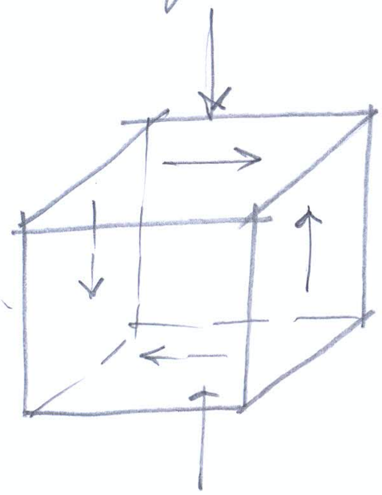
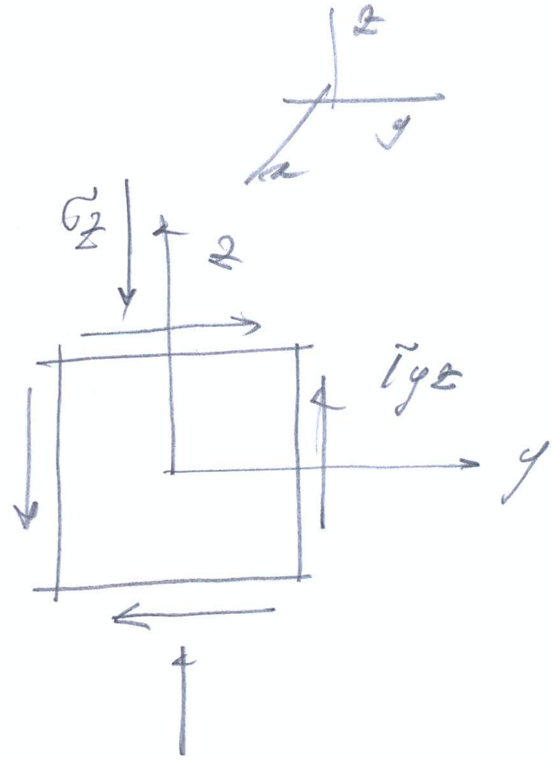
Dato $[\sigma]_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 500 \\ 0 & 500 & -1500 \end{bmatrix}_p$ kg/cmq

determinare tensioni e direzioni principali della tensione

Nello svolgimento è $\sigma_\eta = \sigma_2$, $\sigma_\zeta = \sigma_3$

(y, z) è il piano della tensione



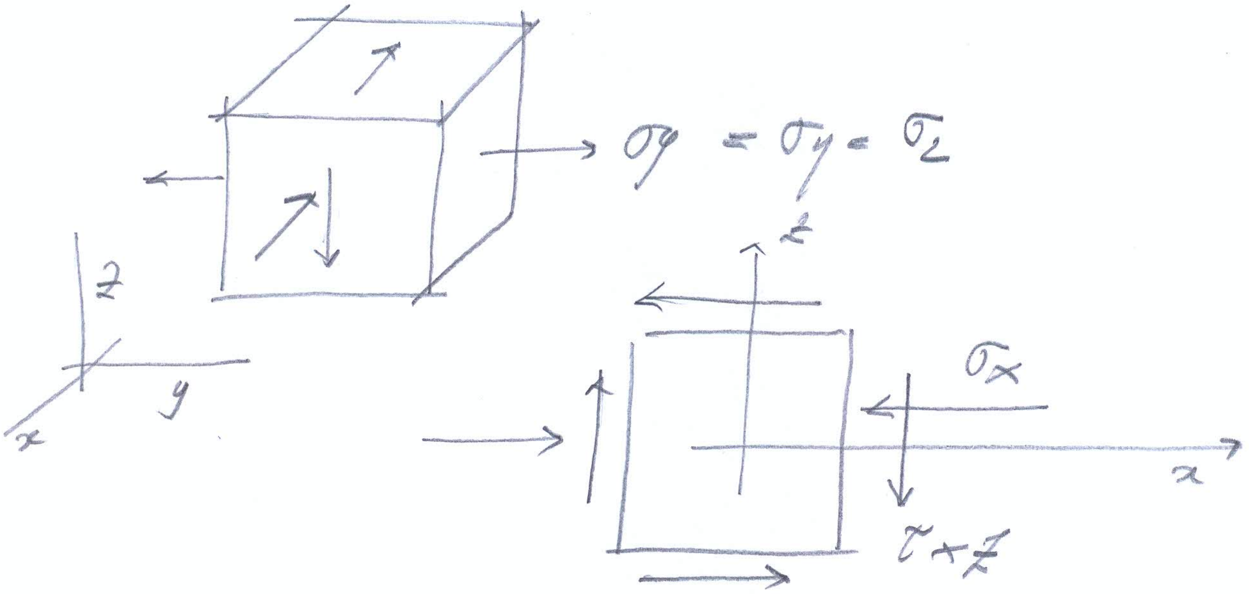


Dato
$$[\sigma]_p = \begin{bmatrix} -1500 & 0 & -800 \\ 0 & 500 & 0 \\ -800 & 0 & -200 \end{bmatrix}_p \ \text{kg/cmq}$$

determinare tensioni e direzioni principali

Nello svolgimento sia $\xi = 1$, $\eta = 2$, $\zeta = 3$

$\sigma_\xi = \sigma_1$, $\sigma_\eta = \sigma_2$, $\sigma_\zeta = \sigma_3$



A.2)

$$[\sigma_P] = \begin{bmatrix} -1500 & 0 & -800 \\ 0 & 500 & 0 \\ -800 & 0 & -2000 \end{bmatrix} \quad kg/cm^2$$

$\sigma_y = 500 = \sigma_\eta$

$\Rightarrow$ determinazione di σ_ξ, σ_ζ

$$\Rightarrow \begin{bmatrix} -1500 & -800 \\ -800 & -2000 \end{bmatrix} \Rightarrow \begin{bmatrix} -1500-\sigma & -800 \\ -800 & -2000-\sigma \end{bmatrix} \begin{bmatrix} n_x \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(-1500-\sigma)(-2000-\sigma) - 800^2 =$

$= 1500 \cdot 2000 + \sigma \cdot 1500 + \sigma \cdot 2000 + \sigma^2 - 800^2 =$

$= 2360000 + 3500\,\sigma + \sigma^2 = 0$

$$\sigma_{\xi,\zeta} = \frac{-3500 \pm \sqrt{3500^2 - 4 \cdot 2360000}}{2} = \begin{matrix} -912 \\ -2588 \end{matrix}$$

$\sigma_x > \sigma_z \rightarrow \sigma_\xi > \sigma_\zeta$

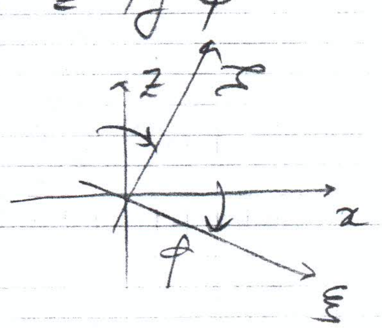
$\underline{\sigma_\xi = -912 \ kg/cm^2}$ $\quad$ $\underline{\sigma_\zeta = -2588 \ kg/cm^2}$

$(-1500 + 912)\, n_x^\xi - 800\, n_z^\xi = 0$

$-588\, n_x^\xi - 800\, n_z^\xi = 0$

$n_z^\xi / n_x^\xi = -\dfrac{588}{800} = -0{,}735 = tg\,\varphi$

$\varphi = arctg(-0{,}735) = -36{,}31^\circ$



stato

$$[\sigma_1]_p = \begin{bmatrix} 100 & -80 & 0 \\ -80 & 150 & 0 \\ 0 & 0 & -100 \end{bmatrix}_p \text{ Kg/cmq}$$

$$[\sigma]_p = \begin{bmatrix} 1600 & 500 & 0 \\ 500 & 800 & 0 \\ 0 & 0 & -1000 \end{bmatrix}_p \text{ Kg/cmq}$$

$$[\sigma]_r = \begin{bmatrix} 500 & -300 & 0 \\ -300 & -300 & 0 \\ 0 & 0 & -500 \end{bmatrix} \text{ Kg/cmq}$$

determinare tensioni principali e direzioni principali

$\xi = 1, \quad \eta = 2, \quad \zeta = 3$

$\sigma_\xi = \sigma_1, \quad \sigma_\eta = \sigma_2, \quad \sigma_\zeta = \sigma_3$

3.A)

$$\begin{vmatrix} 100-\sigma_n & -80 & 0 \\ -80 & 150-\sigma_n & 0 \\ 0 & 0 & -100-\sigma_n \end{vmatrix} = 0$$

$$(-100-\sigma_n)\left[(100-\sigma_n)(150-\sigma_n) - 80^2\right] = 0$$

$\sigma_3 = \sigma_I = -100 \ \text{kg/cm}^2$ $\qquad 3 \equiv I \equiv z$

$$15000 - 100\,\sigma_n - 150\,\sigma_n + \sigma_n^2 - 80^2 = 0$$

$$\sigma_n^2 - 250\,\sigma_n + 8600 = 0$$

$$\sigma_{1,2} = \frac{250 \pm \sqrt{250^2 - 4\cdot 8600}}{2} \left\langle \begin{array}{l} 208{,}81 \ \text{kg/cm}^2 = \sigma_2 \\ 41{,}18 \ \text{kg/cm}^2 = \sigma_1 \end{array} \right.$$

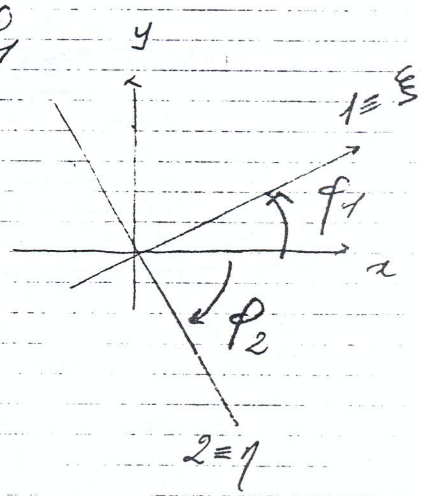
$\sigma_x < \sigma_y \Rightarrow \sigma_1 = 41{,}18 = \sigma_\xi < \sigma_2 = 208{,}81 = \sigma_\eta$

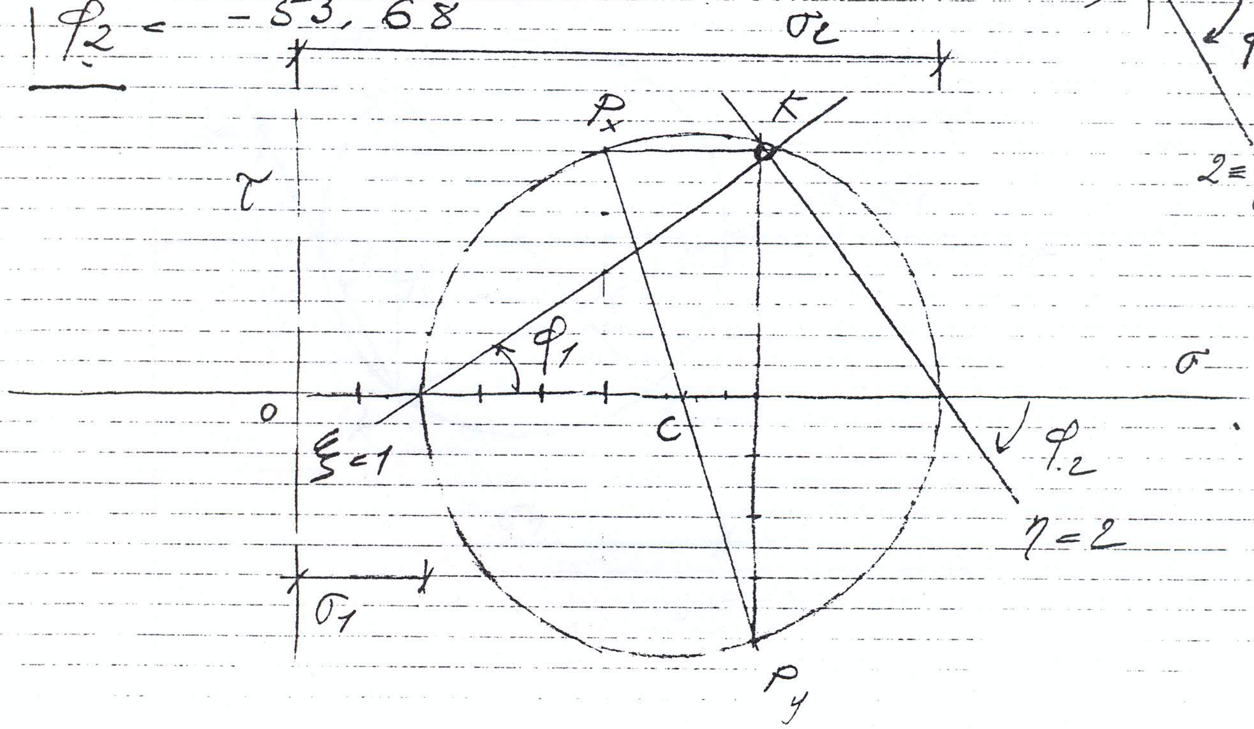
$$(100 - 41{,}18)\, n_x^1 - 80\, n_y^1 = 0$$

$$\frac{n_y^1}{n_x^1} = \frac{58{,}82}{80} = 0{,}73525 = tg\,\varphi_1$$

$\varphi_1 = \text{arctg}\ 0{,}73525 = 36{,}32^\circ$

$\varphi_2 = -53{,}68$





3.B)

$$\begin{vmatrix} 1600-\sigma_n & 500 & 0 \\ 500 & 800-\sigma_n & 0 \\ 0 & 0 & -1000-\sigma_n \end{vmatrix} = 0$$

$\boxed{\sigma_3 = \sigma_z = -1000 \ \text{kg/cmq}}$ $\quad \boxed{3 \equiv z \equiv z}$

$$(800-\sigma_n)(1600-\sigma_n) - 500^2 = 0$$

$$1'280\cdot000 - 800\,\sigma_n - 1600\,\sigma_n + \sigma_n^2 - 500^2 =$$

$$= 1030\cdot000 + \sigma_n^2 - 2400\,\sigma_n = 0$$

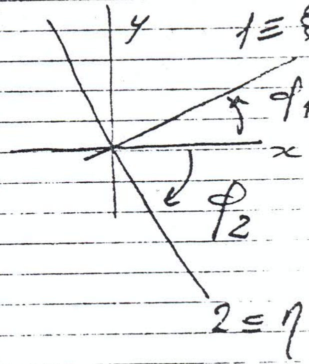
$$\boxed{\sigma_{1,2}} = \frac{2400 \pm \sqrt{2400^2 - 4\cdot1030.000}}{2} = \begin{matrix} 1840,31 = \sigma_1 \\ 559,68 = \sigma_2 \end{matrix}$$

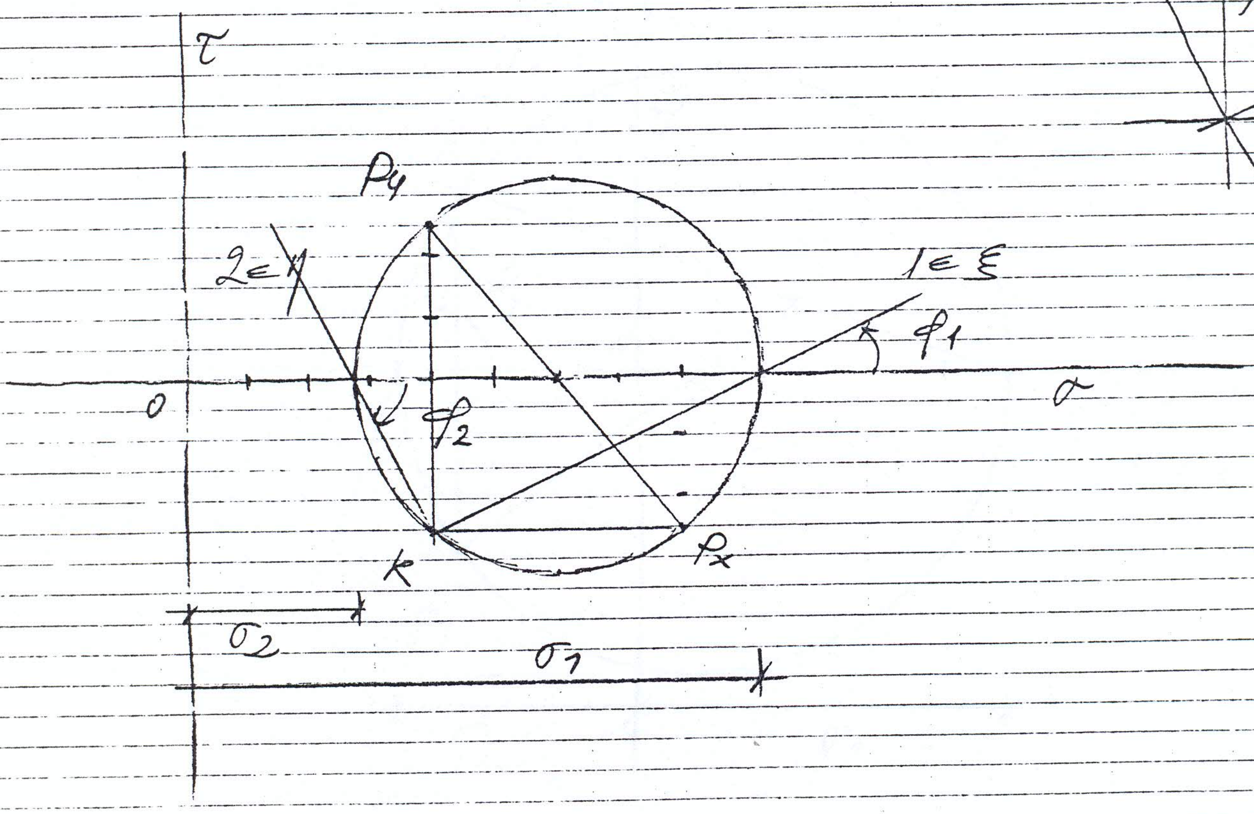
$\sigma_1 = \sigma_\xi \quad , \quad \sigma_2 = \sigma_\eta$

$$\underbrace{(1600 - 1840,31)}_{-240,31} n_x' + 500\, n_y' = 0$$

$$\frac{n_y'}{n_x'} = \frac{240.31}{500} = 0,48062 = tg\,\varphi_1$$

$\boxed{\varphi_1 = \operatorname{arctg} 0,48062 = 25^\circ,67}$ $\quad \boxed{\varphi_2 = -64^\circ,33}$





3.c) $\begin{vmatrix} 500-\sigma_n & -300 & \\ -300 & -300-\sigma_n & \\ & & -500-\sigma_n \end{vmatrix} = 0$

$\boxed{\sigma_3 = \sigma_\zeta = \sigma_z = -500\ kg/cm^2}$ $\quad 3 \equiv \zeta \equiv z$

$(500-\sigma_n)(-300-\sigma_n) - 300^2 = 0$

$-150\,000 - 500\,\sigma_n + 300\,\sigma_n + \sigma_n^2 - 300^2 =$

$= -240.000 + \sigma_n^2 - 200\,\sigma_n = 0$

$\sigma_{1,2} = \dfrac{200 \pm \sqrt{200^2 + 4 \cdot 240.000}}{2}$ $\begin{cases} 600\ kg/cm^2 = \sigma_1 \\ -400\ kg/cm^2 = \sigma_2 \end{cases}$

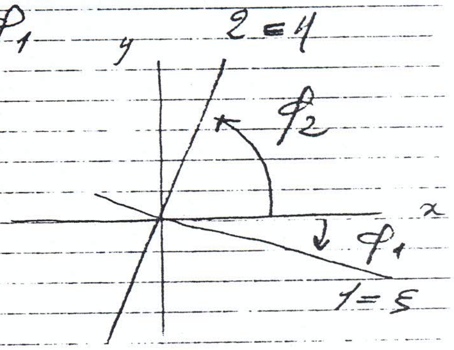
$\sigma_1 = \sigma_\xi \qquad \sigma_2 = \sigma_\eta$

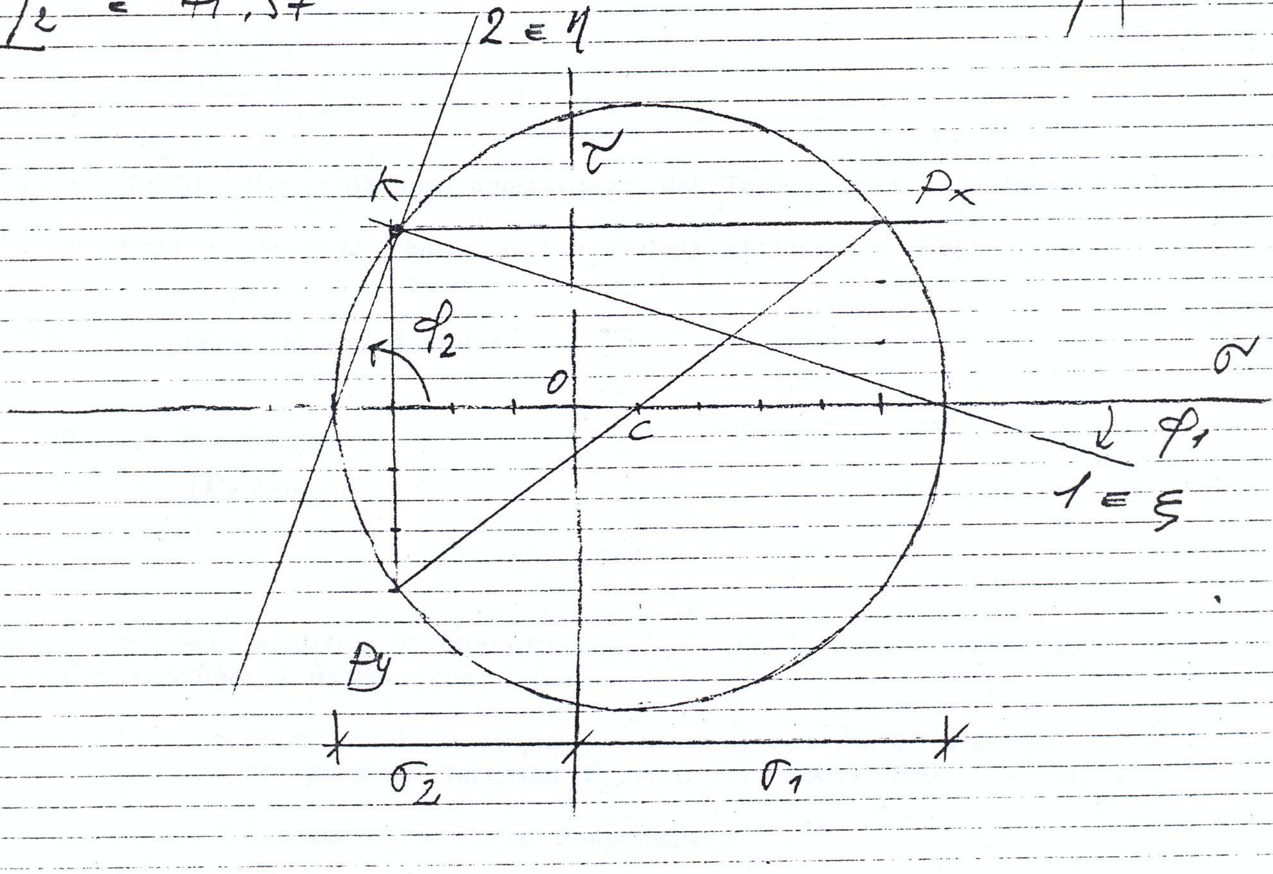
$(500-600)\,n_x^1 - 300\,n_y^1 = 0$

$\dfrac{n_y^1}{n_x^1} = -\dfrac{100}{300} = -0,\bar{3} = tg\,\varphi_1$

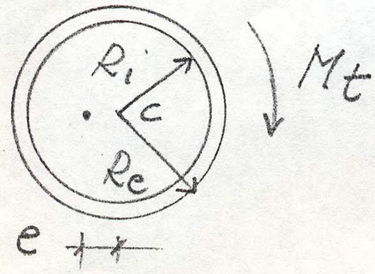
$\varphi_1 = arctg(-0,\bar{3}) = -18{,}43^\circ$

$\varphi_2 = 71{,}57^\circ$





3 Verificare la seguente sezione circolare cava con Re=20cm, Ri=19cm, N=-20000Kg, e=5cm, M_t=1000Kgm.



$= -163{,}2358 \pm 4{,}29\,y$

$\sigma_{z\,max} = -163{,}2358 - 4{,}29 \cdot 20 = -249$ kg/cmq

$\Omega = \pi \cdot 19{,}5^2 = 1194{,}59$ cmq $\qquad s = 1$ cm.

$\tau = \dfrac{Mt}{2\Omega s} = \dfrac{100000}{2 \cdot 119{,}459} \cong 41{,}85$ kg/cmq

$\underline{\sigma_{id}} = \sqrt{\sigma_z^2 + 3\tau^2} = \sqrt{249^2 + 3 \cdot 41{,}85^2} = \sqrt{67256} =$

$= 259$ kg/cmq $< \sigma_{am}$.

AB | 2023 3

3) $A = \pi \left(R_e^2 - R_i^2\right) = 122,5221$ cmq

$$I = \frac{\pi}{4}\left(R_e^4 - R_i^4\right) = 23309 \text{ cm}^4$$

$$\sigma_z = -\frac{20000}{122,5221} \pm \frac{20000 \cdot 5}{23309}\, y =$$

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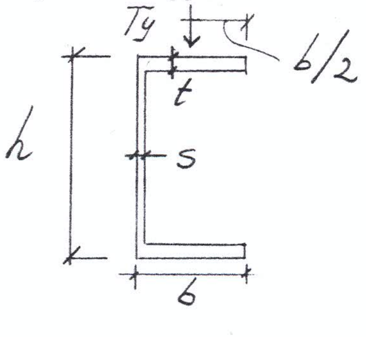
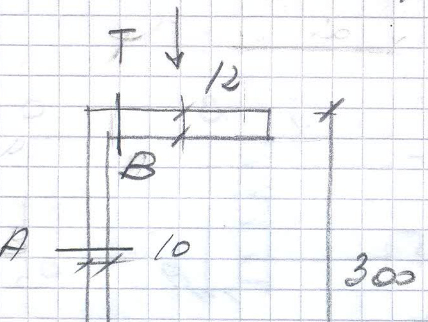
Anno di Corso...**Tests da recuperare:** 1 2 3

C.2 Verificare la seguente sezione in acciaio Fe 360, di dimensioni b = 10 cm, h = 30 cm, s = 1 cm, t = 1,2 cm, soggetta al taglio Ty = 1 t applicato alla metà dell'ala superiore.



$\tau_{CD} = 0,0096029\ldots$

C.2)



$T = 1t$

Fe 360

$$\tau_{zx\,max} = \frac{T\,S_x}{I_x\,b}$$

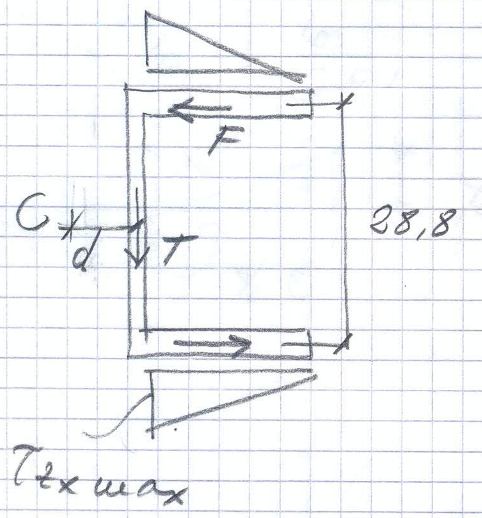
$$S_x = (10-1)\cdot 1,2\cdot(15-0.6) = 155,52 \text{ cm}^3$$

$$\tau_{zx\,max} = \frac{T\cdot 155,52}{8028\cdot 1,2} = 0,01614\ T$$

$$F = \frac{\tau_{zx\,max} \cdot 9 \cdot 1{,}2}{2} = 0{,}08715\,T$$

$$F \cdot (30 - 1{,}2) = F \cdot 28{,}8 = M = 2{,}50992\,T$$

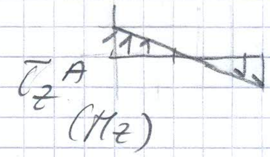
$$T d = M \qquad \boxed{d = \frac{M}{T} \cong 2{,}5}$$



$$M_z = 1000\,(5 - 0{,}5 + 2{,}5) = 7000 \text{ kg cm}$$

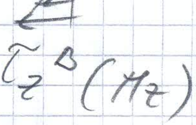
$$\tau_{zy\,(T)}^{A} = \frac{T\,(155{,}52 + (15-1{,}2)^2/2)}{8028 \cdot 1} = 31{,}23 \text{ kg/cm}^2$$

$$\tau_{z\,(M_z)}^{A} = \frac{3 M_z \cdot 1}{\underbrace{(9 \cdot (1{,}2)^3 \cdot 2 + 27{,}6 \cdot 1^3)}_{58{,}704}} \cong 358 \text{ kg/cm}^2$$



$$\tau_{zx\,(T)}^{B} = 16{,}14 \text{ kg/cm}^2$$

$$\tau_{z\,(M_z)}^{B} = \frac{3 M_z \cdot 1{,}2}{58{,}704} = 429 \text{ kg/cm}^2$$



$$\boxed{\tau_{z\,max}} = 429 + 16.14 \cong 445 \text{ kg/cm}^2$$

$$\sigma_{id} = \sqrt{3 \tau_z^2} = 771 \text{ kg/cm}^2 < \sigma_{am} = 1600 \text{ kg/cm}^2$$