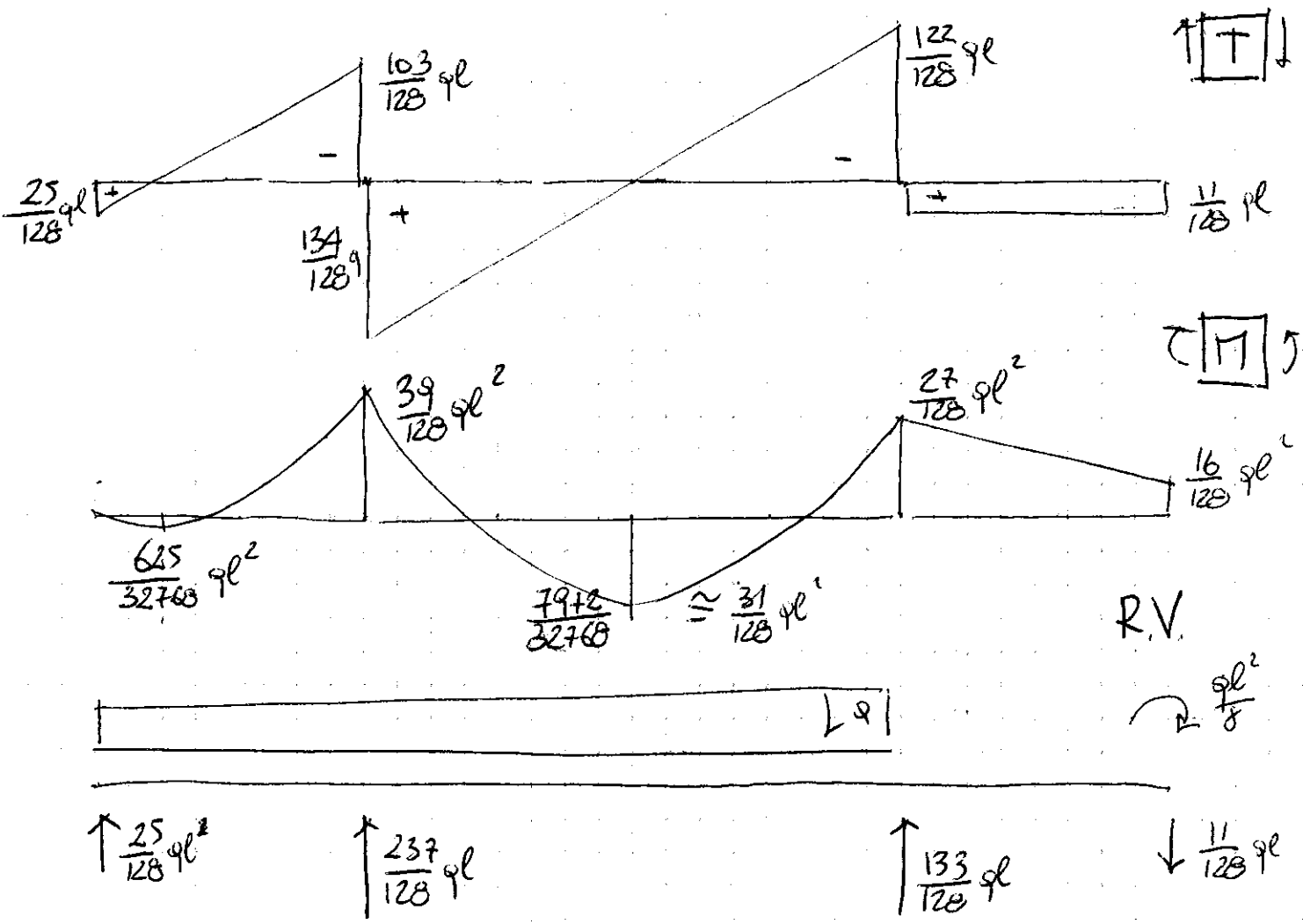
$\uparrow \boxed{+} \downarrow$
a meno di ql

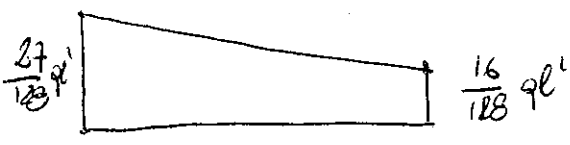
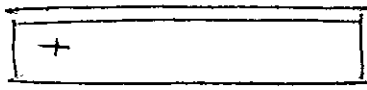
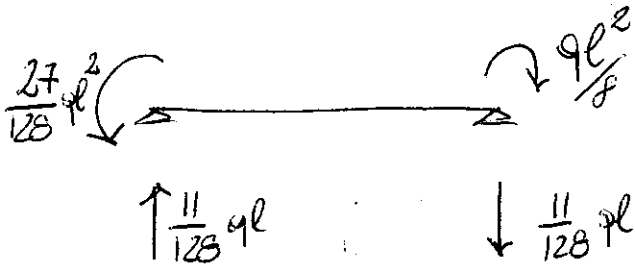


$$M_{max} = \frac{25}{128} ql \cdot \frac{25}{128} l - \frac{q}{2} \left(\frac{25}{128} l \right)^2 = q \left(\frac{25}{128} l \right)^2 - \frac{q}{2} \left(\frac{25}{128} l \right)^2 = \frac{q}{2} \left(\frac{25}{128} l \right)^2 = \frac{625}{32768} ql^2 \approx \frac{2,44}{128} ql^2$$

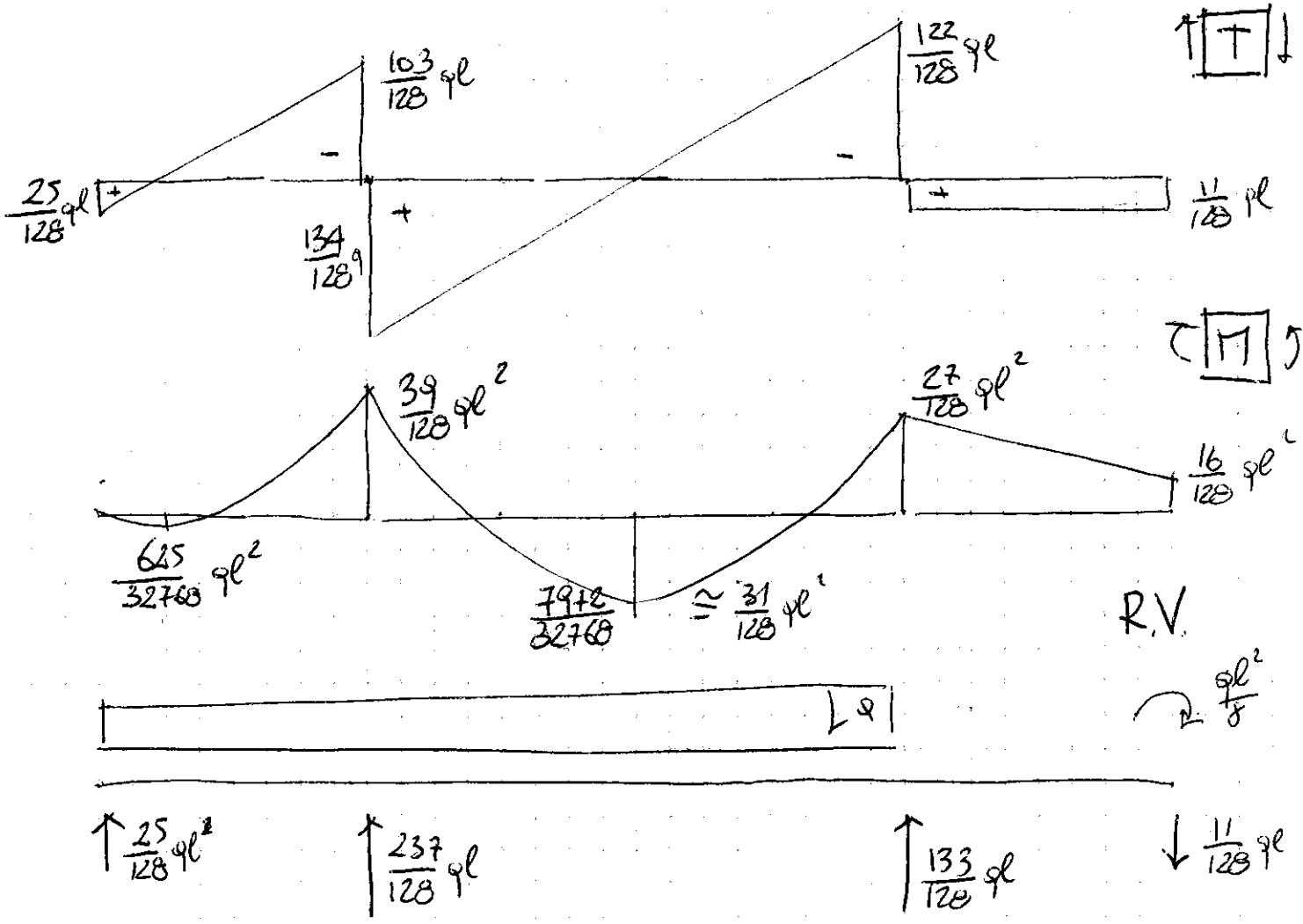


DIAGRAMMI TOTALI

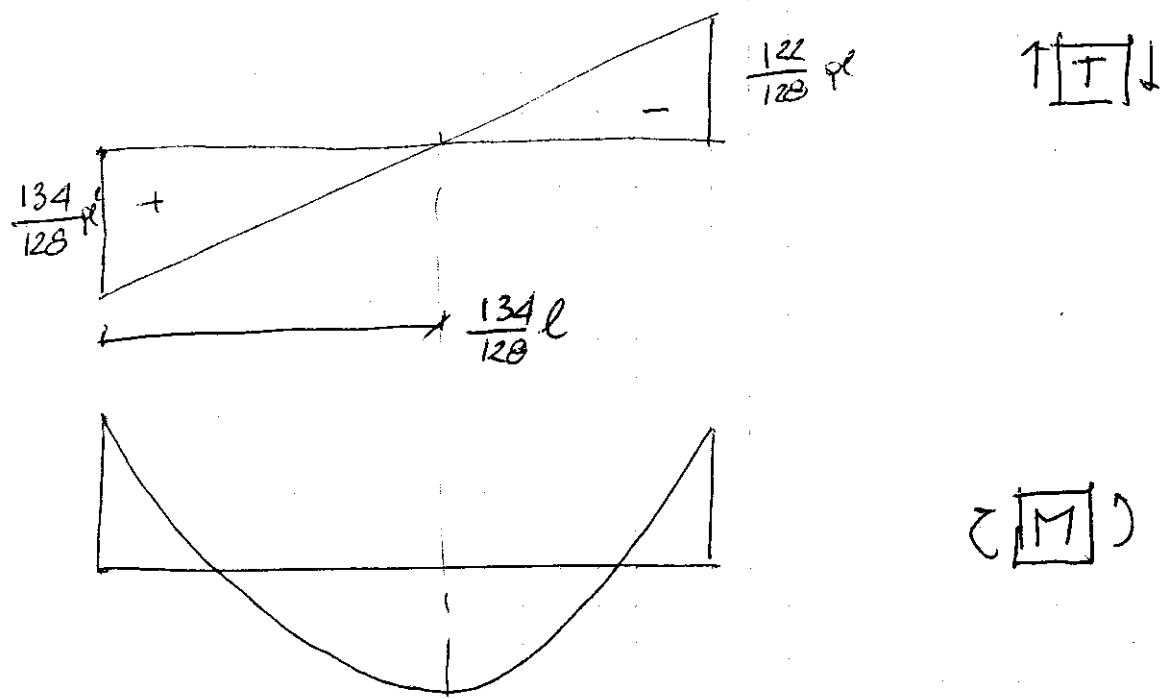
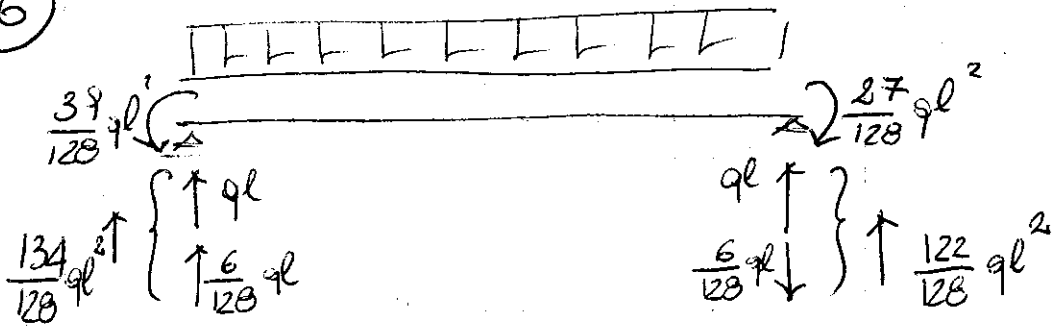




DIAGRAMMI TOTALI



6

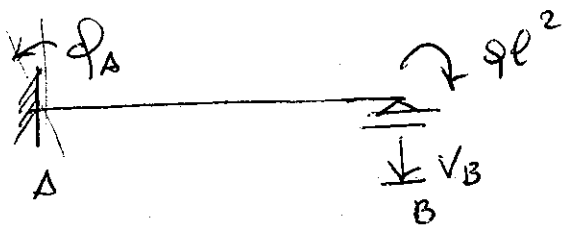


$$M_{max} = -\frac{39}{128} ql^2 + \frac{134}{128} ql \cdot \frac{134}{128} l - \frac{q}{2} \left(\frac{134}{128} l\right)^2$$

$$M_{max} = -\frac{39}{128} ql^2 + \frac{q}{2} \left(\frac{134}{128} l\right)^2 =$$

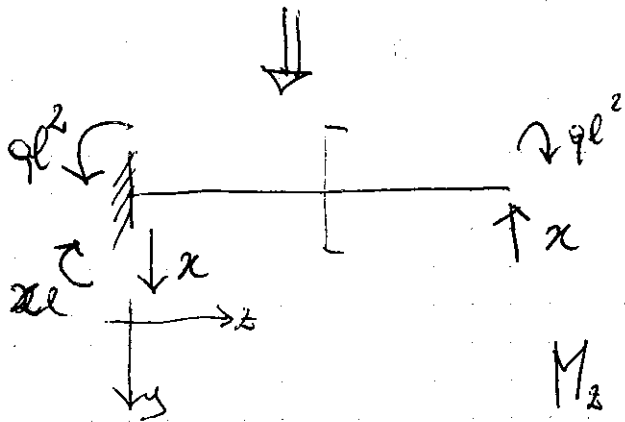
$$\frac{-9984 + 17956}{32768} ql^2 = \frac{7972}{32768} ql^2 \approx \frac{31}{128} ql^2$$

8)



$$V_B = ql^4/64EI$$

$$\varphi_B = ql^3/32EI$$



$$M_z = -ql^2 - xz + xl$$

$$\frac{d^2 V}{dz^2} = \frac{ql^2}{EI} + \frac{xz}{EI} - \frac{xl}{EI}$$

$$\frac{dV}{dz} = \frac{ql^2 z}{EI} + \frac{xz^2}{2EI} - \frac{xlz}{EI} + C_1 = -\varphi(z)$$

$$V(z) = \frac{ql^2 z^2}{2EI} + \frac{xz^3}{6EI} - \frac{xlz^2}{2EI} + C_1 z + C_2$$

CONDIZIONI AL CONTORNO

(A) Per $z=0$

$$\begin{cases} ① V=0 \\ ② \varphi = \frac{ql^3}{32EI} \end{cases}$$

(B) Per $z=l$

$$\begin{cases} ③ V = \frac{ql^4}{64EI} \end{cases}$$

DALLA ① SI RICAVA $C_2 = 0$

DALLA ② SI RICAVA $C_1 = -\frac{ql^3}{32EI}$

DALLA (3)

$$\frac{ql^4}{2EI} + \frac{x l^3}{6EI} - \frac{x l^3}{2EI} - \frac{ql^4}{32EI} = \frac{ql^4}{64EI}$$

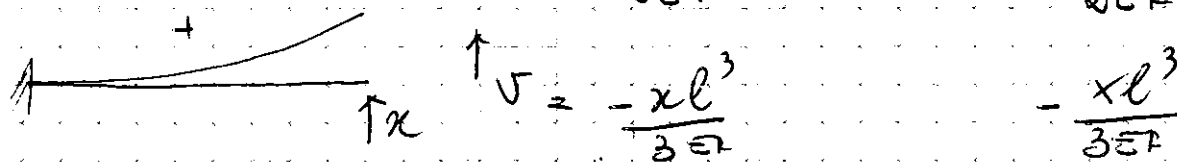
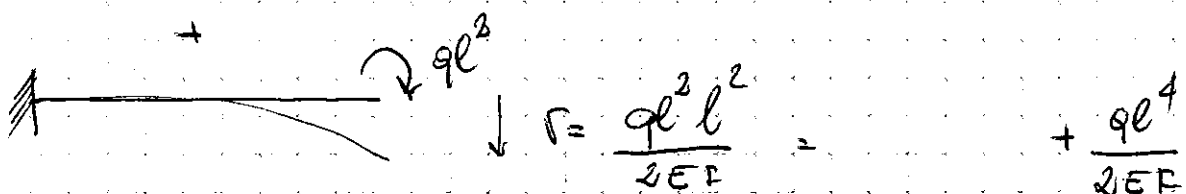
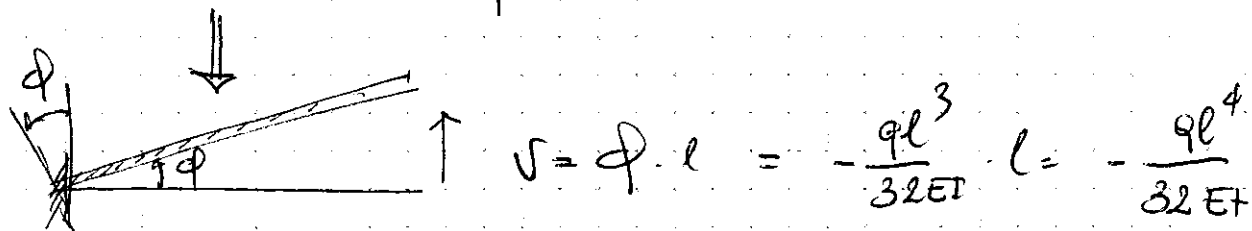
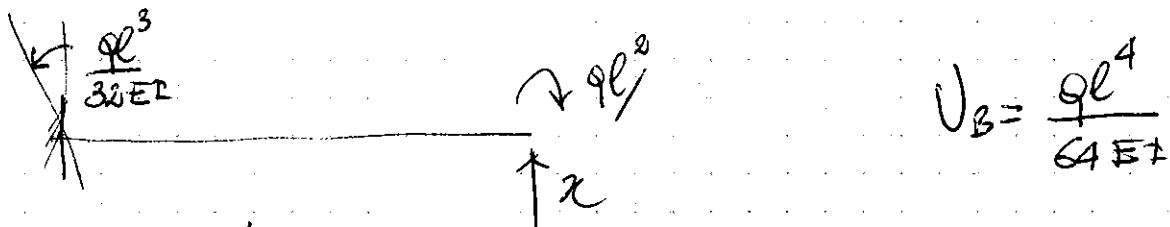
192 x

$$96 ql + 32x - 96x - 6ql = 39l$$

$$-64x = -87ql$$

$$x = \frac{87}{64} ql$$

CON POSIZIONE E CONDIZIONI DEGLI SPORSTAMENTI



$$-\frac{ql^4}{32EI} + \frac{ql^4}{2EI} - \frac{x l^3}{2EI} = \frac{ql^4}{64EI}$$

x 192

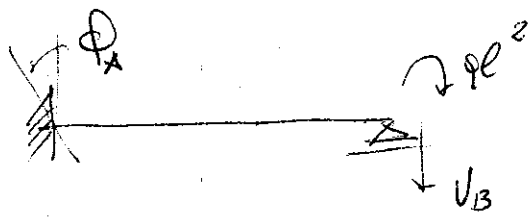
$$-6ql + 96ql - 64x = 39l$$

$$-64x = -87ql$$

$$x = \frac{87}{64} ql$$

10

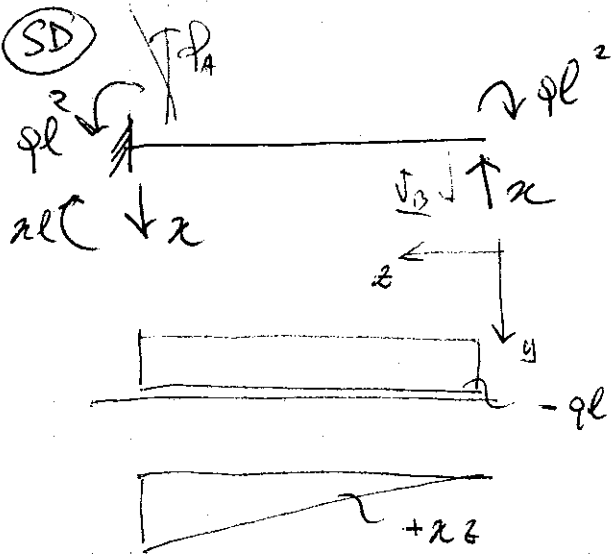
PRINCIPIO DEI LAVORI VIRTUALI



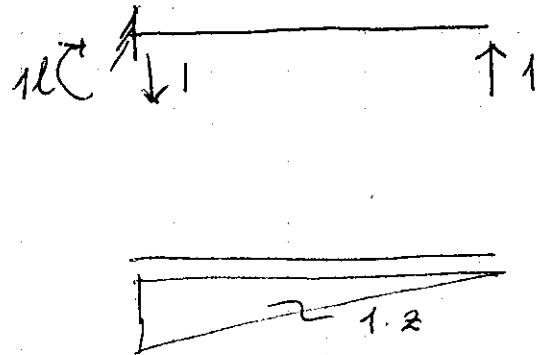
$$\phi_A = \frac{ql^3}{32EI}$$

$$v_B = \frac{ql^4}{64EI}$$

(SD)



(FT)



ASTA	l	M ^F	M ^v	q
BA	l	1.z	kappa.z	-ql ²

$$L_{VE} = -1 \cdot v_B - 1 \cdot l \phi_A = -\frac{1 \cdot ql^4}{64EI} - \frac{1 \cdot ql^3}{32EI} = L_{V1}$$

$$L_{V1} = \frac{1}{EI} \int_0^l (1 \cdot z)(\kappa z - ql^2) dz = \frac{1}{EI} \int_0^l (\kappa z^2 - ql^2 z) dz =$$

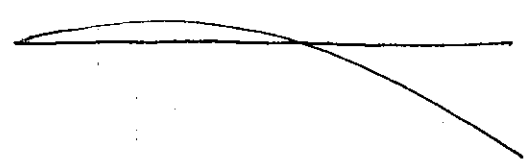
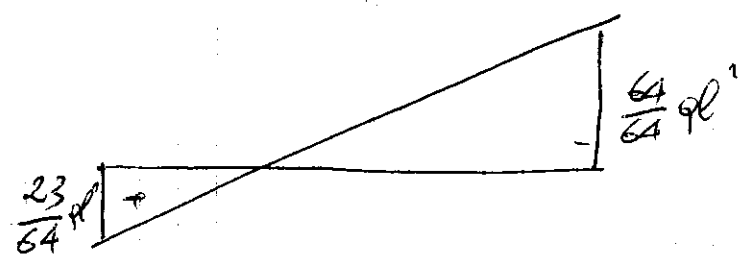
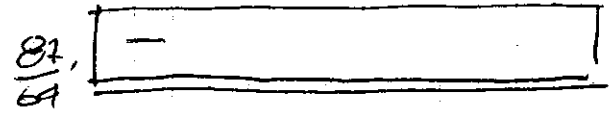
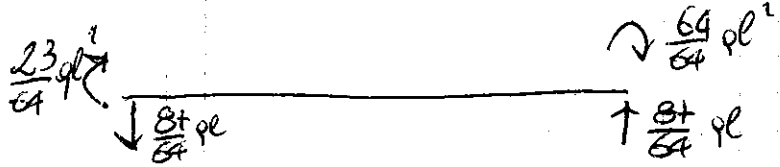
$$\frac{1}{EI} \left[\frac{\kappa z^3}{3} - \frac{ql^2 z^2}{2} \right] = \frac{\kappa l^3}{3EI} - \frac{ql^4}{2EI}$$

$$-\frac{ql^4}{64EI} - \frac{ql^4}{32EI} = \frac{\kappa l^3}{3EI} - \frac{ql^4}{2EI}$$

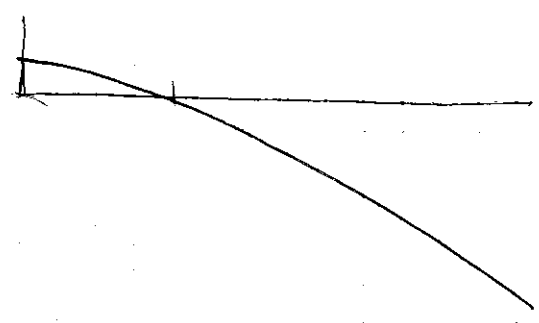
192 x

$$64 \times = 3ql - 6ql - 96ql$$

$$\boxed{\kappa = \frac{87}{64} ql}$$



v_z



ϕ_z