PART 1

VALUE

CHAPTER 5

Net Present Value and Other Investment Criteria

A company's shareholders prefer to be rich rather than poor. Therefore, they want the firm to invest in every project that is worth more than it costs. The difference between a project's value and its cost is its *net present value (NPV)*. Companies can best help their shareholders by investing in all projects with a positive NPV and rejecting those with a negative NPV.

We start this chapter with a review of the net present value rule. We then turn to some other measures that companies may look at when making investment decisions. The first two of these measures, the project's payback period and its book rate of return, are little better than rules of thumb, easy to calculate and easy to communicate. Although there is a place for rules of thumb in this world, an engineer needs something more accurate when designing a 100-story building, and a financial manager needs more than a rule of thumb when making a substantial capital investment decision.

Instead of calculating a project's NPV, companies often compare the expected rate of return from investing in the project with the return that shareholders could earn on equivalent-risk investments in the capital market. The company accepts those projects that provide a higher return than shareholders could earn for themselves. If used correctly, this rate of return rule should always identify projects that increase firm value. However, we shall see that the rule sets several traps for the unwary.

We conclude the chapter by showing how to cope with situations when the firm has only limited capital. This raises two problems. One is computational. In simple cases we just choose those projects that give the highest NPV per dollar invested, but more elaborate techniques are sometimes needed to sort through the possible alternatives. The other problem is to decide whether capital rationing really exists and whether it invalidates the net present value rule. Guess what? NPV, properly interpreted, wins out in the end.

5-1 A Review of the Basics

Vegetron's chief financial officer (CFO) is wondering how to analyze a proposed $1 million investment in a new venture called project X. He asks what you think.

Your response should be as follows: "First, forecast the cash flows generated by project X over its economic life. Second, determine the appropriate opportunity cost of capital (r). This should reflect both the time value of money and the risk involved in project X. Third, use this opportunity cost of capital to discount the project's future cash flows. The sum of the discounted cash flows is called present value (PV). Fourth, calculate *net* present value (NPV) by subtracting the $1 million investment from PV. If we call the cash flows C_0, C_1, and so on, then

$$\text{NPV} = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots$$

We should invest in project X if its NPV is greater than zero."

However, Vegetron's CFO is unmoved by your sagacity. He asks why NPV is so important.

Your reply: "Let us look at what is best for Vegetron stockholders. They want you to make their Vegetron shares as valuable as possible.

"Right now Vegetron's total market value (price per share times the number of shares outstanding) is $10 million. That includes $1 million cash we can invest in project X. The value of Vegetron's other assets and opportunities must therefore be $9 million. We have to decide whether it is better to keep the $1 million cash and reject project X or to spend the cash and accept the project. Let us call the value of the new project PV. Then the choice is as follows:

	Market Value ($ millions)	
Asset	**Reject Project X**	**Accept Project X**
Cash	1	0
Other assets	9	9
Project X	0	PV
	10	9 + PV

"Clearly project X is worthwhile if its present value, PV, is greater than $1 million, that is, if net present value is positive."

CFO: "How do I know that the PV of project X will actually show up in Vegetron's market value?"

Your reply: "Suppose we set up a new, independent firm X, whose only asset is project X. What would be the market value of firm X?

"Investors would forecast the dividends that firm X would pay and discount those dividends by the expected rate of return of securities having similar risks. We know that stock prices are equal to the present value of forecasted dividends.

"Since project X is the only asset, the dividend payments we would expect firm X to pay are exactly the cash flows we have forecasted for project X. Moreover, the rate investors would use to discount firm X's dividends is exactly the rate we should use to discount project X's cash flows.

"I agree that firm X is entirely hypothetical. But if project X is accepted, investors holding Vegetron stock will really hold a portfolio of project X and the firm's other assets. We know the other assets are worth $9 million considered as a separate venture. Since asset values add up, we can easily figure out the portfolio value once we calculate the value of project X as a separate venture.

"By calculating the present value of project X, we are replicating the process by which the common stock of firm X would be valued in capital markets."

CFO: "The one thing I don't understand is where the discount rate comes from."

Your reply: "I agree that the discount rate is difficult to measure precisely. But it is easy to see what we are *trying* to measure. The discount rate is the opportunity cost of investing in the project rather than in the capital market. In other words, instead of accepting a project, the firm can always return the cash to the shareholders and let them invest it in financial assets.

"You can see the trade-off (Figure 5.1). The opportunity cost of taking the project is the return shareholders could have earned had they invested the funds on their own. When we discount the project's cash flows by the expected rate of return on financial assets, we are measuring how much investors would be prepared to pay for your project."

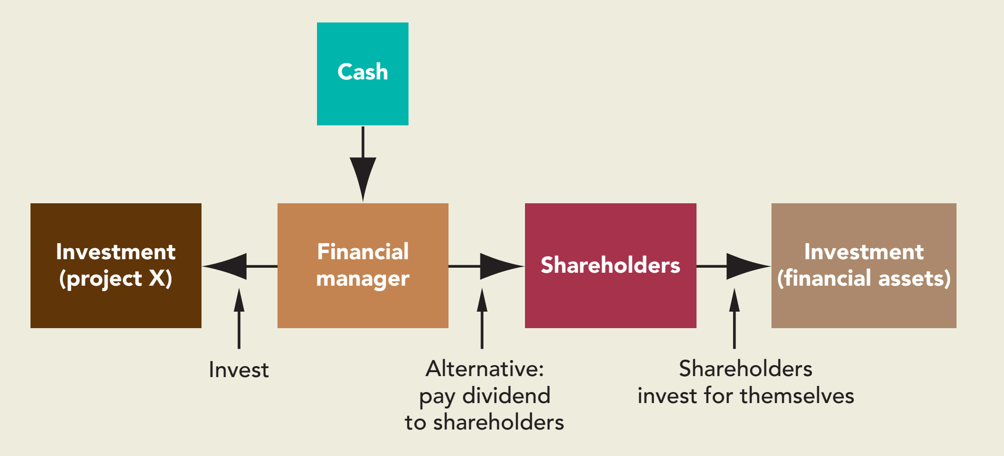

FIGURE 5.1

The firm can either keep and reinvest cash or return it to investors. (Arrows represent possible cash flows or transfers.) If cash is reinvested, the opportunity cost is the expected rate of return that shareholders could have obtained by investing in financial assets.

"But which financial assets?" Vegetron's CFO queries. "The fact that investors expect only 12% on IBM stock does not mean that we should purchase Fly-by-Night Electronics if it offers 13%."

Your reply: "The opportunity-cost concept makes sense only if assets of equivalent risk are compared. In general, you should identify financial assets that have the same risk as your project, estimate the expected rate of return on these assets, and use this rate as the opportunity cost."

Net Present Value's Competitors

When you advised the CFO to calculate the project's NPV, you were in good company. These days 75% of firms always, or almost always, calculate net present value when deciding on investment projects. However, as you can see from Figure 5.2, NPV is not the only investment criterion that companies use, and firms often look at more than one measure of a project's attractiveness.

About three-quarters of firms calculate the project's internal rate of return (or IRR); that is roughly the same proportion as use NPV. The IRR rule is a close relative of NPV and, when used properly, it will give the same answer. You therefore need to understand the IRR rule and how to take care when using it.

A large part of this chapter is concerned with explaining the IRR rule, but first we look at two other measures of a project's attractiveness—the project's payback and its book rate of return. As we will explain, both measures have obvious defects. Few companies rely on them to make their investment decisions, but they do use them as supplementary measures that may help to distinguish the marginal project from the no-brainer.

Later in the chapter we also come across one further investment measure, the profitability index. Figure 5.2 shows that it is not often used, but you will find that there are circumstances in which this measure has some special advantages.

Three Points to Remember about NPV

As we look at these alternative criteria, it is worth keeping in mind the following key features of the net present value rule. First, the NPV rule recognizes that *a dollar today is worth more than a dollar tomorrow,* because the dollar today can be invested to start earning interest immediately. Any investment rule that does not recognize the *time value of money* cannot be sensible. Second, net present value depends solely on the *forecasted cash flows*

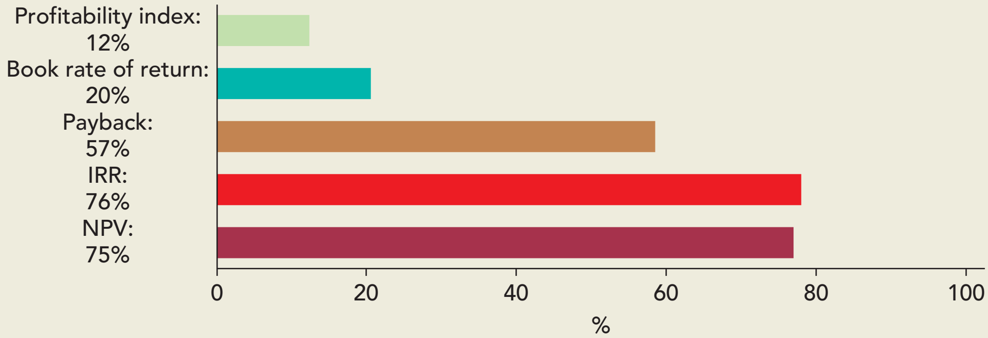

FIGURE 5.2

Survey evidence on the percentage of CFOs who always, or almost always, use a particular technique for evaluating investment projects.

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from the project and the *opportunity cost of capital.* Any investment rule that is affected by the manager's tastes, the company's choice of accounting method, the profitability of the company's existing business, or the profitability of other independent projects will lead to inferior decisions. Third, *because present values are all measured in today's dollars, you can add them up*. Therefore, if you have two projects A and B, the net present value of the combined investment is

$$\text{NPV}(A + B) = \text{NPV}(A) + \text{NPV}(B)$$

This adding-up property has important implications. Suppose project B has a negative NPV. If you tack it onto project A, the joint project (A + B) must have a lower NPV than A on its own. Therefore, you are unlikely to be misled into accepting a poor project (B) just because it is packaged with a good one (A). As we shall see, the alternative measures do not have this property. If you are not careful, you may be tricked into deciding that a package of a good and a bad project is better than the good project on its own.

NPV Depends on Cash Flow, Not on Book Returns

Net present value depends only on the project's cash flows and the opportunity cost of capital. But when companies report to shareholders, they do not simply show the cash flows. They also report book—that is, accounting—income and book assets.

Financial managers sometimes use these numbers to calculate a book (or accounting) rate of return on a proposed investment. In other words, they look at the prospective book income as a proportion of the book value of the assets that the firm is proposing to acquire:

$$\text{Book rate of return} = \frac{\text{book income}}{\text{book assets}}$$

Cash flows and book income are often very different. For example, the accountant labels some cash outflows as *capital investments* and others as *operating expenses.* The operating expenses are, of course, deducted immediately from each year's income. The capital expenditures are put on the firm's balance sheet and then depreciated. The annual depreciation charge is deducted from each year's income. Thus the book rate of return depends on which items the accountant treats as capital investments and how rapidly they are depreciated.[1]

Now the merits of an investment project do not depend on how accountants classify the cash flows[2] and few companies these days make investment decisions just on the basis of the book rate of return. But managers know that the company's shareholders pay considerable attention to book measures of profitability and naturally they think (and worry) about how major projects would affect the company's book return. Those projects that would reduce the company's book return may be scrutinized more carefully by senior management.

You can see the dangers here. The company's book rate of return may not be a good measure of true profitability. It is also an *average* across all of the firm's activities. The average profitability of past investments is not usually the right hurdle for new investments. Think of a firm that has been exceptionally lucky and successful. Say its average book return is 24%, double shareholders' 12% opportunity cost of capital. Should it demand that all *new* investments offer 24% or better? Clearly not: That would mean passing up many positive-NPV opportunities with rates of return between 12 and 24%.

We will come back to the book rate of return in Chapters 12 and 28, when we look more closely at accounting measures of financial performance.

5-2 Payback

We suspect that you have often heard conversations that go something like this: "We are spending $6 a week, or around $300 a year, at the laundromat. If we bought a washing machine for $800, it would pay for itself within three years. That's well worth it." You have just encountered the payback rule.

A project's **payback period** is found by counting the number of years it takes before the cumulative cash flow equals the initial investment. For the washing machine the payback period was just under three years. The **payback *rule*** states that a project should be accepted if its payback period is less than some specified cutoff period. For example, if the cutoff period is four years, the washing machine makes the grade; if the cutoff is two years, it doesn't.

EXAMPLE 5.1 ● The Payback Rule

Consider the following three projects:

	Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	Payback Period (years)	NPV at 10%
A	−2,000	500	500	5,000	3	+2,624
B	−2,000	500	1,800	0	2	−58
C	−2,000	1,800	500	0	2	+50

[1] This chapter's mini-case contains simple illustrations of how book rates of return are calculated and of the difference between accounting income and project cash flow. Read the case if you wish to refresh your understanding of these topics. Better still, do the case calculations.

[2] Of course, the depreciation method used for tax purposes does have cash consequences that should be taken into account in calculating NPV. We cover depreciation and taxes in the next chapter.

Project A involves an initial investment of \$2,000 ($C_0 = -2{,}000$) followed by cash inflows during the next three years. Suppose the opportunity cost of capital is 10%. Then project A has an NPV of +\$2,624:

$$\text{NPV(A)} = -2{,}000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{5{,}000}{1.10^3} = +\$2{,}624$$

Project B also requires an initial investment of \$2,000 but produces a cash inflow of \$500 in year 1 and \$1,800 in year 2. At a 10% opportunity cost of capital project B has an NPV of −\$58:

$$\text{NPV(B)} = -2{,}000 + \frac{500}{1.10} + \frac{1{,}800}{1.10^2} = -\$58$$

The third project, C, involves the same initial outlay as the other two projects but its first-period cash flow is larger. It has an NPV of +\$50.

$$\text{NPV(C)} = -2{,}000 + \frac{1{,}800}{1.10} + \frac{500}{1.10^2} = +\$50$$

The net present value rule tells us to accept projects A and C but to reject project B.

Now look at how rapidly each project pays back its initial investment. With project A you take three years to recover the \$2,000 investment; with projects B and C you take only two years. If the firm used the *payback rule* with a cutoff period of two years, it would accept only projects B and C; if it used the payback rule with a cutoff period of three or more years, it would accept all three projects. Therefore, regardless of the choice of cutoff period, the payback rule gives different answers from the net present value rule.

You can see why payback can give misleading answers as illustrated in Example 5.1:

1. *The payback rule ignores all cash flows after the cutoff date.* If the cutoff date is two years, the payback rule rejects project A regardless of the size of the cash inflow in year 3.
2. *The payback rule gives equal weight to all cash flows before the cutoff date.* The payback rule says that projects B and C are equally attractive, but because C's cash inflows occur earlier, C has the higher net present value at any discount rate.

In order to use the payback rule, a firm has to decide on an appropriate cutoff date. If it uses the same cutoff regardless of project life, it will tend to accept many poor short-lived projects and reject many good long-lived ones.

We have had little good to say about the payback rule. So why do many companies continue to use it? Senior managers don't truly believe that all cash flows after the payback period are irrelevant. We suggest three explanations. First, payback may be used because it is the simplest way to *communicate* an idea of project profitability. Investment decisions require discussion and negotiation between people from all parts of the firm, and it is important to have a measure that everyone can understand. Second, managers of larger corporations may opt for projects with short paybacks because they believe that quicker profits mean quicker promotion. That takes us back to Chapter 1 where we discussed the need to align the objectives of managers with those of shareholders. Finally, owners of family firms with limited access to capital may worry about their future ability to raise capital. These worries may lead them to favor rapid payback projects even though a longer-term venture may have a higher NPV.

Discounted Payback

Occasionally companies discount the cash flows before they compute the payback period. The discounted cash flows for our three projects are as follows:

Discounted Cash Flows ($)						
Project	C_0	C_1	C_2	C_3	Discounted Payback Period (years)	NPV at 20%
A	−2,000	500/1.10 = 455	$500/1.10^2$ = 413	$5{,}000/1.10^3$ = 3,757	3	+2,624
B	−2,000	500/1.10 = 455	$1{,}800/1.10^2$ = 1,488		—	−58
C	−2,000	1,800/1.10 = 1,636	$500/1.10^2$ = 413		2	+50

The *discounted payback rule* asks, How many years does the project have to last in order for it to make sense in terms of net present value? You can see that the value of the cash inflows from project B never exceeds the initial outlay and would always be rejected under the discounted payback rule. Thus the discounted payback rule will never accept a negative-NPV project. On the other hand, it still takes no account of cash flows after the cutoff date, so that good long-term projects such as A continue to risk rejection.

Rather than automatically rejecting any project with a long discounted payback period, many managers simply use the measure as a warning signal. These managers don't unthinkingly reject a project with a long discounted-payback period. Instead they check that the proposer is not unduly optimistic about the project's ability to generate cash flows into the distant future. They satisfy themselves that the equipment has a long life and that competitors will not enter the market and eat into the project's cash flows.

5-3 Internal (or Discounted-Cash-Flow) Rate of Return

Whereas payback and return on book are ad hoc measures, internal rate of return has a much more respectable ancestry and is recommended in many finance texts. If, therefore, we dwell more on its deficiencies, it is not because they are more numerous but because they are less obvious.

In Chapter 2 we noted that the net present value rule could also be expressed in terms of rate of return, which would lead to the following rule: "Accept investment opportunities offering rates of return in excess of their opportunity costs of capital." That statement, properly interpreted, is absolutely correct. However, interpretation is not always easy for long-lived investment projects.

There is no ambiguity in defining the true rate of return of an investment that generates a single payoff after one period:

$$\text{Rate of return} = \frac{\text{payoff}}{\text{investment}} - 1$$

Alternatively, we could write down the NPV of the investment and find the discount rate that makes NPV = 0.

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{discount rate}} = 0$$

implies

$$\text{Discount rate} = \frac{C_1}{-C_0} - 1$$

Of course C_1 is the payoff and $-C_0$ is the required investment, and so our two equations say exactly the same thing. *The discount rate that makes NPV = 0 is also the rate of return.*

How do we calculate return when the project produces cash flows in several periods? Answer: we use the same definition that we just developed for one-period projects—*the project rate of return is the discount rate that gives a zero NPV.* This discount rate is known as the **discounted-cash-flow (DCF) rate of return** or **internal rate of return (IRR).** The internal rate of return is used frequently in finance. It can be a handy measure, but, as we shall see, it can also be a misleading measure. You should, therefore, know how to calculate it and how to use it properly.

Calculating the IRR

The internal rate of return is defined as the rate of discount that makes NPV = 0. So to find the IRR for an investment project lasting T years, we must solve for IRR in the following expression:

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \cdots + \frac{C_T}{(1 + \text{IRR})^T} = 0$$

Actual calculation of IRR usually involves trial and error. For example, consider a project that produces the following flows:

Cash Flows ($)		
C_0	C_1	C_2
−4,000	+2,000	+4,000

The internal rate of return is IRR in the equation

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1 + \text{IRR}} + \frac{4{,}000}{(1 + \text{IRR})^2} = 0$$

Let us arbitrarily try a zero discount rate. In this case NPV is not zero but +$2,000:

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1.0} + \frac{4{,}000}{(1.0)^2} = +\$2{,}000$$

The NPV is positive; therefore, the IRR must be greater than zero. The next step might be to try a discount rate of 50%. In this case net present value is −$889:

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1.50} + \frac{4{,}000}{(1.50)^2} = -\$889$$

The NPV is negative; therefore, the IRR must be less than 50%. In Figure 5.3 we have plotted the net present values implied by a range of discount rates. From this we can see that a discount rate of 28% gives the desired net present value of zero. Therefore IRR is 28%.

The easiest way to calculate IRR, if you have to do it by hand, is to plot three or four combinations of NPV and discount rate on a graph like Figure 5.3, connect the points with a smooth line, and read off the discount rate at which NPV = 0. It is of course quicker and more accurate to use a computer spreadsheet or a specially programmed calculator, and in practice this is what financial managers do. The Useful Spreadsheet Functions box near the end of the chapter presents Excel functions for doing so.

Some people confuse the internal rate of return and the opportunity cost of capital because both appear as discount rates in the NPV formula. The internal rate of return is a *profitability measure* that depends solely on the amount and timing of the project cash flows. The opportunity cost of capital is a *standard of profitability* that we use to calculate how much the project is worth. The opportunity cost of capital is established in capital markets. It is the expected rate of return offered by other assets with the same risk as the project being evaluated.

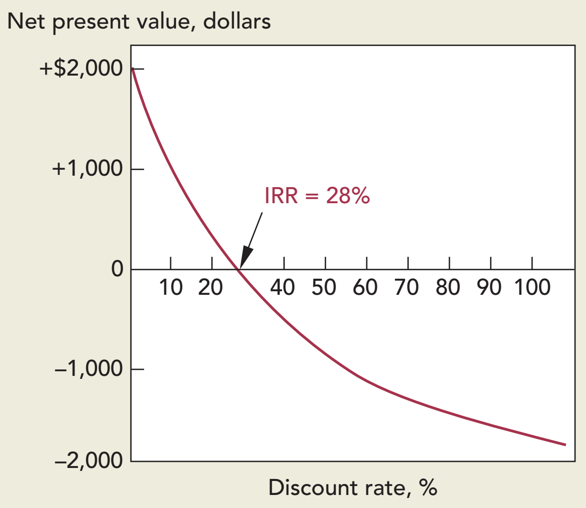

FIGURE 5.3

This project costs $4,000 and then produces cash inflows of $2,000 in year 1 and $4,000 in year 2. Its internal rate of return (IRR) is 28%, the rate of discount at which NPV is zero.

The IRR Rule

The internal rate of return rule is to accept an investment project if the opportunity cost of capital is less than the internal rate of return. You can see the reasoning behind this idea if you look again at Figure 5.3. If the opportunity cost of capital is less than the 28% IRR, then the project has a *positive* NPV when discounted at the opportunity cost of capital. If it is equal to the IRR, the project has a *zero* NPV. And if it is greater than the IRR, the project has a *negative* NPV. Therefore, when we compare the opportunity cost of capital with the IRR on our project, we are effectively asking whether our project has a positive NPV. This is true not only for our example. The rule will give the same answer as the net present value rule *whenever the NPV of a project is a smoothly declining function of the discount rate.*

Many firms use internal rate of return as a criterion in preference to net present value. We think that this is a pity. Although, properly stated, the two criteria are formally equivalent, the internal rate of return rule contains several pitfalls.

Pitfall 1—Lending or Borrowing?

Not all cash-flow streams have NPVs that decline as the discount rate increases. Consider the following projects A and B:

	Cash Flows ($)			
Project	C_0	C_1	**IRR**	**NPV at 10%**
A	−1,000	+1,500	+50%	+364
B	+1,000	−1,500	+50%	−364

Each project has an IRR of 50%. (In other words, $-1{,}000 + 1{,}500/1.50 = 0$ *and* $+ 1{,}000 - 1{,}500/1.50 = 0$.)

Does this mean that they are equally attractive? Clearly not, for in the case of A, where we are initially paying out $1,000, we are *lending* money at 50%, in the case of B, where we are initially receiving $1,000, we are *borrowing* money at 50%. When we lend money, we want a *high* rate of return; when we borrow money, we want a *low* rate of return.

If you plot a graph like Figure 5.3 for project B, you will find that NPV increases as the discount rate increases. Obviously the internal rate of return rule, as we stated it above, won't work in this case; we have to look for an IRR *less* than the opportunity cost of capital.

Pitfall 2—Multiple Rates of Return

Helmsley Iron is proposing to develop a new strip mine in Western Australia. The mine involves an initial investment of A$3 billion and is expected to produce a cash inflow of A$1 billion a year for the next nine years. At the end of that time the company will incur A$6.5 billion of cleanup costs. Thus the cash flows from the project are:

Cash Flows (billions of Australian dollars)				
C_0	C_1	. . .	C_9	C_{10}
−3	1		1	−6.5

Helmsley calculates the project's IRR and its NPV as follows:

IRR (%)	NPV at 10%
+3.50 *and* 19.54	$A253 million

Note that there are *two* discount rates that make NPV = 0. That is, *each* of the following statements holds:

$$NPV = -3 + \frac{1}{1.035} + \frac{1}{1.035^2} + \cdots + \frac{1}{1.035^9} - \frac{6.5}{1.035^{10}} = 0$$

$$NPV = -3 + \frac{1}{1.1954} + \frac{1}{1.1954^2} + \cdots + \frac{1}{1.1954^9} - \frac{6.5}{1.1954^{10}} = 0$$

In other words, the investment has an IRR of both 3.50 *and* 19.54%. Figure 5.4 shows how this comes about. As the discount rate increases, NPV initially rises and then declines. The reason for this is the double change in the sign of the cash-flow stream. There can be as many internal rates of return for a project as there are changes in the sign of the cash flows.[3]

Decommissioning and clean-up costs can sometimes be huge. Phillips Petroleum has estimated that it will need to spend $1 billion to remove its Norwegian offshore oil platforms. It can cost over $300 million to decommission a nuclear power plant. These are obvious instances where cash flows go from positive to negative, but you can probably think of a number of other cases where the company needs to plan for later expenditures. Ships periodically need to go into dry dock for a refit, hotels may receive a major face-lift, machine parts may need replacement, and so on.

Whenever the cash-flow stream is expected to change sign more than once, the company typically sees more than one IRR.

As if this is not difficult enough, there are also cases in which *no* internal rate of return exists. For example, project C has a positive net present value at all discount rates:

	Cash Flows ($)				
Project	C_0	C_1	C_2	IRR (%)	NPV at 10%
C	+1,000	−3,000	+2,500	None	+339

[3] By Descartes's "rule of signs" there can be as many different solutions to a polynomial as there are changes of sign.

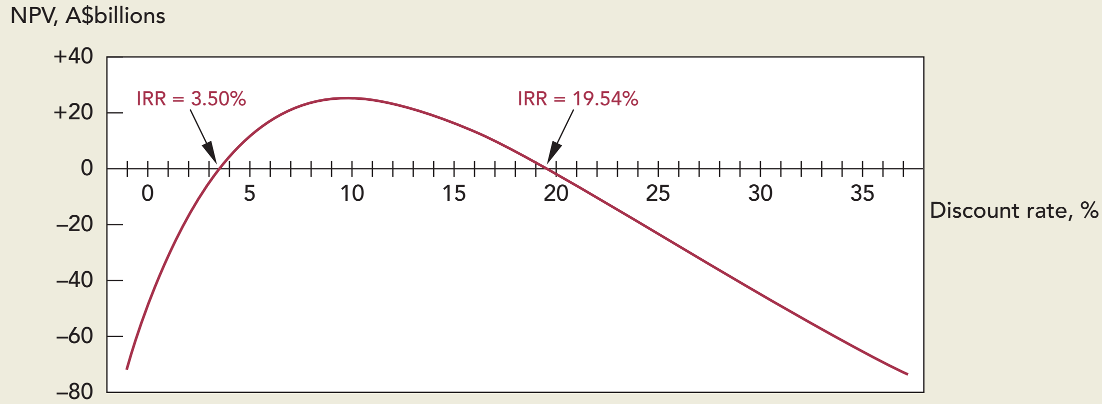

FIGURE 5.4

Helmsley Iron's mine has two internal rates of return. NPV = 0 when the discount rate is +3.50% *and* when it is +19.54%.

A number of adaptations of the IRR rule have been devised for such cases. Not only are they inadequate, but they also are unnecessary, for the simple solution is to use net present value.[4]

Pitfall 3—Mutually Exclusive Projects

Firms often have to choose from among several alternative ways of doing the same job or using the same facility. In other words, they need to choose from among **mutually exclusive projects.** Here too the IRR rule can be misleading.

Consider projects D and E:

	Cash Flows ($)			
Project	C_0	C_1	IRR (%)	NPV at 10%
D	−10,000	+20,000	100	+ 8,182
E	−20,000	+35,000	75	+11,818

Perhaps project D is a manually controlled machine tool and project E is the same tool with the addition of computer control. Both are good investments, but E has the higher NPV and is, therefore, better. However, the IRR rule seems to indicate that if you have to choose, you should go for D since it has the higher IRR. If you follow the IRR rule, you have the satisfaction of earning a 100% rate of return; if you follow the NPV rule, you are $11,818 richer.

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[4] Companies sometimes get around the problem of multiple rates of return by discounting the later cash flows back at the cost of capital until there remains only one change in the sign of the cash flows. A *modified internal rate of return* (MIRR) can then be calculated on this revised series. In our example, the MIRR is calculated as follows:

1. Calculate the present value in year 5 of all the subsequent cash flows:

$$\text{PV in year } 5 = 1/1.1 + 1/1.1^2 + 1/1.1^3 + +1/1.1^4 - 6.5/1.1^5 = -.866$$

2. Add to the year 5 cash flow the present value of subsequent cash flows:

$$C_5 + \text{PV(subsequent cash flows)} = 1 - .866 = .134$$

3. Since there is now only one change in the sign of the cash flows, the revised series has a unique rate of return, which is 13.7%

$$\text{NPV} = 1/1.137 + 1/1.137^2 + 1/1.137^3 + 1/1.137^4 + .134/1.137^5 = 0$$

Since the MIRR of 13.7% is greater than the cost of capital (and the initial cash flow is negative), the project has a positive NPV when valued at the cost of capital.

Of course, it would be much easier in such cases to abandon the IRR rule and just calculate project NPV.

You can salvage the IRR rule in these cases by looking at the internal rate of return on the *incremental* flows. Here is how to do it: First, consider the smaller project (D in our example). It has an IRR of 100%, which is well in excess of the 10% opportunity cost of capital. You know, therefore, that D is acceptable. You now ask yourself whether it is worth making the additional $10,000 investment in E. The incremental flows from undertaking E rather than D are as follows:

	Cash Flows ($)			
Project	C_0	C_1	**IRR (%)**	**NPV at 10%**
E − D	−10,000	+15,000	50	+3,636

The IRR on the incremental investment is 50%, which is also well in excess of the 10% opportunity cost of capital. So you should prefer project E to project D.[5]

Unless you look at the incremental expenditure, IRR is unreliable in ranking projects of different scale. It is also unreliable in ranking projects that offer different patterns of cash flow over time. For example, suppose the firm can take project F *or* project G but not both (ignore H for the moment):

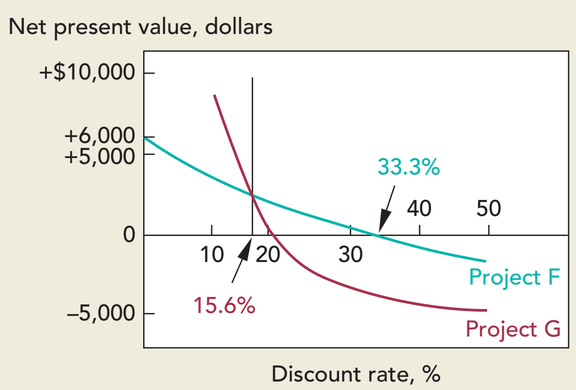
	Cash Flows ($)								
Project	C_0	C_1	C_2	C_3	C_4	C_5	**Etc.**	**IRR (%)**	**NPV at 10%**
F	−9,000	+6,000	+5,000	+4,000	0	0	. . .	33	3,592
G	−9,000	+1,800	+1,800	+1,800	+1,800	+1,800	. . .	20	9,000
H		−6,000	+1,200	+1,200	+1,200	+1,200	. . .	20	6,000

Project F has a higher IRR, but project G, which is a perpetuity, has the higher NPV. Figure 5.5 shows why the two rules give different answers. The green line gives the net present value of project F at different rates of discount. Since a discount rate of 33% produces a net present value of zero, this is the internal rate of return for project F. Similarly, the brown line shows the net present value of project G at different discount rates. The IRR of project G is 20%. (We assume project G's cash flows continue indefinitely.) Note, however, that project G has a higher NPV as long as the opportunity cost of capital is less than 15.6%.

The reason that IRR is misleading is that the total cash inflow of project G is larger but tends to occur later. Therefore, when the discount rate is low, G has the higher NPV; when the discount rate is high, F has the higher NPV. (You can see from Figure 5.5 that the two projects have the *same* NPV when the discount rate is 15.6%.) The internal rates of return on the two projects tell us that at a discount rate of 20% G has a zero NPV (IRR = 20%) and F has a positive NPV. Thus if the opportunity cost of capital were 20%, investors would place a higher value on the shorter-lived project F. But in our example the opportunity cost of capital is not 20% but 10%. So investors will

FIGURE 5.5

The IRR of project F exceeds that of project G, but the NPV of project F is higher *only* if the discount rate is greater than 15.6%.



[5] You may, however, find that you have jumped out of the frying pan into the fire. The series of incremental cash flows may involve several changes in sign. In this case there are likely to be multiple IRRs and you will be forced to use the NPV rule after all.

pay a relatively high price for the longer-lived project. At a 10% cost of capital, an investment in G has an NPV of $9,000 and an investment in F has an NPV of only $3,592.[6]

This is a favorite example of ours. We have gotten many businesspeople's reaction to it. When asked to choose between F and G, many choose F. The reason seems to be the rapid payback generated by project F. In other words, they believe that if they take F, they will also be able to take a later project like H (note that H can be financed using the cash flows from F), whereas if they take G, they won't have money enough for H. In other words they implicitly assume that it is a *shortage of capital* that forces the choice between F and G. When this implicit assumption is brought out, they usually admit that G is better if there is no capital shortage.

But the introduction of capital constraints raises two further questions. The first stems from the fact that most of the executives preferring F to G work for firms that would have no difficulty raising more capital. Why would a manager at IBM, say, choose F on the grounds of limited capital? IBM can raise plenty of capital and can take project H regardless of whether F or G is chosen; therefore H should not affect the choice between F and G. The answer seems to be that large firms usually impose capital budgets on divisions and subdivisions as a part of the firm's planning and control system. Since the system is complicated and cumbersome, the budgets are not easily altered, and so they are perceived as real constraints by middle management.

The second question is this. If there is a capital constraint, either real or self-imposed, should IRR be used to rank projects? The answer is no. The problem in this case is to find the package of investment projects that satisfies the capital constraint and has the largest net present value. The IRR rule will not identify this package. As we will show in the next section, the only practical and general way to do so is to use the technique of linear programming.

When we have to choose between projects F and G, it is easiest to compare the net present values. But if your heart is set on the IRR rule, you can use it as long as you look at the internal rate of return on the incremental flows. The procedure is exactly the same as we showed above. First, you check that project F has a satisfactory IRR. Then you look at the return on the incremental cash flows from G.

	Cash Flows ($)							
Project	C_0	C_1	C_2	C_3	C_4	C_5	**Etc.**	**IRR (%)**	**NPV at 10%**
G − F	0	−4,200	−3,200	−2,200	+1,800	+1,800	. . .	15.6	+5,408

The IRR on the incremental cash flows from G is 15.6%. Since this is greater than the opportunity cost of capital, you should undertake G rather than F.[7]

Pitfall 4—What Happens When There Is More than One Opportunity Cost of Capital?

We have simplified our discussion of capital budgeting by assuming that the opportunity cost of capital is the same for all the cash flows, C_1, C_2, C_3, etc. Remember our most general formula for calculating net present value:

$$\text{NPV} = C_0 + \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \frac{C_3}{(1 + r_3)^3} + \cdots$$

[6] It is often suggested that the choice between the net present value rule and the internal rate of return rule should depend on the probable reinvestment rate. This is wrong. The prospective return on another *independent* investment should *never* be allowed to influence the investment decision.

[7] Because F and G had the same 10% cost of capital, we could choose between the two projects by asking whether the IRR on the incremental cash flows was greater or less than 10%. But suppose that F and G had different risks and therefore different costs of capital. In that case there would be no simple yardstick for assessing whether the IRR on the incremental cash flows was adequate.

In other words, we discount C_1 at the opportunity cost of capital for one year, C_2 at the opportunity cost of capital for two years, and so on. The IRR rule tells us to accept a project if the IRR is greater than the opportunity cost of capital. But what do we do when we have several opportunity costs? Do we compare IRR with $r_1, r_2, r_3, \ldots$? Actually we would have to compute a complex weighted average of these rates to obtain a number comparable to IRR.

What does this mean for capital budgeting? It means trouble for the IRR rule whenever there is more than one opportunity cost of capital. Many firms use the IRR, thereby implicitly assuming that there is no difference between short-term and long-term discount rates. They do this for the same reason that we have so far finessed the issue: simplicity.[8]

The Verdict on IRR

We have given four examples of things that can go wrong with IRR. We spent much less space on payback or return on book. Does this mean that IRR is worse than the other two measures? Quite the contrary. There is little point in dwelling on the deficiencies of payback or return on book. They are clearly ad hoc measures that often lead to silly conclusions. The IRR rule has a much more respectable ancestry. It is less easy to use than NPV, but, used properly, it gives the same answer.

Nowadays few large corporations use the payback period or return on book as their primary measure of project attractiveness. Most use discounted cash flow or "DCF," and for many companies DCF means IRR, not NPV. For "normal" investment projects with an initial cash outflow followed by a series of cash inflows, there is no difficulty in using the internal rate of return to make a simple accept/reject decision. However, we think that financial managers need to worry more about Pitfall 3. Financial managers never see all possible projects. Most projects are proposed by operating managers. A company that instructs nonfinancial managers to look first at project IRRs prompts a search for those projects with the highest IRRs rather than the highest NPVs. It also encourages managers to *modify* projects so that their IRRs are higher. Where do you typically find the highest IRRs? In short-lived projects requiring little up-front investment. Such projects may not add much to the value of the firm.

We don't know why so many companies pay such close attention to the internal rate of return, but we suspect that it may reflect the fact that management does not trust the forecasts it receives. Suppose that two plant managers approach you with proposals for two new investments. Both have a positive NPV of $1,400 at the company's 8% cost of capital, but you nevertheless decide to accept project A and reject B. Are you being irrational?

The cash flows for the two projects and their NPVs are set out in the table below. You can see that, although both proposals have the same NPV, project A involves an investment of $9,000, while B requires an investment of $9 million. Investing $9,000 to make $1,400 is clearly an attractive proposition, and this shows up in A's IRR of nearly 16%. Investing $9 million to make $1,400 might also be worth doing if you could be *sure* of the plant manager's forecasts, but there is almost no room for error in project B. You could spend time and money checking the cash-flow forecasts, but is it really worth the effort? Most managers would look at the IRR and decide that, if the cost of capital is 8%, a project that offers a return of 8.01% is not worth the worrying time.

Alternatively, management may conclude that project A is a clear winner that is worth undertaking right away, but in the case of project B it may make sense to wait and see

[8] In Chapter 9 we look at some other cases in which it would be misleading to use the same discount rate for both short-term and long-term cash flows.

whether the decision looks more clear-cut in a year's time.[9] Management postpones the decision on projects such as B by setting a hurdle rate for the IRR that is higher than the cost of capital.

	Cash Flows ($ thousands)					
Project	C_0	C_1	C_2	C_3	NPV at 8%	IRR (%)
A	−9.0	2.9	4.0	5.4	1.4	15.58
B	−9,000	2,560	3,540	4,530	1.4	8.01

5-4 Choosing Capital Investments When Resources Are Limited

Our entire discussion of methods of capital budgeting has rested on the proposition that the wealth of a firm's shareholders is highest if the firm accepts *every* project that has a positive net present value. Suppose, however, that there are limitations on the investment program that prevent the company from undertaking all such projects. Economists call this *capital rationing.* When capital is rationed, we need a method of selecting the package of projects that is within the company's resources yet gives the highest possible net present value.

An Easy Problem in Capital Rationing

Let us start with a simple example. The opportunity cost of capital is 10%, and our company has the following opportunities:

	Cash Flows ($ millions)			
Project	C_0	C_1	C_2	NPV at 10%
A	−10	+30	+5	21
B	−5	+5	+20	16
C	−5	+5	+15	12

All three projects are attractive, but suppose that the firm is limited to spending $10 million. In that case, it can invest *either* in project A *or* in projects B and C, but it cannot invest in all three. Although individually B and C have lower net present values than project A, when taken together they have the higher net present value. Here we cannot choose between projects solely on the basis of net present values. When funds are limited, we need to concentrate on getting the biggest bang for our buck. In other words, we must pick the projects that offer the highest net present value per dollar of initial outlay. This ratio is known as the **profitability index:**[10]

$$\text{Profitability index} = \frac{\text{net present value}}{\text{investment}}$$

[9] In Chapter 22 we discuss when it may pay a company to delay undertaking a positive-NPV project. We will see that when projects are "deep-in-the-money" (project A), it generally pays to invest right away and capture the cash flows. However, in the case of projects that are close-to-the-money (project B) it makes more sense to wait and see.

[10] If a project requires outlays in two or more periods, the denominator should be the present value of the outlays. A few companies do not discount the benefits or costs before calculating the profitability index. The less said about these companies the better.

For our three projects the profitability index is calculated as follows:[11]

Project	Investment ($ millions)	NPV ($ millions)	Profitability Index
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4

Project B has the highest profitability index and C has the next highest. Therefore, if our budget limit is $10 million, we should accept these two projects.[12]

Unfortunately, there are some limitations to this simple ranking method. One of the most serious is that it breaks down whenever more than one resource is rationed.[13] For example, suppose that the firm can raise only $10 million for investment in *each* of years 0 and 1 and that the menu of possible projects is expanded to include an investment next year in project D:

	Cash Flows ($ millions)				
Project	C_0	C_1	C_2	**NPV at 10%**	**Profitability Index**
A	−10	+30	+5	21	2.1
B	−5	+5	+20	16	3.2
C	−5	+5	+15	12	2.4
D	0	−40	+60	13	0.4

One strategy is to accept projects B and C; however, if we do this, we cannot also accept D, which costs more than our budget limit for period 1. An alternative is to accept project A in period 0. Although this has a lower net present value than the combination of B and C, it provides a $30 million positive cash flow in period 1. When this is added to the $10 million budget, we can also afford to undertake D next year. A and D have *lower* profitability indexes than B and C, but they have a *higher* total net present value.

The reason that ranking on the profitability index fails in this example is that resources are constrained in each of two periods. In fact, this ranking method is inadequate whenever there is *any* other constraint on the choice of projects. This means that it cannot cope with cases in which two projects are mutually exclusive or in which one project is dependent on another.

For example, suppose that you have a long menu of possible projects starting this year and next. There is a limit on how much you can invest in each year. Perhaps also you can't undertake both project alpha and beta (they both require the same piece of land), and you can't invest in project gamma unless you invest in delta (gamma is simply an add-on to

[11] Sometimes the profitability index is defined as the ratio of the present value to initial outlay, that is, as PV/investment. This measure is also known as the *benefit–cost ratio*. To calculate the benefit–cost ratio, simply add 1.0 to each profitability index. Project rankings are unchanged.

[12] If a project has a positive profitability index, it must also have a positive NPV. Therefore, firms sometimes use the profitability index to select projects when capital is *not* limited. However, like the IRR, the profitability index can be misleading when used to choose between mutually exclusive projects. For example, suppose you were forced to choose between (1) investing $100 in a project whose payoffs have a present value of $200 or (2) investing $1 million in a project whose payoffs have a present value of $1.5 million. The first investment has the higher profitability index; the second makes you richer.

[13] It may also break down if it causes some money to be left over. It might be better to spend all the available funds even if this involves accepting a project with a slightly lower profitability index.

delta). You need to find the package of projects that satisfies all these constraints and gives the highest NPV.

One way to tackle such a problem is to work through all possible combinations of projects. For each combination you first check whether the projects satisfy the constraints and then calculate the net present value. But it is smarter to recognize that linear programming (LP) techniques are specially designed to search through such possible combinations.[14]

Uses of Capital Rationing Models

Linear programming models seem tailor-made for solving capital budgeting problems when resources are limited. Why then are they not universally accepted either in theory or in practice? One reason is that these models can turn out to be very complex. Second, as with any sophisticated long-range planning tool, there is the general problem of getting good data. It is just not worth applying costly, sophisticated methods to poor data. Furthermore, these models are based on the assumption that all future investment opportunities are known. In reality, the discovery of investment ideas is an unfolding process.

Our most serious misgivings center on the basic assumption that capital is limited. When we come to discuss company financing, we shall see that most large corporations do not face capital rationing and can raise large sums of money on fair terms. Why then do many company presidents tell their subordinates that capital is limited? If they are right, the capital market is seriously imperfect. What then are they doing maximizing NPV?[15] We might be tempted to suppose that if capital is not rationed, they do not *need* to use linear programming and, if it is rationed, then surely they *ought* not to use it. But that would be too quick a judgment. Let us look at this problem more deliberately.

Soft Rationing Many firms' capital constraints are "soft." They reflect no imperfections in capital markets. Instead they are provisional limits adopted by management as an aid to financial control.

Some ambitious divisional managers habitually overstate their investment opportunities. Rather than trying to distinguish which projects really are worthwhile, headquarters may find it simpler to impose an upper limit on divisional expenditures and thereby force the divisions to set their own priorities. In such instances budget limits are a rough but effective way of dealing with biased cash-flow forecasts. In other cases management may believe that very rapid corporate growth could impose intolerable strains on management and the organization. Since it is difficult to quantify such constraints explicitly, the budget limit may be used as a proxy.

Because such budget limits have nothing to do with any inefficiency in the capital market, there is no contradiction in using an LP model in the division to maximize net present value subject to the budget constraint. On the other hand, there is not much point in elaborate selection procedures if the cash-flow forecasts of the division are seriously biased.

Even if capital is not rationed, other resources may be. The availability of management time, skilled labor, or even other capital equipment often constitutes an important constraint on a company's growth.

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[14] On our Web site at **www.mhhe.com/bma** we show how linear programming can be used to select from the four projects in our earlier example.

[15] Don't forget that in Chapter 1 we had to assume perfect capital markets to derive the NPV rule.

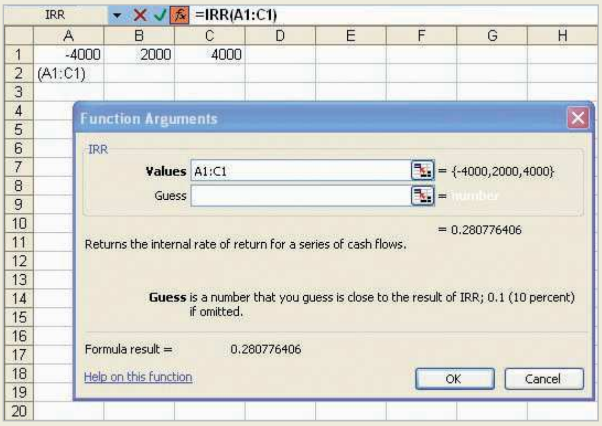
USEFUL SPREADSHEET FUNCTIONS

Internal Rate of Return

Spreadsheet programs such as Excel provide built-in functions to solve for internal rates of return. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel will guide you through the inputs that are required. At the bottom left of the function box there is a Help facility with an example of how the function is used.

Here is a list of useful functions for calculating internal rates of return, together with some points to remember when entering data:

- **IRR:** Internal rate of return on a series of regularly spaced cash flows.
- **XIRR:** The same as IRR, but for irregularly spaced flows.



Note the following:

- For these functions, you must enter the addresses of the cells that contain the input values.
- The IRR functions calculate only one IRR even when there are multiple IRRs.

SPREADSHEET QUESTIONS

The following questions provide an opportunity to practice each of the above functions:

1. (IRR) Check the IRRs on projects F and G in Section 5-3.

2. (IRR) What is the IRR of a project with the following cash flows:

C_0	C_1	C_2	C_3
−\$5,000	+\$2,200	+\$4,650	+\$3,330

3. (IRR) Now use the function to calculate the IRR on Helmsley Iron's mining project in Section 5-3. There are really two IRRs to this project (why?). How many IRRs does the function calculate?

4. (XIRR) What is the IRR of a project with the following cash flows:

C_0		C_4		C_5		C_6
−\$215,000	. . .	+\$185,000	. . .	+\$85,000	. . .	+\$43,000

(All other cash flows are 0.)

Hard Rationing Soft rationing should never cost the firm anything. If capital constraints become tight enough to hurt—in the sense that projects with significant positive NPVs are passed up—then the firm raises more money and loosens the constraint. But what if it *can't* raise more money—what if it faces *hard* rationing?

Hard rationing implies market imperfections, but that does not necessarily mean we have to throw away net present value as a criterion for capital budgeting. It depends on the nature of the imperfection.

Arizona Aquaculture, Inc. (AAI), borrows as much as the banks will lend it, yet it still has good investment opportunities. This is not hard rationing so long as AAI can issue stock. But perhaps it can't. Perhaps the founder and majority shareholder vetoes the idea from fear of losing control of the firm. Perhaps a stock issue would bring costly red tape or legal complications.[16]

This does not invalidate the NPV rule. AAI's *shareholders* can borrow or lend, sell their shares, or buy more. They have free access to security markets. The type of portfolio they hold is independent of AAI's financing or investment decisions. The only way AAI can help its shareholders is to make them richer. Thus AAI should invest its available cash in the package of projects having the largest aggregate net present value.

A barrier between the firm and capital markets does not undermine net present value so long as the barrier is the *only* market imperfection. The important thing is that the firm's *shareholders* have free access to well-functioning capital markets.

The net present value rule *is* undermined when imperfections restrict shareholders' portfolio choice. Suppose that Nevada Aquaculture, Inc. (NAI), is solely owned by its founder, Alexander Turbot. Mr. Turbot has no cash or credit remaining, but he is convinced that expansion of his operation is a high-NPV investment. He has tried to sell stock but has found that prospective investors, skeptical of prospects for fish farming in the desert, offer him much less than he thinks his firm is worth. For Mr. Turbot capital markets hardly exist. It makes little sense for him to discount prospective cash flows at a market opportunity cost of capital.

[16] A majority owner who is "locked in" and has much personal wealth tied up in AAI may be effectively cut off from capital markets. The NPV rule may not make sense to such an owner, though it will to the other shareholders.

SUMMARY

If you are going to persuade your company to use the net present value rule, you must be prepared to explain why other rules may *not* lead to correct decisions. That is why we have examined three alternative investment criteria in this chapter.

Some firms look at the book rate of return on the project. In this case the company decides which cash payments are capital expenditures and picks the appropriate rate to depreciate these expenditures. It then calculates the ratio of book income to the book value of the investment. Few companies nowadays base their investment decision simply on the book rate of return, but shareholders pay attention to book measures of firm profitability and some managers therefore look with a jaundiced eye on projects that would damage the company's book rate of return.

Some companies use the payback method to make investment decisions. In other words, they accept only those projects that recover their initial investment within some specified period. Payback is an ad hoc rule. It ignores the timing of cash flows within the payback period, and it ignores subsequent cash flows entirely. It therefore takes no account of the opportunity cost of capital.

The internal rate of return (IRR) is defined as the rate of discount at which a project would have zero NPV. It is a handy measure and widely used in finance; you should therefore know how to calculate it. The IRR rule states that companies should accept any investment offering an IRR in excess of the opportunity cost of capital. The IRR rule is, like net present value, a technique based on discounted cash flows. It will therefore give the correct answer if properly used. The problem is that it is easily misapplied. There are four things to look out for:

1. *Lending or borrowing?* If a project offers positive cash flows followed by negative flows, NPV can *rise* as the discount rate is increased. You should accept such projects if their IRR is *less* than the opportunity cost of capital.

2. *Multiple rates of return.* If there is more than one change in the sign of the cash flows, the project may have several IRRs or no IRR at all.
3. *Mutually exclusive projects.* The IRR rule may give the wrong ranking of mutually exclusive projects that differ in economic life or in scale of required investment. If you insist on using IRR to rank mutually exclusive projects, you must examine the IRR on each incremental investment.
4. *The cost of capital for near-term cash flows may be different from the cost for distant cash flows.* The IRR rule requires you to compare the project's IRR with the opportunity cost of capital. But sometimes there is an opportunity cost of capital for one-year cash flows, a different cost of capital for two-year cash flows, and so on. In these cases there is no simple yardstick for evaluating the IRR of a project.

In developing the NPV rule, we assumed that the company can maximize shareholder wealth by accepting every project that is worth more than it costs. But, if capital is strictly limited, then it may not be possible to take every project with a positive NPV. If capital is rationed in only one period, then the firm should follow a simple rule: Calculate each project's profitability index, which is the project's net present value per dollar of investment. Then pick the projects with the highest profitability indexes until you run out of capital. Unfortunately, this procedure fails when capital is rationed in more than one period or when there are other constraints on project choice. The only general solution is linear programming.

Hard capital rationing always reflects a market imperfection—a barrier between the firm and capital markets. If that barrier also implies that the firm's shareholders lack free access to a well-functioning capital market, the very foundations of net present value crumble. Fortunately, hard rationing is rare for corporations in the United States. Many firms do use soft capital rationing, however. That is, they set up self-imposed limits as a means of financial planning and control.

FURTHER READING

For a survey of capital budgeting procedures, see:

J. Graham and C. Harvey, "How CFOs Make Capital Budgeting and Capital Structure Decisions," *Journal of Applied Corporate Finance* 15 (spring 2002), pp. 8–23.

PROBLEM SETS

BASIC

1. a. What is the payback period on each of the following projects?

Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	C_4
A	−5,000	+1,000	+1,000	+3,000	0
B	−1,000	0	+1,000	+2,000	+3,000
C	−5,000	+1,000	+1,000	+3,000	+5,000

b. Given that you wish to use the payback rule with a cutoff period of two years, which projects would you accept?

c. If you use a cutoff period of three years, which projects would you accept?

d. If the opportunity cost of capital is 10%, which projects have positive NPVs?

e. "If a firm uses a single cutoff period for all projects, it is likely to accept too many short-lived projects." True or false?

f. If the firm uses the discounted-payback rule, will it accept any negative-NPV projects? Will it turn down positive-NPV projects? Explain.

2. Write down the equation defining a project's internal rate of return (IRR). In practice how is IRR calculated?

3. a. Calculate the net present value of the following project for discount rates of 0, 50, and 100%:

Cash Flows ($)		
C_0	C_1	C_2
−6,750	+4,500	+18,000

b. What is the IRR of the project?

4. You have the chance to participate in a project that produces the following cash flows:

Cash Flows ($)		
C_0	C_1	C_2
+5,000	+4,000	−11,000

The internal rate of return is 13%. If the opportunity cost of capital is 10%, would you accept the offer?

5. Consider a project with the following cash flows:

C_0	C_1	C_2
−100	+200	−75

a. How many internal rates of return does this project have?

b. Which of the following numbers is the project IRR:
(i) −50%; (ii) −12%; (iii) +5%; (iv) +50%?

c. The opportunity cost of capital is 20%. Is this an attractive project? Briefly explain.

6. Consider projects Alpha and Beta:

Cash Flows ($)				
Project	C_0	C_1	C_2	**IRR (%)**
Alpha	−400,000	+241,000	+293,000	21
Beta	−200,000	+131,000	+172,000	31

The opportunity cost of capital is 8%.

Suppose you can undertake Alpha or Beta, but not both. Use the IRR rule to make the choice. (*Hint:* What's the incremental investment in Alpha?)

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7. Suppose you have the following investment opportunities, but only $90,000 available for investment. Which projects should you take?

Project	NPV	Investment
1	5,000	10,000
2	5,000	5,000
3	10,000	90,000
4	15,000	60,000
5	15,000	75,000
6	3,000	15,000

INTERMEDIATE

8. Consider the following projects:

	Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	C_4	C_5
A	−1,000	+1,000	0	0	0	0
B	−2,000	+1,000	+1,000	+4,000	+1,000	+1,000
C	−3,000	+1,000	+1,000	0	+1,000	+1,000

a. If the opportunity cost of capital is 10%, which projects have a positive NPV?
b. Calculate the payback period for each project.
c. Which project(s) would a firm using the payback rule accept if the cutoff period were three years?
d. Calculate the discounted payback period for each project.
e. Which project(s) would a firm using the discounted payback rule accept if the cutoff period were three years?

9. Respond to the following comments:
a. "I like the IRR rule. I can use it to rank projects without having to specify a discount rate."
b. "I like the payback rule. As long as the minimum payback period is short, the rule makes sure that the company takes no borderline projects. That reduces risk."

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10. Calculate the IRR (or IRRs) for the following project:

C_0	C_1	C_2	C_3
−3,000	+3,500	+4,000	−4,000

For what range of discount rates does the project have positive NPV?

11. Consider the following two mutually exclusive projects:

	Cash Flows ($)			
Project	C_0	C_1	C_2	C_3
A	−100	+60	+60	0
B	−100	0	0	+140

a. Calculate the NPV of each project for discount rates of 0, 10, and 20%. Plot these on a graph with NPV on the vertical axis and discount rate on the horizontal axis.
b. What is the approximate IRR for each project?

c. In what circumstances should the company accept project A?

d. Calculate the NPV of the incremental investment (B − A) for discount rates of 0, 10, and 20%. Plot these on your graph. Show that the circumstances in which you would accept A are also those in which the IRR on the incremental investment is less than the opportunity cost of capital.

12. Mr. Cyrus Clops, the president of Giant Enterprises, has to make a choice between two possible investments:

Cash Flows ($ thousands)				
Project	C_0	C_1	C_2	**IRR (%)**
A	−400	+250	+300	23
B	−200	+140	+179	36

The opportunity cost of capital is 9%. Mr. Clops is tempted to take B, which has the higher IRR.

a. Explain to Mr. Clops why this is not the correct procedure.

b. Show him how to adapt the IRR rule to choose the best project.

c. Show him that this project also has the higher NPV.

13. The Titanic Shipbuilding Company has a noncancelable contract to build a small cargo vessel. Construction involves a cash outlay of $250,000 at the end of each of the next two years. At the end of the third year the company will receive payment of $650,000. The company can speed up construction by working an extra shift. In this case there will be a cash outlay of $550,000 at the end of the first year followed by a cash payment of $650,000 at the end of the second year. Use the IRR rule to show the (approximate) range of opportunity costs of capital at which the company should work the extra shift.

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14. Look again at projects D and E in Section 5.3. Assume that the projects are mutually exclusive and that the opportunity cost of capital is 10%.

a. Calculate the profitability index for each project.

b. Show how the profitability-index rule can be used to select the superior project.

15. Borghia Pharmaceuticals has $1 million allocated for capital expenditures. Which of the following projects should the company accept to stay within the $1 million budget? How much does the budget limit cost the company in terms of its market value? The opportunity cost of capital for each project is 11%.

Project	Investment ($ thousands)	NPV ($ thousands)	IRR (%)
1	300	66	17.2
2	200	−4	10.7
3	250	43	16.6
4	100	14	12.1
5	100	7	11.8
6	350	63	18.0
7	400	48	13.5

CHALLENGE

16. Some people believe firmly, even passionately, that ranking projects on IRR is OK if each project's cash flows can be reinvested at the project's IRR. They also say that the NPV rule "assumes that cash flows are reinvested at the opportunity cost of capital." Think carefully about these statements. Are they true? Are they helpful?

17. Look again at the project cash flows in Problem 10. Calculate the modified IRR as defined in Footnote 4 in Section 5.3. Assume the cost of capital is 12%.

Now try the following variation on the MIRR concept. Figure out the fraction x such that x times C_1 and C_2 has the same present value as (minus) C_3.

$$xC_1 + \frac{xC_2}{1.12} = -\frac{C_3}{1.12^2}$$

Define the modified project IRR as the solution of

$$C_0 + \frac{(1-x)C_1}{1+\text{IRR}} + \frac{(1-x)C_2}{(1+\text{IRR})^2} = 0$$

Now you have two MIRRs. Which is more meaningful? If you can't decide, what do you conclude about the usefulness of MIRRs?

18. Consider the following capital rationing problem:

Project	C_0	C_1	C_2	NPV
W	−10,000	−10,000	0	+6,700
X	0	−20,000	+5,000	+9,000
Y	−10,000	+5,000	+5,000	+0
Z	−15,000	+5,000	+4,000	−1,500
Financing available	20,000	20,000	20,000	

Set up this problem as a linear program and solve it.

You can allow partial investments, that is, $0 \leq x \leq 1$. Calculate and interpret the shadow prices[17] on the capital constraints.

MINI-CASE

Vegetron's CFO Calls Again

(The first episode of this story was presented in Section 5.1.)

Later that afternoon, Vegetron's CFO bursts into your office in a state of anxious confusion. The problem, he explains, is a last-minute proposal for a change in the design of the fermentation tanks that Vegetron will build to extract hydrated zirconium from a stockpile of powdered ore. The CFO has brought a printout (Table 5.1) of the forecasted revenues, costs, income, and book rates of return for the standard, low-temperature design. Vegetron's engineers have just proposed an alternative high-temperature design that will extract most of the hydrated zirconium over a shorter period, five instead of seven years. The forecasts for the high-temperature method are given in Table 5.2.[18]

[17] A shadow price is the marginal change in the objective for a marginal change in the constraint.

[18] For simplicity we have ignored taxes. There will be plenty about taxes in Chapter 6.

CFO: Why do these engineers always have a bright idea at the last minute? But you've got to admit the high-temperature process looks good. We'll get a faster payback, and the rate of return beats Vegetron's 9% cost of capital in every year except the first. Let's see, income is $30,000 per year. Average investment is half the $400,000 capital outlay, or $200,000, so the average rate of return is 30,000/200,000, or 15%—a lot better than the 9% hurdle rate. The average rate of return for the low-temperature process is not that good, only 28,000/200,000, or 14%. Of course we might get a higher rate of return for the low-temperature proposal if we depreciated the investment faster—do you think we should try that?

You: Let's not fixate on book accounting numbers. Book income is not the same as cash flow to Vegetron or its investors. Book rates of return don't measure the true rate of return.

CFO: But people use accounting numbers all the time. We have to publish them in our annual report to investors.

You: Accounting numbers have many valid uses, but they're not a sound basis for capital investment decisions. Accounting changes can have big effects on book income or rate of return, even when cash flows are unchanged.

Here's an example. Suppose the accountant depreciates the capital investment for the low-temperature process over six years rather than seven. Then income for years 1 to 6 goes down, because depreciation is higher. Income for year 7 goes up because the depreciation for that year becomes zero. But there is no effect on year-to-year cash flows, because depreciation is not a cash outlay. It is simply the accountant's device for spreading out the "recovery" of the up-front capital outlay over the life of the project.

CFO: So how do we get cash flows?

You: In these cases it's easy. Depreciation is the only noncash entry in your spreadsheets (Tables 5.1 and 5.2), so we can just leave it out of the calculation. Cash flow equals revenue minus operating costs. For the high-temperature process, annual cash flow is:

$$\text{Cash flow} = \text{revenue} - \text{operating cost} = 180 - 70 = 110, \text{ or } \$110{,}000$$

CFO: In effect you're adding back depreciation, because depreciation is a noncash accounting expense.

You: Right. You could also do it that way:

$$\text{Cash flow} = \text{net income} + \text{depreciation} = 30 + 80 = 110, \text{ or } \$110{,}000$$

CFO: Of course. I remember all this now, but book returns seem important when someone shoves them in front of your nose.

	Year						
	1	2	3	4	5	6	7
1. Revenue	140	140	140	140	140	140	140
2. Operating costs	55	55	55	55	55	55	55
3. Depreciation*	57	57	57	57	57	57	57
4. Net income	28	28	28	28	28	28	28
5. Start-of-year book value†	400	343	286	229	171	114	57
6. Book rate of return (4 ÷ 5)	7%	8.2%	9.8%	12.2%	16.4%	24.6%	49.1%

TABLE 5.1 Income statement and book rates of return for low-temperature extraction of hydrated zirconium ($ thousands).

* Rounded. Straight-line depreciation over seven years is 400/7 = 57.14, or $57,140 per year.
† Capital investment is $400,000 in year 0.

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	Year				
	1	2	3	4	5
1. Revenue	180	180	180	180	180
2. Operating costs	70	70	70	70	70
3. Depreciation*	80	80	80	80	80
4. Net income	30	30	30	30	30
5. Start-of-year book value†	400	320	240	160	80
6. Book rate of return (4 ÷ 5)	7.5%	9.4%	12.5%	18.75%	37.5%

TABLE 5.2 Income statement and book rates of return for high-temperature extraction of hydrated zirconium ($ thousands).

* Straight-line depreciation over five years is 400/5 = 80, or $80,000 per year.
† Capital investment is $400,000 in year 0.

You: It's not clear which project is better. The high-temperature process appears to be less efficient. It has higher operating costs and generates less total revenue over the life of the project, but of course it generates more cash flow in years 1 to 5.

CFO: Maybe the processes are equally good from a financial point of view. If so we'll stick with the low-temperature process rather than switching at the last minute.

You: We'll have to lay out the cash flows and calculate NPV for each process.

CFO: OK, do that. I'll be back in a half hour—and I also want to see each project's true, DCF rate of return.

QUESTIONS

1. Are the book rates of return reported in Tables 5.1 and 5.2 useful inputs for the capital investment decision?

2. Calculate NPV and IRR for each process. What is your recommendation? Be ready to explain to the CFO.

PART 1

CHAPTER

6

VALUE

Making Investment Decisions with the Net Present Value Rule

In late 2003 Boeing announced its intention to produce and market the 787 Dreamliner. The decision committed Boeing and its partners to a $10 billion capital investment, involving 3 million square feet of additional facilities. If the technical glitches that have delayed production can be sorted out, it looks as if Boeing will earn a good return on this investment. As we write this in August 2009, Boeing has booked orders for 865 Dreamliners, making it one of the most successful aircraft launches in history.

How does a company, such as Boeing, decide to go ahead with the launch of a new airliner? We know the answer in principle. The company needs to forecast the project's cash flows and discount them at the opportunity cost of capital to arrive at the project's NPV. A project with a positive NPV increases shareholder value.

But those cash flow forecasts do not arrive on a silver platter. First, the company's managers need answers to a number of basic questions. How soon can the company get the plane into production? How many planes are likely to be sold each year and at what price? How much does the firm need to invest in new production facilities, and what is the likely production cost? How long will the model stay in production, and what happens to the plant and equipment at the end of that time?

These predictions need to be checked for completeness and accuracy, and then pulled together to produce a single set of cash-flow forecasts. That requires careful tracking of taxes, changes in working capital, inflation, and the end-of-project salvage values of plant, property, and equipment. The financial manager must also ferret out hidden cash flows and take care to reject accounting entries that look like cash flows but truly are not.

Our first task in this chapter is to look at how to develop a set of project cash flows. We will then work through a realistic and comprehensive example of a capital investment analysis.

We conclude the chapter by looking at how the financial manager should apply the present value rule when choosing between investment in plant and equipment with different economic lives. For example, suppose you must decide between machine Y with a 5-year useful life and Z with a 10-year life. The present value of Y's lifetime investment and operating costs is naturally less than Z's because Z will last twice as long. Does that necessarily make Y the better choice? Of course not. You will find that, when you are faced with this type of problem, the trick is to transform the present value of the cash flow into an *equivalent annual* flow, that is, the total cash per year from buying and operating the asset.

6-1 Applying the Net Present Value Rule

Many projects require a heavy initial outlay on new production facilities. But often the largest investments involve the acquisition of intangible assets. Consider, for example, the expenditure by major banks on information technology. These projects can soak up hundreds of millions of dollars. Yet much of the expenditure goes to intangibles such as system design, programming, testing, and training. Think also of the huge expenditure by pharmaceutical companies on research and development (R&D). Pfizer, one of the largest pharmaceutical companies, spent $7.9 billion on R&D in 2008. The R&D cost of bringing *one* new prescription drug to market has been estimated at $800 million.

Expenditures on intangible assets such as IT and R&D are investments just like expenditures on new plant and equipment. In each case the company is spending money today in the expectation that it will generate a stream of future profits. Ideally, firms should apply the same criteria to all capital investments, regardless of whether they involve a tangible or intangible asset.

We have seen that an investment in any asset creates wealth if the discounted value of the future cash flows exceeds the up-front cost. But up to this point we have glossed over the problem of *what* to discount. When you are faced with this problem, you should stick to three general rules:

1. Only cash flow is relevant.
2. Always estimate cash flows on an incremental basis.
3. Be consistent in your treatment of inflation.

We discuss each of these rules in turn.

Rule 1: Only Cash Flow Is Relevant

The first and most important point: Net present value depends on future cash flows. Cash flow is the simplest possible concept; it is just the difference between cash received and cash paid out. Many people nevertheless confuse cash flow with accounting income.

Income statements are intended to show how well the company is performing. Therefore, accountants *start* with "dollars in" and "dollars out," but to obtain accounting income they adjust these inputs in two ways. First, they try to show profit as it is *earned* rather than when the company and its customers get around to paying their bills. Second, they sort cash outflows into two categories: current expenses and capital expenses. They deduct current expenses when calculating income but do not deduct capital expenses. There is a good reason for this. If the firm lays out a large amount of money on a big capital project, you do not conclude that the firm is performing poorly, even though a lot of cash is going out the door. Therefore, the accountant does not deduct capital expenditure when calculating the year's income but, instead, depreciates it over several years.

As a result of these adjustments, income includes some cash flows and excludes others, and it is reduced by depreciation charges, which are not cash flows at all. It is not always easy to translate the customary accounting data back into actual dollars—dollars you can buy beer with. If you are in doubt about what is a cash flow, simply count the dollars coming in and take away the dollars going out. Don't assume without checking that you can find cash flow by routine manipulations of accounting data.

Always estimate cash flows on an after-tax basis. Some firms do not deduct tax payments. They try to offset this mistake by discounting the cash flows before taxes at a rate higher than the opportunity cost of capital. Unfortunately, there is no reliable formula for making such adjustments to the discount rate.

You should also make sure that cash flows are recorded *only when they occur* and not when work is undertaken or a liability is incurred. For example, taxes should be discounted from their actual payment date, not from the time when the tax liability is recorded in the firm's books.

Rule 2: Estimate Cash Flows on an Incremental Basis

The value of a project depends on *all* the additional cash flows that follow from project acceptance. Here are some things to watch for when you are deciding which cash flows to include:

Do Not Confuse Average with Incremental Payoffs Most managers naturally hesitate to throw good money after bad. For example, they are reluctant to invest more money in a losing division. But occasionally you will encounter turnaround opportunities in which the *incremental* NPV from investing in a loser is strongly positive.

Conversely, it does not always make sense to throw good money after good. A division with an outstanding past profitability record may have run out of good opportunities. You would not pay a large sum for a 20-year-old horse, sentiment aside, regardless of how many races that horse had won or how many champions it had sired.

Here is another example illustrating the difference between average and incremental returns: Suppose that a railroad bridge is in urgent need of repair. With the bridge the railroad can continue to operate; without the bridge it can't. In this case the payoff from the repair work consists of all the benefits of operating the railroad. The incremental NPV of such an investment may be enormous. Of course, these benefits should be net of all other costs and all subsequent repairs; otherwise the company may be misled into rebuilding an unprofitable railroad piece by piece.

Include All Incidental Effects It is important to consider a project's effects on the remainder of the firm's business. For example, suppose Sony proposes to launch PlayStation 4, a new version of its video game console. Demand for the new product will almost certainly cut into sales of Sony's existing consoles. This incidental effect needs to be factored into the incremental cash flows. Of course, Sony may reason that it needs to go ahead with the new product because its existing product line is likely to come under increasing threat from competitors. So, even if it decides not to produce the new PlayStation, there is no guarantee that sales of the existing consoles will continue at their present level. Sooner or later they will decline.

Sometimes a new project will *help* the firm's existing business. Suppose that you are the financial manager of an airline that is considering opening a new short-haul route from Peoria, Illinois, to Chicago's O'Hare Airport. When considered in isolation, the new route may have a negative NPV. But once you allow for the additional business that the new route brings to your other traffic out of O'Hare, it may be a very worthwhile investment.

Forecast Sales Today and Recognize After-Sales Cash Flows to Come Later Financial managers should forecast all incremental cash flows generated by an investment. Sometimes these incremental cash flows last for decades. When GE commits to the design and production of a new jet engine, the cash inflows come first from the sale of engines and then from service and spare parts. A jet engine will be in use for 30 years. Over that period revenues from service and spare parts will be roughly seven times the engine's purchase price. GE's revenue in 2008 from commercial engine services was $6.8 billion versus $5.2 billion from commercial engine sales.[1]

Many manufacturing companies depend on the revenues that come *after* their products are sold. The consulting firm Accenture estimates that services and parts typically account for about 25% of revenues and 50% of profits for industrial companies.

[1] P. Glader, "GE's Focus on Services Faces Test," *The Wall Street Journal*, March 3, 2009, p. B1. The following estimate from Accenture also comes from this article.

Do Not Forget Working Capital Requirements **Net working capital** (often referred to simply as *working capital*) is the difference between a company's short-term assets and liabilities. The principal short-term assets are accounts receivable (customers' unpaid bills) and inventories of raw materials and finished goods. The principal short-term liabilities are accounts payable (bills that *you* have not paid). Most projects entail an additional investment in working capital. This investment should, therefore, be recognized in your cash-flow forecasts. By the same token, when the project comes to an end, you can usually recover some of the investment. This is treated as a cash inflow. We supply a numerical example of working-capital investment later in this chapter.

Include Opportunity Costs The cost of a resource may be relevant to the investment decision even when no cash changes hands. For example, suppose a new manufacturing operation uses land that could otherwise be sold for $100,000. This resource is not free: It has an opportunity cost, which is the cash it could generate for the company if the project were rejected and the resource were sold or put to some other productive use.

This example prompts us to warn you against judging projects on the basis of "before versus after." The proper comparison is "with or without." A manager comparing before versus after might not assign any value to the land because the firm owns it both before and after:

Before	Take Project	After	Cash Flow, Before versus After
Firm owns land	→	Firm still owns land	0

The proper comparison, with or without, is as follows:

With	Take Project	After	Cash Flow, with Project
Firm owns land	→	Firm still owns land	0

Without	Do Not Take Project	After	Cash Flow, without Project
	→	Firm sells land for $100,000	$100,000

Comparing the two possible "afters," we see that the firm gives up $100,000 by undertaking the project. This reasoning still holds if the land will not be sold but is worth $100,000 to the firm in some other use.

Sometimes opportunity costs may be very difficult to estimate; however, where the resource can be freely traded, its opportunity cost is simply equal to the market price. Why? It cannot be otherwise. If the value of a parcel of land to the firm is less than its market price, the firm will sell it. On the other hand, the opportunity cost of using land in a particular project cannot exceed the cost of buying an equivalent parcel to replace it.

Forget Sunk Costs Sunk costs are like spilled milk: They are past and irreversible outflows. Because sunk costs are bygones, they cannot be affected by the decision to accept or reject the project, and so they should be ignored.

For example, when Lockheed sought a federal guarantee for a bank loan to continue development of the TriStar airplane, the company and its supporters argued it would be foolish to abandon a project on which nearly $1 billion had already been spent. Some of Lockheed's critics countered that it would be equally foolish to continue with a project that offered no prospect of a satisfactory return on that $1 billion. Both groups were guilty of the *sunk-cost fallacy;* the $1 billion was irrecoverable and, therefore, irrelevant.

Beware of Allocated Overhead Costs We have already mentioned that the accountant's objective is not always the same as the investment analyst's. A case in point is the allocation of overhead costs. Overheads include such items as supervisory salaries, rent, heat, and light. These overheads may not be related to any particular project, but they have to be paid for somehow. Therefore, when the accountant assigns costs to the firm's projects, a charge for overhead is usually made. Now our principle of incremental cash flows says that in investment appraisal we should include only the *extra* expenses that would result from the project. A project may generate extra overhead expenses; then again, it may not. We should be cautious about assuming that the accountant's allocation of overheads represents the true extra expenses that would be incurred.

Remember Salvage Value When the project comes to an end, you may be able to sell the plant and equipment or redeploy the assets elsewhere in the business. If the equipment is sold, you must pay tax on the difference between the sale price and the book value of the asset. The salvage value (net of any taxes) represents a positive cash flow to the firm.

Some projects have significant shut-down costs, in which case the final cash flows may be *negative.* For example, the mining company, FCX, has earmarked over $430 million to cover the future reclamation and closure costs of its New Mexico mines.

Rule 3: Treat Inflation Consistently

As we pointed out in Chapter 3, interest rates are usually quoted in *nominal* rather than *real* terms. For example, if you buy an 8% Treasury bond, the government promises to pay you $80 interest each year, but it does not promise what that $80 will buy. Investors take inflation into account when they decide what is an acceptable rate of interest.

If the discount rate is stated in nominal terms, then consistency requires that cash flows should also be estimated in nominal terms, taking account of trends in selling price, labor and materials costs, etc. This calls for more than simply applying a single assumed inflation rate to all components of cash flow. Labor costs per hour of work, for example, normally increase at a faster rate than the consumer price index because of improvements in productivity. Tax savings from depreciation do *not* increase with inflation; they are constant in nominal terms because tax law in the United States allows only the original cost of assets to be depreciated.

Of course, there is nothing wrong with discounting real cash flows at a real discount rate. In fact this is standard procedure in countries with high and volatile inflation. Here is a simple example showing that real and nominal discounting, properly applied, always give the same present value.

Suppose your firm usually forecasts cash flows in nominal terms and discounts at a 15% nominal rate. In this particular case, however, you are given project cash flows in real terms, that is, current dollars:

Real Cash Flows ($ thousands)			
C_0	C_1	C_2	C_3
−100	+35	+50	+30

It would be inconsistent to discount these real cash flows at the 15% nominal rate. You have two alternatives: Either restate the cash flows in nominal terms and discount at 15%, or restate the discount rate in real terms and use it to discount the real cash flows.

Assume that inflation is projected at 10% a year. Then the cash flow for year 1, which is $35,000 in current dollars, will be 35,000 × 1.10 = $38,500 in year-1 dollars. Similarly the cash flow for year 2 will be 50,000 × $(1.10)^2$ = \$60,500 in year-2 dollars, and so on. If we discount these nominal cash flows at the 15% nominal discount rate, we have

$$\text{NPV} = -100 + \frac{38.5}{1.15} + \frac{60.5}{(1.15)^2} + \frac{39.9}{(1.15)^3} = 5.5, \text{ or } \$5{,}500$$

Instead of converting the cash-flow forecasts into nominal terms, we could convert the discount rate into real terms by using the following relationship:

$$\text{Real discount rate} = \frac{1 + \text{nominal discount rate}}{1 + \text{inflation rate}} - 1$$

In our example this gives

$$\text{Real discount rate} = \frac{1.15}{1.10} - 1 = .045, \text{ or } 4.5\%$$

If we now discount the real cash flows by the real discount rate, we have an NPV of \$5,500, just as before:

$$\text{NPV} = -100 + \frac{35}{1.045} + \frac{50}{(1.045)^2} + \frac{30}{(1.045)^3} = 5.5, \text{ or } \$5{,}500$$

The message of all this is quite simple. Discount nominal cash flows at a nominal discount rate. Discount real cash flows at a real rate. *Never* mix real cash flows with nominal discount rates or nominal flows with real rates.

6-2 Example—IM&C's Fertilizer Project

As the newly appointed financial manager of International Mulch and Compost Company (IM&C), you are about to analyze a proposal for marketing guano as a garden fertilizer. (IM&C's planned advertising campaign features a rustic gentleman who steps out of a vegetable patch singing, "All my troubles have guano way.")[2]

You are given the forecasts shown in Table 6.1.[3] The project requires an investment of \$10 million in plant and machinery (line 1). This machinery can be dismantled and sold for net proceeds estimated at \$1.949 million in year 7 (line 1, column 7). This amount is your forecast of the plant's *salvage value.*

Whoever prepared Table 6.1 depreciated the capital investment over six years to an arbitrary salvage value of \$500,000, which is less than your forecast of salvage value. *Straight-line depreciation* was assumed. Under this method annual depreciation equals a constant proportion of the initial investment less salvage value (\$9.5 million). If we call the depreciable life T, then the straight-line depreciation in year t is

$$\text{Depreciation in year } t = 1/T \times \text{depreciable amount} = 1/6 \times 9.5 = \$1.583 \text{ million}$$

Lines 6 through 12 in Table 6.1 show a simplified income statement for the guano project.[4] This will be our starting point for estimating cash flow. All the entries in the table are nominal amounts. In other words, IM&C's managers have taken into account the likely effect of inflation on prices and costs.

[2] Sorry.

[3] "Live" Excel versions of Tables 6.1, 6.2, 6.4, 6.5, and 6.6 are available on the book's Web site, **www.mhhe.com/bma**.

[4] We have departed from the usual income-statement format by separating depreciation from costs of goods sold.

Table 6.2 derives cash-flow forecasts from the investment and income data given in Table 6.1. The project's net cash flow is the sum of three elements:

$$\begin{aligned}\text{Net cash flow} &= \text{cash flow from capital investment and disposal}\\ &+ \text{cash flow from changes in working capital}\\ &+ \text{operating cash flow}\end{aligned}$$

		Period							
		0	1	2	3	4	5	6	7
1	Capital investment	10,000							-1,949[a]
2	Accumulated depreciation		1,583	3,167	4,750	6,333	7,917	9,500	0
3	Year-end book value	10,000	8,417	6,833	5,250	3,667	2,083	500	0
4	Working capital		550	1,289	3,261	4,890	3,583	2,002	0
5	Total book value (3 + 4)		8,967	8,122	8,511	8,557	5,666	2,502	0
6	Sales		523	12,887	32,610	48,901	35,834	19,717	
7	Cost of goods sold[b]		837	7,729	19,552	29,345	21,492	11,830	
8	Other costs[c]	4,000	2,200	1,210	1,331	1,464	1,611	1,772	
9	Depreciation		1,583	1,583	1,583	1,583	1,583	1,583	0
10	Pretax profit (6 − 7 − 8 − 9)	−4,000	−4,097	2,365	10,144	16,509	11,148	4,532	1,449[d]
11	Tax at 35%	−1,400	−1,434	828	3,550	5,778	3,902	1,586	507
12	Profit after tax (10 − 11)	−2,600	−2,663	1,537	6,593	10,731	7,246	2,946	942

TABLE 6.1 IM&C's guano project—projections ($ thousands) reflecting inflation and assuming straight-line depreciation.

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[a] Salvage value.
[b] We have departed from the usual income-statement format by *not* including depreciation in cost of goods sold. Instead, we break out depreciation separately (see line 9).
[c] Start-up costs in years 0 and 1, and general and administrative costs in years 1 to 6.
[d] The difference between the salvage value and the ending book value of $500 is a taxable profit.

		Period							
		0	1	2	3	4	5	6	7
1	Capital investment and disposal	−10,000	0	0	0	0	0	0	1,442[a]
2	Change in working capital		−550	−739	−1,972	−1,629	1,307	1,581	2,002
3	Sales	0	523	12,887	32,610	48,901	35,834	19,717	0
4	Cost of goods sold	0	837	7,729	19,552	29,345	21,492	11,830	0
5	Other costs	4,000	2,200	1,210	1,331	1,464	1,611	1,772	0
6	Tax	−1,400	−1,434	828	3,550	5,778	3,902	1,586	
7	Operating cash flow (3 − 4 − 5 − 6)	−2,600	−1,080	3,120	8,177	12,314	8,829	4,529	
8	Net cash flow (1 + 2 + 7)	−12,600	−1,630	2,381	6,205	10,685	10,136	6,110	3,444
9	Present value at 20%	−12,600	−1,358	1,654	3,591	5,153	4,074	2,046	961
10	Net present value =	+3,520	(sum of 9)						

TABLE 6.2 IM&C's guano project—initial cash-flow analysis assuming straight-line depreciation ($ thousands).

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[a] Salvage value of $1,949 less tax of $507 on the difference between salvage value and ending book value.

Each of these items is shown in the table. Row 1 shows the initial capital investment and the estimated salvage value of the equipment when the project comes to an end. If, as you expect, the salvage value is higher than the depreciated value of the machinery, you will have to pay tax on the difference. So the salvage value in row 1 is shown after payment of this tax. Row 2 of the table shows the changes in working capital, and the remaining rows calculate the project's operating cash flows.

Notice that in calculating the operating cash flows we did not deduct depreciation. Depreciation is an accounting entry. It affects the tax that the company pays, but the firm does not send anyone a check for depreciation. The operating cash flow is simply the dollars coming in less the dollars going out:[5]

$$\text{Operating cash flow} = \text{revenues} - \text{cash expenses} - \text{taxes}$$

For example, in year 6 of the guano project:

$$\text{Operating cash flow} = 19{,}717 - (11{,}830 + 1{,}772) - 1{,}586 = 4{,}529$$

IM&C estimates the nominal opportunity cost of capital for projects of this type as 20%. When all cash flows are added up and discounted, the guano project is seen to offer a net present value of about $3.5 million:

$$\text{NPV} = -12{,}600 - \frac{1{,}630}{1.20} + \frac{2{,}381}{(1.20)^2} + \frac{6{,}205}{(1.20)^3} + \frac{10{,}685}{(1.20)^4} + \frac{10{,}136}{(1.20)^5}$$
$$+ \frac{6{,}110}{(1.20)^6} + \frac{3{,}444}{(1.20)^7} = +3{,}520, \text{ or } \$3{,}520{,}000$$

Separating Investment and Financing Decisions

Our analysis of the guano project takes no notice of how that project is financed. It may be that IM&C will decide to finance partly by debt, but if it does we will not subtract the debt proceeds from the required investment, nor will we recognize interest and principal payments as cash outflows. We analyze the project as if it were all-equity-financed, treating all cash outflows as coming from stockholders and all cash inflows as going to them.

We approach the problem in this way so that we can separate the analysis of the investment decision from the financing decision. But this does not mean that the financing decision can be ignored. We explain in Chapter 19 how to recognize the effect of financing choices on project values.

Investments in Working Capital

Now here is an important point. You can see from line 2 of Table 6.2 that working capital increases in the early and middle years of the project. What is working capital, you may ask, and why does it increase?

Working capital summarizes the net investment in short-term assets associated with a firm, business, or project. Its most important components are *inventory, accounts receivable,*

---

[5] There are several alternative ways to calculate operating cash flow. For example, you can add depreciation back to the after-tax profit:

$$\text{Operating cash flow} = \text{after-tax profit} + \text{depreciation}$$

Thus, in year 6 of the guano project:

$$\text{Operating cash flow} = 2{,}946 + 1{,}583 = 4{,}529$$

Another alternative is to calculate after-tax profit assuming *no* depreciation, and then to add back the tax saving provided by the depreciation allowance:

$$\text{Operating cash flow} = (\text{revenues} - \text{expenses}) \times (1 - \text{tax rate}) + (\text{depreciation} \times \text{tax rate})$$

Thus, in year 6 of the guano project:

$$\text{Operating cash flow} = (19{,}717 - 11{,}830 - 1{,}772) \times (1 - .35) + (1{,}583 \times .35) = 4{,}529$$

and *accounts payable.* The guano project's requirements for working capital in year 2 might be as follows:

$$\text{Working capital} = \text{inventory} + \text{accounts receivable} - \text{accounts payable}$$
$$\$1{,}289 = 635 + 1{,}030 - 376$$

Why does working capital increase? There are several possibilities:

1. Sales recorded on the income statement overstate actual cash receipts from guano shipments because sales are increasing and customers are slow to pay their bills. Therefore, accounts receivable increase.
2. It takes several months for processed guano to age properly. Thus, as projected sales increase, larger inventories have to be held in the aging sheds.
3. An offsetting effect occurs if payments for materials and services used in guano production are delayed. In this case accounts payable will increase.

The additional investment in working capital from year 2 to 3 might be

$$\text{Additional investment in working capital} = \text{increase in inventory} + \text{increase in accounts receivable} - \text{increase in accounts payable}$$
$$\$1{,}972 = 972 + 1{,}500 - 500$$

A more detailed cash-flow forecast for year 3 would look like Table 6.3.

Working capital is one of the most common sources of confusion in estimating project cash flows. Here are the most common mistakes:

1. *Forgetting about working capital entirely.* We hope you never fall into that trap.
2. *Forgetting that working capital may change during the life of the project.* Imagine that you sell $100,000 of goods one year and that customers pay six months late. You will therefore have $50,000 of unpaid bills. Now you increase prices by 10%, so revenues increase to $110,000. If customers continue to pay six months late, unpaid bills increase to $55,000, and therefore you need to make an *additional* investment in working capital of $5,000.
3. *Forgetting that working capital is recovered at the end of the project.* When the project comes to an end, inventories are run down, any unpaid bills are paid off (you hope) and you recover your investment in working capital. This generates a cash *inflow.*

There is an alternative to worrying about changes in working capital. You can estimate cash flow directly by counting the dollars coming in from customers and deducting the dollars

Cash Flows		Data from Forecasted Income Statement		Working-Capital Changes
Cash inflow	=	Sales	−	Increase in accounts receivable
$31,110	=	32,610	−	1,500
Cash outflow	=	Cost of goods sold, other costs, and taxes	+	Increase in inventory net of increase in accounts payable
$24,905	=	(19,552 + 1,331 + 3,550)	+	(972 − 500)
		Net cash flow = cash inflow − cash outflow $6,205 = 31,110 − 24,905		

TABLE 6.3 Details of cash-flow forecast for IM&C's guano project in year 3 ($ thousands).

going out to suppliers. You would also deduct all cash spent on production, including cash spent for goods held in inventory. In other words,

1. If you replace each year's sales with that year's cash payments received from customers, you don't have to worry about accounts receivable.
2. If you replace cost of goods sold with cash payments for labor, materials, and other costs of production, you don't have to keep track of inventory or accounts payable.

However, you would still have to construct a projected income statement to estimate taxes.

We discuss the links between cash flow and working capital in much greater detail in Chapter 30.

A Further Note on Depreciation

Depreciation is a noncash expense; it is important only because it reduces taxable income. It provides an annual *tax shield* equal to the product of depreciation and the marginal tax rate:

$$\begin{aligned}\text{Tax shield} &= \text{depreciation} \times \text{tax rate}\\ &= 1{,}583 \times .35 = 554, \text{ or } \$554{,}000\end{aligned}$$

The present value of the tax shields ($554,000 for six years) is $1,842,000 at a 20% discount rate.

Now if IM&C could just get those tax shields sooner, they would be worth more, right? Fortunately tax law allows corporations to do just that: It allows *accelerated depreciation.*

The current rules for tax depreciation in the United States were set by the Tax Reform Act of 1986, which established a Modified Accelerated Cost Recovery System (MACRS). Table 6.4 summarizes the tax depreciation schedules. Note that there are six schedules, one for each recovery period class. Most industrial equipment falls into the five- and seven-year classes. To keep things simple, we assume that all the guano project's investment goes into five-year assets. Thus, IM&C can write off 20% of its depreciable investment in year 1, as soon as the assets are placed in service, then 32% of depreciable investment in year 2, and so on. Here are the tax shields for the guano project:

	Year					
	1	2	3	4	5	6
Tax depreciation (MACRS percentage × depreciable investment)	2,000	3,200	1,920	1,152	1,152	576
Tax shield (tax depreciation × tax rate, T_c = .35)	700	1,120	672	403	403	202

The present value of these tax shields is $2,174,000, about $331,000 higher than under the straight-line method.

Table 6.5 recalculates the guano project's impact on IM&C's future tax bills, and Table 6.6 shows revised after-tax cash flows and present value. This time we have incorporated realistic assumptions about taxes as well as inflation. We arrive at a higher NPV than in Table 6.2, because that table ignored the additional present value of accelerated depreciation.

There is one possible additional problem lurking in the woodwork behind Table 6.5: In the United States there is an *alternative minimum tax,* which can limit or defer the tax shields of accelerated depreciation or other *tax preference* items. Because the alternative minimum tax can be a motive for leasing, we discuss it in Chapter 25, rather than here. But make a mental note not to sign off on a capital budgeting analysis without checking whether your company is subject to the alternative minimum tax.

Tax Depreciation Schedules by Recovery-Period Class							
	Year(s)	3-year	5-year	7-year	10-year	15-year	20-year
1	1	33.33	20.00	14.29	10.00	5.00	3.75
2	2	44.45	32.00	24.49	18.00	9.50	7.22
3	3	14.81	19.20	17.49	14.40	8.55	6.68
4	4	7.41	11.52	12.49	11.52	7.70	6.18
5	5		11.52	8.93	9.22	6.93	5.71
6	6		5.76	8.92	7.37	6.23	5.28
7	7			8.93	6.55	5.90	4.89
8	8			4.46	6.55	5.90	4.52
9	9				6.56	5.91	4.46
10	10				6.55	5.90	4.46
11	11				3.28	5.91	4.46
12	12					5.90	4.46
13	13					5.91	4.46
14	14					5.90	4.46
15	15					5.91	4.46
16	16					2.95	4.46
17	17-20						4.46
18	21						2.23

TABLE 6.4 Tax depreciation allowed under the modified accelerated cost recovery system (MACRS) (figures in percent of depreciable investment).

Notes:

1. Tax depreciation is lower in the first and last years because assets are assumed to be in service for only six months.
2. Real property is depreciated straight-line over 27.5 years for residential property and 39 years for nonresidential property.

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		Period							
		0	1	2	3	4	5	6	7
1	Sales[a]		523	12,887	32,610	48,901	35,834	19,717	
2	Cost of goods sold[a]		837	7,729	19,552	29,345	21,492	11,830	
3	Other costs[a]	4,000	2,200	1,210	1,331	1,464	1,611	1,772	
4	Tax depreciation		2,000	3,200	1,920	1,152	1,152	576	
5	Pretax profit (1 − 2 − 3 − 4)	−4,000	−4,514	748	9,807	16,940	11,579	5,539	1,949[b]
6	Tax at 35%[c]	−1,400	−1,580	262	3,432	5,929	4,053	1,939	682

TABLE 6.5 Tax payments on IM&C's guano project ($ thousands).

[a] From Table 6.1.
[b] Salvage value is zero, for tax purposes, after all tax depreciation has been taken. Thus, IM&C will have to pay tax on the full salvage value of $1,949.
[c] A negative tax payment means a cash *inflow,* assuming IM&C can use the tax loss on its guano project to shield income from other projects.

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A Final Comment on Taxes

All large U.S. corporations keep two separate sets of books, one for stockholders and one for the Internal Revenue Service. It is common to use straight-line depreciation on the stockholder books and accelerated depreciation on the tax books. The IRS doesn't object to this, and it makes the firm's reported earnings higher than if accelerated depreciation were used everywhere. There are many other differences between tax books and shareholder books.[6]

[6] This separation of tax accounts from shareholder accounts is not found worldwide. In Japan, for example, taxes reported to shareholders must equal taxes paid to the government; ditto for France and many other European countries.

		Period							
		0	1	2	3	4	5	6	7
1	Capital investment and disposal	−10,000	0	0	0	0	0	0	1,949
2	Change in working capital		−550	−739	−1,972	−1,629	1,307	1,581	2,002
3	Sales[a]	0	523	12,887	32,610	48,901	35,834	19,717	0
4	Cost of goods sold[a]	0	837	7,729	19,552	29,345	21,492	11,830	0
5	Other costs[a]	4,000	2,200	1,210	1,331	1,464	1,611	1,772	0
6	Tax[b]	−1,400	−1,580	262	3,432	5,929	4,053	1,939	682
7	Operating cash flow (3 − 4 − 5 − 6)	−2,600	−934	3,686	8,295	12,163	8,678	4,176	−682
8	Net cash flow (1 + 2 + 7)	−12,600	−1,484	2,947	6,323	10,534	9,985	5,757	3,269
9	Present value at 20%	−12,600	−1,237	2,047	3,659	5,080	4,013	1,928	912
10	Net present value =	3,802	(sum of 9)						

TABLE 6.6 IM&C's guano project—revised cash-flow analysis ($ thousands).

[a] From Table 6.1.
[b] From Table 6.5.

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The financial analyst must be careful to remember which set of books he or she is looking at. In capital budgeting only the tax books are relevant, but to an outside analyst only the shareholder books are available.

Project Analysis

Let us review. Several pages ago, you embarked on an analysis of IM&C's guano project. You started with a simplified statement of assets and income for the project that you used to develop a series of cash-flow forecasts. Then you remembered accelerated depreciation and had to recalculate cash flows and NPV.

You were lucky to get away with just two NPV calculations. In real situations, it often takes several tries to purge all inconsistencies and mistakes. Then you may want to analyze some alternatives. For example, should you go for a larger or smaller project? Would it be better to market the fertilizer through wholesalers or directly to the consumer? Should you build 90,000-square-foot aging sheds for the guano in northern South Dakota rather than the planned 100,000-square-foot sheds in southern North Dakota? In each case your choice should be the one offering the highest NPV. Sometimes the alternatives are not immediately obvious. For example, perhaps the plan calls for two costly high-speed packing lines. But, if demand for guano is seasonal, it may pay to install just one high-speed line to cope with the base demand and two slower but cheaper lines simply to cope with the summer rush. You won't know the answer until you have compared NPVs.

You will also need to ask some "what if" questions. How would NPV be affected if inflation rages out of control? What if technical problems delay start-up? What if gardeners prefer chemical fertilizers to your natural product? Managers employ a variety of techniques to develop a better understanding of how such unpleasant surprises could damage NPV. For example, they might undertake a *sensitivity analysis,* in which they look at how far the project could be knocked off course by bad news about one of the variables. Or they might construct different *scenarios* and estimate the effect of each on NPV. Another technique, known as *break-even analysis,* is to explore how far sales could fall short of forecast before the project went into the red.

In Chapter 10 we practice using each of these "what if" techniques. You will find that project analysis is much more than one or two NPV calculations.[7]

Calculating NPV in Other Countries and Currencies

Our guano project was undertaken in the United States by a U.S. company. But the principles of capital investment are the same worldwide. For example, suppose that you are the financial manager of the German company, K.G.R. Ökologische Naturdüngemittel GmbH (KGR), that is faced with a similar opportunity to make a €10 million investment in Germany. What changes?

1. KGR must also produce a set of cash-flow forecasts, but in this case the project cash flows are stated in euros, the Eurozone currency.
2. In developing these forecasts, the company needs to recognize that prices and costs will be influenced by the German inflation rate.
3. Profits from KGR's project are liable to the German rate of corporate tax.
4. KGR must use the German system of depreciation allowances. In common with many other countries, Germany allows firms to choose between two methods of depreciation—the straight-line system and the declining-balance system. KGR opts for the declining-balance method and writes off 30% of the depreciated value of the equipment each year (the maximum allowed under current German tax rules). Thus, in the first year KGR writes off $.30 \times 10 = €3$ million and the written-down value of the equipment falls to $10 - 3 = €7$ million. In year 2, KGR writes off $.30 \times 7 = €2.1$ million and the written-down value is further reduced to $7 - 2.1 = €4.9$ million. In year 4 KGR observes that depreciation would be higher if it could switch to straight-line depreciation and write off the balance of €3.43 million over the remaining three years of the equipment's life. Fortunately, German tax law allows it to do this. Therefore, KGR's depreciation allowance each year is calculated as follows:

	Year					
	1	2	3	4	5	6
Written-down value, start of year (€ millions)	10	7	4.9	3.43	2.29	1.14
Depreciation (€ millions)	$.3 \times 10 = 3$	$.3 \times 7 = 2.1$	$.3 \times 4.9 = 1.47$	$3.43/3 = 1.14$	$3.43/3 = 1.14$	$3.43/3 = 1.14$
Written-down value, end of year (€ millions)	$10 - 3 = 7$	$7 - 2.1 = 4.9$	$4.9 - 1.47 = 3.43$	$3.43 - 1.14 = 2.29$	$2.29 - 1.14 = 1.14$	$1.14 - 1.14 = 0$

Notice that KGR's depreciation deduction declines for the first few years and then flattens out. That is also the case with the U.S. MACRS system of depreciation. In fact, MACRS is just another example of the declining-balance method with a later switch to straight-line.

[7] In the meantime you might like to get ahead of the game by viewing the live spreadsheets for the guano project and seeing how NPV would change with a shortfall in sales or an unexpected rise in costs.

6-3 Investment Timing

The fact that a project has a positive NPV does not mean that it is best undertaken now. It might be even more valuable if undertaken in the future.

The question of optimal timing is not difficult when the cash flows are certain. You must first examine alternative start dates (t) for the investment and calculate the net *future* value at each of these dates. Then, to find which of the alternatives would add most to the firm's *current* value, you must discount these net future values back to the present:

$$\text{Net present value of investment if undertaken at date } t = \frac{\text{Net future value at date } t}{(1 + r)^t}$$

For example, suppose you own a large tract of inaccessible timber. To harvest it, you have to invest a substantial amount in access roads and other facilities. The longer you wait, the higher the investment required. On the other hand, lumber prices will rise as you wait, and the trees will keep growing, although at a gradually decreasing rate.

Let us suppose that the net present value of the harvest at different *future* dates is as follows:

	Year of Harvest					
	0	1	2	3	4	5
Net *future* value ($ thousands)	50	64.4	77.5	89.4	100	109.4
Change in value from previous year (%)		+28.8	+20.3	+15.4	+11.9	+9.4

As you can see, the longer you defer cutting the timber, the more money you will make. However, your concern is with the date that maximizes the net *present* value of your investment, that is, its contribution to the value of your firm *today*. You therefore need to discount the net future value of the harvest back to the present. Suppose the appropriate discount rate is 10%. Then, if you harvest the timber in year 1, it has a net *present* value of $58,500:

$$\text{NPV if harvested in year 1} = \frac{64.4}{1.10} = 58.5, \text{ or } \$58{,}500$$

The net present value for other harvest dates is as follows:

	Year of Harvest					
	0	1	2	3	4	5
Net present value ($ thousands)	50	58.5	64.0	67.2	68.3	67.9

The optimal point to harvest the timber is year 4 because this is the point that maximizes NPV.

Notice that before year 4 the net future value of the timber increases by more than 10% a year: The gain in value is greater than the cost of the capital tied up in the project. After year

4 the gain in value is still positive but less than the cost of capital. So delaying the harvest further just reduces shareholder wealth.[8]

The investment-timing problem is much more complicated when you are unsure about future cash flows. We return to the problem of investment timing under uncertainty in Chapters 10 and 22.

6-4 Equivalent Annual Cash Flows

When you calculate NPV, you transform future, year-by-year cash flows into a lump-sum value expressed in today's dollars (or euros, or other relevant currency). But sometimes it's helpful to reverse the calculation, transforming an investment today into an equivalent stream of future cash flows. Consider the following example.

Investing to Produce Reformulated Gasoline at California Refineries

In the early 1990s, the California Air Resources Board (CARB) started planning its "Phase 2" requirements for reformulated gasoline (RFG). RFG is gasoline blended to tight specifications designed to reduce pollution from motor vehicles. CARB consulted with refiners, environmentalists, and other interested parties to design these specifications.

As the outline for the Phase 2 requirements emerged, refiners realized that substantial capital investments would be required to upgrade California refineries. What might these investments mean for the retail price of gasoline? A refiner might ask: "Suppose my company invests $400 million to upgrade our refinery to meet Phase 2. How much extra revenue would we need every year to recover that cost?" Let's see if we can help the refiner out.

Assume $400 million of capital investment and a real (inflation-adjusted) cost of capital of 7%. The new equipment lasts for 25 years, and does not change raw-material and operating costs.

How much additional revenue does it take to cover the $400 million investment? The answer is simple: Just find the 25-year annuity payment with a present value equal to $400 million.

$$\text{PV of annuity} = \text{annuity payment} \times \text{25-year annuity factor}$$

At a 7% cost of capital, the 25-year annuity factor is 11.65.

$$\$400 \text{ million} = \text{annuity payment} \times 11.65$$
$$\text{Annuity payment} = \$34.3 \text{ million per year}^{9}$$

[8] Our timber-cutting example conveys the right idea about investment timing, but it misses an important practical point: The sooner you cut the first crop of trees, the sooner the second crop can start growing. Thus, the value of the second crop depends on when you cut the first. The more complex and realistic problem can be solved in one of two ways:

1. Find the cutting dates that maximize the present value of a series of harvests, taking into account the different growth rates of young and old trees.
2. Repeat our calculations, counting the future market value of cut-over land as part of the payoff to the first harvest. The value of cut-over land includes the present value of all subsequent harvests.

The second solution is far simpler if you can figure out what cut-over land will be worth.

[9] For simplicity we have ignored taxes. Taxes would enter this calculation in two ways. First, the $400 million investment would generate depreciation tax shields. The easiest way to handle these tax shields is to calculate their PV and subtract it from the initial outlay. For example, if the PV of depreciation tax shields is $83 million, equivalent annual cost would be calculated on an after-tax investment base of $400 − 83 = $317 million. Second, our annuity payment is after-tax. To actually achieve after-tax revenues of, say, $34.3 million, the refiner would have to achieve pretax revenue sufficient to pay tax and have $34.3 million left over. If the tax rate is 35%, the required pretax revenue is 34.3/(1 − .35) = $52.8 million. Note how the after-tax figure is "grossed up" by dividing by one minus the tax rate.

This annuity is called an **equivalent annual cash flow.** It is the annual cash flow sufficient to recover a capital investment, including the cost of capital for that investment, over the investment's economic life. In our example the refiner would need to generate an extra $34.3 million for each of the next 25 years to recover the initial investment of $400 million.

Equivalent annual cash flows are handy—and sometimes essential—tools of finance. Here is a further example.

Choosing between Long- and Short-Lived Equipment

Suppose the firm is forced to choose between two machines, A and B. The two machines are designed differently but have identical capacity and do exactly the same job. Machine A costs $15,000 and will last three years. It costs $5,000 per year to run. Machine B is an economy model costing only $10,000, but it will last only two years and costs $6,000 per year to run. These are real cash flows: the costs are forecasted in dollars of constant purchasing power.

Because the two machines produce exactly the same product, the only way to choose between them is on the basis of cost. Suppose we compute the present value of cost:

	Costs ($ thousands)				
Machine	C_0	C_1	C_2	C_3	**PV at 6% ($ thousands)**
A	+15	+5	+5	+5	28.37
B	+10	+6	+6		21.00

Should we take machine B, the one with the lower present value of costs? Not necessarily, because B will have to be replaced a year earlier than A. In other words, the timing of a future investment decision depends on today's choice of A or B.

So, a machine with total PV(costs) of $21,000 spread over three years (0, 1, and 2) is not necessarily better than a competing machine with PV(costs) of $28,370 spread over four years (0 through 3). We have to convert total PV(costs) to a cost per year, that is, to an equivalent annual cost. For machine A, the annual cost turns out to be 10.61, or $10,610 per year:

	Costs ($ thousands)				
Machine	C_0	C_1	C_2	C_3	**PV at 6% ($ thousands)**
Machine A	+15	+5	+5	+5	28.37
Equivalent annual cost		+10.61	+10.61	+10.61	28.37

We calculated the equivalent annual cost by finding the three-year annuity with the same present value as A's lifetime costs.

$$\text{PV of annuity} = \text{PV of A's costs} = 28.37$$
$$= \text{annuity payment} \times \text{3-year annuity factor}$$

The annuity factor is 2.673 for three years and a 6% real cost of capital, so

$$\text{Annuity payment} = \frac{28.37}{2.673} = 10.61$$

A similar calculation for machine B gives:

	Costs ($ thousands)			
	C_0	C_1	C_2	**PV at 6% ($ thousands)**
Machine B	+10	+6	+6	21.00
Equivalent annual cost		+11.45	+11.45	21.00

Machine A is better, because its equivalent annual cost is less ($10,610 versus $11,450 for machine B).

You can think of the equivalent annual cost of machine A or B as an annual rental charge. Suppose the financial manager is asked to *rent* machine A to the plant manager actually in charge of production. There will be three equal rental payments starting in year 1. The three payments must recover both the original cost of machine A in year 0 and the cost of running it in years 1 to 3. Therefore the financial manager has to make sure that the rental payments are worth $28,370, the total PV(costs) of machine A. You can see that the financial manager would calculate a fair rental payment equal to machine A's equivalent annual cost.

Our rule for choosing between plant and equipment with different economic lives is, therefore, to select the asset with the lowest fair rental charge, that is, the lowest equivalent annual cost.

Equivalent Annual Cash Flow and Inflation

The equivalent annual costs we just calculated are *real* annuities based on forecasted *real* costs and a 6% *real* discount rate. We could, of course, restate the annuities in nominal terms. Suppose the expected inflation rate is 5%; we multiply the first cash flow of the annuity by 1.05, the second by $(1.05)^2 = 1.1025$, and so on.

		C_0	C_1	C_2	C_3
A	Real annuity		10.61	10.61	10.61
	Nominal cash flow		11.14	11.70	12.28
B	Real annuity		11.45	11.45	
	Nominal cash flow		12.02	12.62	

Note that B is still inferior to A. Of course the present values of the nominal and real cash flows are identical. Just remember to discount the real annuity at the real rate and the equivalent nominal cash flows at the consistent nominal rate.[10]

When you use equivalent annual costs simply for comparison of costs per period, as we did for machines A and B, we strongly recommend doing the calculations in real terms.[11] But if you actually rent out the machine to the plant manager, or anyone else, be careful to specify that the rental payments be "indexed" to inflation. If inflation runs on at 5% per year and rental payments do not increase proportionally, then the real value of the rental payments must decline and will not cover the full cost of buying and operating the machine.

Equivalent Annual Cash Flow and Technological Change

So far we have the following simple rule: Two or more streams of cash outflows with different lengths or time patterns can be compared by converting their present values to equivalent annual cash flows. Just remember to do the calculations in real terms.

Now any rule this simple cannot be completely general. For example, when we evaluated machine A versus machine B, we implicitly assumed that their fair rental charges would *continue* at $10,610 versus $11,450. This will be so only if the *real* costs of buying and operating the machines stay the same.

[10] The nominal discount rate is

$$r_{\text{nominal}} = (1 + r_{\text{real}})(1 + \text{inflation rate}) - 1$$
$$= (1.06)(1.05) - 1 = .113, \text{ or } 11.3\%$$

Discounting the nominal annuities at this rate gives the same present values as discounting the real annuities at 6%.

[11] Do *not* calculate equivalent annual cash flows as level *nominal* annuities. This procedure can give incorrect rankings of true equivalent annual flows at high inflation rates. See Challenge Question 32 at the end of this chapter for an example.

Suppose that this is not the case. Suppose that thanks to technological improvements new machines each year cost 20% less in real terms to buy and operate. In this case future owners of brand-new, lower-cost machines will be able to cut their rental cost by 20%, and owners of old machines will be forced to match this reduction. Thus, we now need to ask: if the real level of rents declines by 20% a year, how much will it cost to rent each machine?

If the rent for year 1 is rent_1, rent for year 2 is $\text{rent}_2 = .8 \times \text{rent}_1$. Rent_3 is $.8 \times \text{rent}_2$, or $.64 \times \text{rent}_1$. The owner of each machine must set the rents sufficiently high to recover the present value of the costs. In the case of machine A,

$$\text{PV of renting machine A} = \frac{\text{rent}_1}{1.06} + \frac{\text{rent}_2}{(1.06)^2} + \frac{\text{rent}_3}{(1.06)^3} = 28.37$$

$$= \frac{\text{rent}_1}{1.06} + \frac{.8(\text{rent}_1)}{(1.06)^2} + \frac{.64(\text{rent}_1)}{(1.06)^3} = 28.37$$

$$\text{rent}_1 = 12.94, \text{ or } \$12{,}940$$

For machine B,

$$\text{PV of renting machine B} = \frac{\text{rent}_1}{1.06} + \frac{.8(\text{rent}_1)}{(1.06)^2} = 21.00$$

$$\text{rent}_1 = 12.69, \text{ or } \$12{,}690$$

The merits of the two machines are now reversed. Once we recognize that technology is expected to reduce the real costs of new machines, then it pays to buy the shorter-lived machine B rather than become locked into an aging technology with machine A in year 3.

You can imagine other complications. Perhaps machine C will arrive in year 1 with an even lower equivalent annual cost. You would then need to consider scrapping or selling machine B at year 1 (more on this decision below). The financial manager could not choose between machines A and B in year 0 without taking a detailed look at what each machine could be replaced with.

Comparing equivalent annual cash flow should never be a mechanical exercise; always think about the assumptions that are implicit in the comparison. Finally, remember why equivalent annual cash flows are necessary in the first place. The reason is that A and B will be replaced at different future dates. The choice between them therefore affects future investment decisions. If subsequent decisions are not affected by the initial choice (for example, because neither machine will be replaced) then we do *not need to take future decisions into account.*[12]

Equivalent Annual Cash Flow and Taxes We have not mentioned taxes. But you surely realized that machine A and B's lifetime costs should be calculated after-tax, recognizing that operating costs are tax-deductible and that capital investment generates depreciation tax shields.

Deciding When to Replace an Existing Machine

The previous example took the life of each machine as fixed. In practice the point at which equipment is replaced reflects economic considerations rather than total physical collapse. *We* must decide when to replace. The machine will rarely decide for us.

Here is a common problem. You are operating an elderly machine that is expected to produce a net cash *inflow* of $4,000 in the coming year and $4,000 next year. After that it

[12] However, if neither machine will be replaced, then we have to consider the extra revenue generated by machine A in its third year, when it will be operating but B will not.

will give up the ghost. You can replace it now with a new machine, which costs $15,000 but is much more efficient and will provide a cash inflow of $8,000 a year for three years. You want to know whether you should replace your equipment now or wait a year.

We can calculate the NPV of the new machine and also its *equivalent annual cash flow,* that is, the three-year annuity that has the same net present value:

Cash Flows ($ thousands)					
	C_0	C_1	C_2	C_3	NPV at 6% ($ thousands)
New machine	− 15	+8	+8	+8	6.38
Equivalent annual cash flow		+2.387	+2.387	+2.387	6.38

In other words, the cash flows of the new machine are equivalent to an annuity of $2,387 per year. So we can equally well ask at what point we would want to replace our old machine with a new one producing $2,387 a year. When the question is put this way, the answer is obvious. As long as your old machine can generate a cash flow of $4,000 a year, who wants to put in its place a new one that generates only $2,387 a year?

It is a simple matter to incorporate salvage values into this calculation. Suppose that the current salvage value is $8,000 and next year's value is $7,000. Let us see where you come out next year if you wait and then sell. On one hand, you gain $7,000, but you lose today's salvage value *plus* a year's return on that money. That is, 8,000 × 1.06 = $8,480. Your net loss is 8,480 − 7,000 = $1,480, which only partly offsets the operating gain. You should not replace yet.

Remember that the logic of such comparisons requires that the new machine be the best of the available alternatives and that it in turn be replaced at the optimal point.

Cost of Excess Capacity Any firm with a centralized information system (computer servers, storage, software, and telecommunication links) encounters many proposals for using it. Recently installed systems tend to have excess capacity, and since the immediate marginal costs of using them seem to be negligible, management often encourages new uses. Sooner or later, however, the load on a system increases to the point at which management must either terminate the uses it originally encouraged or invest in another system several years earlier than it had planned. Such problems can be avoided if a proper charge is made for the use of spare capacity.

Suppose we have a new investment project that requires heavy use of an existing information system. The effect of adopting the project is to bring the purchase date of a new, more capable system forward from year 4 to year 3. This new system has a life of five years, and at a discount rate of 6% the present value of the cost of buying and operating it is $500,000.

We begin by converting the $500,000 present value of the cost of the new system to an equivalent annual cost of $118,700 for each of five years.[13] Of course, when the new system in turn wears out, we will replace it with another. So we face the prospect of future information-system expenses of $118,700 a year. If we undertake the new project, the series of expenses begins in year 4; if we do not undertake it, the series begins in year 5. The new project, therefore, results in an *additional* cost of $118,700 in year 4. This has a present value of $118{,}700/(1.06)^4$, or about $94,000. This cost is properly charged against the new project. When we recognize it, the NPV of the project may prove to be negative. If so, we still need to check whether it is worthwhile undertaking the project now and abandoning it later, when the excess capacity of the present system disappears.

[13] The present value of $118,700 a year for five years discounted at 6% is $500,000.

SUMMARY

By now present value calculations should be a matter of routine. However, forecasting project cash flows will never be routine. Here is a checklist that will help you to avoid mistakes:

1. Discount cash flows, not profits.
   a. Remember that depreciation is not a cash flow (though it may affect tax payments).
   b. Concentrate on cash flows after taxes. Stay alert for differences between tax depreciation and depreciation used in reports to shareholders.
   c. Exclude debt interest or the cost of repaying a loan from the project cash flows. This enables you to separate the investment from the financing decision.
   d. Remember the investment in working capital. As sales increase, the firm may need to make additional investments in working capital, and as the project comes to an end, it will recover those investments.
   e. Beware of allocated overhead charges for heat, light, and so on. These may not reflect the incremental costs of the project.

2. Estimate the project's *incremental* cash flows—that is, the difference between the cash flows with the project and those without the project.
   a. Include all indirect effects of the project, such as its impact on the sales of the firm's other products.
   b. Forget sunk costs.
   c. Include *opportunity costs,* such as the value of land that you would otherwise sell.

3. Treat inflation consistently.
   a. If cash flows are forecasted in nominal terms, use a nominal discount rate.
   b. Discount real cash flows at a real rate.

These principles of valuing capital investments are the same worldwide, but inputs and assumptions vary by country and currency. For example, cash flows from a project in Germany would be in euros, not dollars, and would be forecasted after German taxes.

When we assessed the guano project, we transformed the series of future cash flows into a single measure of their present value. Sometimes it is useful to reverse this calculation and to convert the present value into a stream of annual cash flows. For example, when choosing between two machines with unequal lives, you need to compare equivalent annual cash flows. Remember, though, to calculate equivalent annual cash flows in real terms and adjust for technological change if necessary.

PROBLEM SETS

BASIC

1. Which of the following should be treated as incremental cash flows when deciding whether to invest in a new manufacturing plant? The site is already owned by the company, but existing buildings would need to be demolished.
   a. The market value of the site and existing buildings.
   b. Demolition costs and site clearance.
   c. The cost of a new access road put in last year.
   d. Lost earnings on other products due to executive time spent on the new facility.
   e. A proportion of the cost of leasing the president's jet airplane.

f. Future depreciation of the new plant.
g. The reduction in the corporation's tax bill resulting from tax depreciation of the new plant.
h. The initial investment in inventories of raw materials.
i. Money already spent on engineering design of the new plant.

2. Mr. Art Deco will be paid \$100,000 one year hence. This is a nominal flow, which he discounts at an 8% nominal discount rate:

$$PV = \frac{100{,}000}{1.08} = \$92{,}593$$

The inflation rate is 4%.

Calculate the PV of Mr. Deco's payment using the equivalent *real* cash flow and *real* discount rate. (You should get exactly the same answer as he did.)

3. True or false?

a. A project's depreciation tax shields depend on the actual future rate of inflation.
b. Project cash flows should take account of interest paid on any borrowing undertaken to finance the project.
c. In the U.S., income reported to the tax authorities must equal income reported to shareholders.
d. Accelerated depreciation reduces near-term project cash flows and therefore reduces project NPV.

4. How does the PV of depreciation tax shields vary across the recovery-period classes shown in Table 6.4? Give a general answer; then check it by calculating the PVs of depreciation tax shields in the five-year and seven-year classes. The tax rate is 35% and the discount rate is 10%.

5. The following table tracks the main components of working capital over the life of a four-year project.

	2010	2011	2012	2013	2014
Accounts receivable	0	150,000	225,000	190,000	0
Inventory	75,000	130,000	130,000	95,000	0
Accounts payable	25,000	50,000	50,000	35,000	0

Calculate net working capital and the cash inflows and outflows due to investment in working capital.

6. When appraising mutually exclusive investments in plant and equipment, financial managers calculate the investments' equivalent annual costs and rank the investments on this basis. Why is this necessary? Why not just compare the investments' NPVs? Explain briefly.

7. Air conditioning for a college dormitory will cost \$1.5 million to install and \$200,000 per year to operate. The system should last 25 years. The real cost of capital is 5%, and the college pays no taxes. What is the equivalent annual cost?

8. Machines A and B are mutually exclusive and are expected to produce the following real cash flows:

	Cash Flows (\$ thousands)			
Machine	C_0	C_1	C_2	C_3
A	−100	+110	+121	
B	−120	+110	+121	+133

The real opportunity cost of capital is 10%.

a. Calculate the NPV of each machine.

b. Calculate the equivalent annual cash flow from each machine.

c. Which machine should you buy?

9. Machine C was purchased five years ago for $200,000 and produces an annual real cash flow of $80,000. It has no salvage value but is expected to last another five years. The company can replace machine C with machine B (see Problem 8) *either* now *or* at the end of five years. Which should it do?

INTERMEDIATE

10. Restate the net cash flows in Table 6.6 in real terms. Discount the restated cash flows at a real discount rate. Assume a 20% *nominal* rate and 10% expected inflation. NPV should be unchanged at +3,802, or $3,802,000.

11. CSC is evaluating a new project to produce encapsulators. The initial investment in plant and equipment is $500,000. Sales of encapsulators in year 1 are forecasted at $200,000 and costs at $100,000. Both are expected to increase by 10% a year in line with inflation. Profits are taxed at 35%. Working capital in each year consists of inventories of raw materials and is forecasted at 20% of sales in the following year.

The project will last five years and the equipment at the end of this period will have no further value. For tax purposes the equipment can be depreciated straight-line over these five years. If the nominal discount rate is 15%, show that the net present value of the project is the same whether calculated using real cash flows or nominal flows.

12. In 1898 Simon North announced plans to construct a funeral home on land he owned and rented out as a storage area for railway carts. (A local newspaper commended Mr. North for not putting the cart before the hearse.) Rental income from the site barely covered real estate taxes, but the site was valued at $45,000. However, Mr. North had refused several offers for the land and planned to continue renting it out if for some reason the funeral home was not built. Therefore he did not include the value of the land as an outlay in his NPV analysis of the funeral home. Was this the correct procedure? Explain.

13. Each of the following statements is true. Explain why they are consistent.

a. When a company introduces a new product, or expands production of an existing product, investment in net working capital is usually an important cash outflow.

b. Forecasting changes in net working capital is not necessary if the timing of *all* cash inflows and outflows is carefully specified.

14. Ms. T. Potts, the treasurer of Ideal China, has a problem. The company has just ordered a new kiln for $400,000. Of this sum, $50,000 is described by the supplier as an installation cost. Ms. Potts does not know whether the Internal Revenue Service (IRS) will permit the company to treat this cost as a tax-deductible current expense or as a capital investment. In the latter case, the company could depreciate the $50,000 using the five-year MACRS tax depreciation schedule. How will the IRS's decision affect the after-tax cost of the kiln? The tax rate is 35% and the opportunity cost of capital is 5%.

15. After spending $3 million on research, Better Mousetraps has developed a new trap. The project requires an initial investment in plant and equipment of $6 million. This investment will be depreciated straight-line over five years to a value of zero, but, when the project comes to an end in five years, the equipment can in fact be sold for $500,000. The firm believes that working capital at each date must be maintained at 10% of next year's forecasted sales. Production costs are estimated at $1.50 per trap and the traps will be sold for $4 each. (There are no marketing expenses.) Sales forecasts are given in the following table. The firm pays tax at 35% and the required return on the project is 12%. What is the NPV?

Year:	0	1	2	3	4	5
Sales (millions of traps)	0	.5	.6	1.0	1.0	.6

16. A project requires an initial investment of $100,000 and is expected to produce a cash inflow before tax of $26,000 per year for five years. Company A has substantial accumulated tax losses and is unlikely to pay taxes in the foreseeable future. Company B pays corporate taxes at a rate of 35% and can depreciate the investment for tax purposes using the five-year MACRS tax depreciation schedule. Suppose the opportunity cost of capital is 8%. Ignore inflation.

a. Calculate project NPV for each company.

b. What is the IRR of the after-tax cash flows for each company? What does comparison of the IRRs suggest is the effective corporate tax rate?

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17. Go to the "live" Excel spreadsheet versions of Tables 6.1, 6.5, and 6.6 at **www.mhhe.com/bma** and answer the following questions.

a. How does the guano project's NPV change if IM&C is forced to use the seven-year MACRS tax depreciation schedule?

b. New engineering estimates raise the possibility that capital investment will be more than $10 million, perhaps as much as $15 million. On the other hand, you believe that the 20% cost of capital is unrealistically high and that the true cost of capital is about 11%. Is the project still attractive under these alternative assumptions?

c. Continue with the assumed $15 million capital investment and the 11% cost of capital. What if sales, cost of goods sold, and net working capital are each 10% higher in every year? Recalculate NPV. (*Note:* Enter the revised sales, cost, and working-capital forecasts in the spreadsheet for Table 6.1.)

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18. A widget manufacturer currently produces 200,000 units a year. It buys widget lids from an outside supplier at a price of $2 a lid. The plant manager believes that it would be cheaper to make these lids rather than buy them. Direct production costs are estimated to be only $1.50 a lid. The necessary machinery would cost $150,000 and would last 10 years. This investment could be written off for tax purposes using the seven-year tax depreciation schedule. The plant manager estimates that the operation would require additional working capital of $30,000 but argues that this sum can be ignored since it is recoverable at the end of the 10 years. If the company pays tax at a rate of 35% and the opportunity cost of capital is 15%, would you support the plant manager's proposal? State clearly any additional assumptions that you need to make.

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19. Reliable Electric is considering a proposal to manufacture a new type of industrial electric motor which would replace most of its existing product line. A research breakthrough has given Reliable a two-year lead on its competitors. The project proposal is summarized in Table 6.7 on the next page.

a. Read the notes to the table carefully. Which entries make sense? Which do not? Why or why not?

b. What additional information would you need to construct a version of Table 6.7 that makes sense?

c. Construct such a table and recalculate NPV. Make additional assumptions as necessary.

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20. Marsha Jones has bought a used Mercedes horse transporter for her Connecticut estate. It cost $35,000. The object is to save on horse transporter rentals.

Marsha had been renting a transporter every other week for $200 per day plus $1.00 per mile. Most of the trips are 80 or 100 miles in total. Marsha usually gives the driver a $40 tip. With the new transporter she will only have to pay for diesel fuel and maintenance, at about $.45 per mile. Insurance costs for Marsha's transporter are $1,200 per year.

The transporter will probably be worth $15,000 (in real terms) after eight years, when Marsha's horse Nike will be ready to retire. Is the transporter a positive-NPV investment? Assume a nominal discount rate of 9% and a 3% forecasted inflation rate. Marsha's transporter is a personal outlay, not a business or financial investment, so taxes can be ignored.

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	2009	2010	2011	2012–2019
1. Capital expenditure	−10,400			
2. Research and development	−2,000			
3. Working capital	−4,000			
4. Revenue		8,000	16,000	40,000
5. Operating costs		−4,000	−8,000	−20,000
6. Overhead		−800	−1,600	−4,000
7. Depreciation		−1,040	−1,040	−1,040
8. Interest		−2,160	−2,160	−2,160
9. Income	−2,000	0	3,200	12,800
10. Tax	0	0	420	4,480
11. Net cash flow	−16,400	0	2,780	8,320
12. Net present value = +13,932				

TABLE 6.7 Cash flows and present value of Reliable Electric's proposed investment ($ thousands). See Problem 19.

Notes:
1. *Capital expenditure:* $8 million for new machinery and $2.4 million for a warehouse extension. The full cost of the extension has been charged to this project, although only about half of the space is currently needed. Since the new machinery will be housed in an existing factory building, no charge has been made for land and building.
2. *Research and development:* $1.82 million spent in 2008. This figure was corrected for 10% inflation from the time of expenditure to date. Thus $1.82 \times 1.1 = \$2$ million.
3. *Working capital:* Initial investment in inventories.
4. *Revenue:* These figures assume sales of 2,000 motors in 2010, 4,000 in 2011, and 10,000 per year from 2012 through 2019. The initial unit price of $4,000 is forecasted to remain constant in real terms.
5. *Operating costs:* These include all direct and indirect costs. Indirect costs (heat, light, power, fringe benefits, etc.) are assumed to be 200% of direct labor costs. Operating costs per unit are forecasted to remain constant in real terms at $2,000.
6. *Overhead:* Marketing and administrative costs, assumed equal to 10% of revenue.
7. *Depreciation:* Straight-line for 10 years.
8. *Interest:* Charged on capital expenditure and working capital at Reliable's current borrowing rate of 15%.
9. *Income:* Revenue less the sum of research and development, operating costs, overhead, depreciation, and interest.
10. *Tax:* 35% of income. However, income is negative in 2009. This loss is carried forward and deducted from taxable income in 2011.
11. *Net cash flow:* Assumed equal to income less tax.
12. *Net present value:* NPV of net cash flow at a 15% discount rate.

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21. United Pigpen is considering a proposal to manufacture high-protein hog feed. The project would make use of an existing warehouse, which is currently rented out to a neighboring firm. The next year's rental charge on the warehouse is $100,000, and thereafter the rent is expected to grow in line with inflation at 4% a year. In addition to using the warehouse, the proposal envisages an investment in plant and equipment of $1.2 million. This could be depreciated for tax purposes straight-line over 10 years. However, Pigpen expects to terminate the project at the end of eight years and to resell the plant and equipment in year 8 for $400,000. Finally, the project requires an initial investment in working capital of $350,000. Thereafter, working capital is forecasted to be 10% of sales in each of years 1 through 7.

Year 1 sales of hog feed are expected to be $4.2 million, and thereafter sales are forecasted to grow by 5% a year, slightly faster than the inflation rate. Manufacturing costs are expected to be 90% of sales, and profits are subject to tax at 35%. The cost of capital is 12%. What is the NPV of Pigpen's project?

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22. Hindustan Motors has been producing its Ambassador car in India since 1948. As the company's Web site explains, the Ambassador's "dependability, spaciousness and comfort factor have made it the most preferred car for generations of Indians." Hindustan is now considering producing the car in China. This will involve an initial investment of RMB 4 billion.[14] The plant will start production after one year. It is expected to last for five years and have a salvage value at the end of this period of RMB 500 million in real terms. The plant will produce 100,000 cars a year. The firm anticipates that in the first year it will be able to sell each car for RMB 65,000, and thereafter the price is expected to increase by 4% a year. Raw materials for each car are forecasted to cost RMB 18,000 in the first year and these costs are predicted to increase by 3% annually. Total labor costs for the plant are expected to be RMB 1.1 billion in the first year and thereafter will increase by 7% a year. The land on which the plant is built can be rented for five years at a fixed cost of RMB 300 million a year payable at the *beginning* of each year. Hindustan's discount rate for this type of project is 12% (nominal). The expected rate of inflation is 5%. The plant can be depreciated straight-line over the five-year period and profits will be taxed at 25%. Assume all cash flows occur at the end of each year except where otherwise stated. What is the NPV of the plant?

23. In the International Mulch and Compost example (Section 6.2), we assumed that losses on the project could be used to offset taxable profits elswhere in the corporation. Suppose that the losses had to be carried forward and offset against future taxable profits from the project. How would the project NPV change? What is the value of the company's ability to use the tax deductions immediately?

24. As a result of improvements in product engineering, United Automation is able to sell one of its two milling machines. Both machines perform the same function but differ in age. The newer machine could be sold today for $50,000. Its operating costs are $20,000 a year, but in five years the machine will require a $20,000 overhaul. Thereafter operating costs will be $30,000 until the machine is finally sold in year 10 for $5,000.

The older machine could be sold today for $25,000. If it is kept, it will need an immediate $20,000 overhaul. Thereafter operating costs will be $30,000 a year until the machine is finally sold in year 5 for $5,000.

Both machines are fully depreciated for tax purposes. The company pays tax at 35%. Cash flows have been forecasted in real terms. The real cost of capital is 12%. Which machine should United Automation sell? Explain the assumptions underlying your answer.

25. Low-energy lightbulbs cost $3.50, have a life of nine years, and use about $1.60 of electricity a year. Conventional lightbulbs cost only $.50, but last only about a year and use about $6.60 of energy. If the real discount rate is 5%, what is the equivalent annual cost of the two products?

26. Hayden Inc. has a number of copiers that were bought four years ago for $20,000. Currently maintenance costs $2,000 a year, but the maintenance agreement expires at the end of two years and thereafter the annual maintenance charge will rise to $8,000. The machines have a current resale value of $8,000, but at the end of year 2 their value will have fallen to $3,500. By the end of year 6 the machines will be valueless and would be scrapped.

Hayden is considering replacing the copiers with new machines that would do essentially the same job. These machines cost $25,000, and the company can take out an eight-year maintenance contract for $1,000 a year. The machines will have no value by the end of the eight years and will be scrapped.

Both machines are depreciated by using seven-year MACRS, and the tax rate is 35%. Assume for simplicity that the inflation rate is zero. The real cost of capital is 7%. When should Hayden replace its copiers?

27. Return to the start of Section 6-4, where we calculated the equivalent annual cost of producing reformulated gasoline in California. Capital investment was $400 million. Suppose this amount can be depreciated for tax purposes on the 10-year MACRS schedule from Table 6.4. The marginal tax rate, including California taxes, is 39%, the cost of

[14] The Renminbi (RMB) is the Chinese currency.

capital is 7%, and there is no inflation. The refinery improvements have an economic life of 25 years.

a. Calculate the after-tax equivalent annual cost. (*Hint:* It's easiest to use the PV of depreciation tax shields as an offset to the initial investment).

b. How much extra would retail gasoline customers have to pay to cover this equivalent annual cost? (*Note:* Extra income from higher retail prices would be taxed.)

28. The Borstal Company has to choose between two machines that do the same job but have different lives. The two machines have the following costs:

Year	Machine A	Machine B
0	$40,000	$50,000
1	10,000	8,000
2	10,000	8,000
3	10,000 + replace	8,000
4		8,000 + replace

These costs are expressed in real terms.

a. Suppose you are Borstal's financial manager. If you had to buy one or the other machine and rent it to the production manager for that machine's economic life, what annual rental payment would you have to charge? Assume a 6% real discount rate and ignore taxes.

b. Which machine should Borstal buy?

c. Usually the rental payments you derived in part (a) are just hypothetical—a way of calculating and interpreting equivalent annual cost. Suppose you actually do buy one of the machines and rent it to the production manager. How much would you actually have to charge in each future year if there is steady 8% per year inflation? (*Note:* The rental payments calculated in part (a) are real cash flows. You would have to mark up those payments to cover inflation.)

29. Look again at your calculations for Problem 28 above. Suppose that technological change is expected to reduce costs by 10% per year. There will be new machines in year 1 that cost 10% less to buy and operate than A and B. In year 2 there will be a second crop of new machines incorporating a further 10% reduction, and so on. How does this change the equivalent annual costs of machines A and B?

30. The president's executive jet is not fully utilized. You judge that its use by other officers would increase direct operating costs by only $20,000 a year and would save $100,000 a year in airline bills. On the other hand, you believe that with the increased use the company will need to replace the jet at the end of three years rather than four. A new jet costs $1.1 million and (at its current low rate of use) has a life of six years. Assume that the company does not pay taxes. All cash flows are forecasted in real terms. The real opportunity cost of capital is 8%. Should you try to persuade the president to allow other officers to use the plane?

CHALLENGE

31. One measure of the effective tax rate is the difference between the IRRs of pretax and after-tax cash flows, divided by the pretax IRR. Consider, for example, an investment I generating a perpetual stream of pretax cash flows C. The pretax IRR is C/I, and the after-tax IRR is $C(1 - T_C)/I$, where T_C is the statutory tax rate. The effective rate, call it T_E, is

$$T_E = \frac{C/I - C(1 - T_c)/I}{C/I} = T_c$$

In this case the effective rate equals the statutory rate.

a. Calculate T_E for the guano project in Section 6.2.

b. How does the effective rate depend on the tax depreciation schedule? On the inflation rate?

c. Consider a project where all of the up-front investment is treated as an expense for tax purposes. For example, R&D and marketing outlays are always expensed in the United States. They create no tax depreciation. What is the effective tax rate for such a project?

32. We warned that equivalent annual costs should be calculated in real terms. We did not fully explain why. This problem will show you.

Look back to the cash flows for machines A and B (in "Choosing between Long- and Short-Lived Equipment"). The present values of purchase and operating costs are 28.37 (over three years for A) and 21.00 (over two years for B). The real discount rate is 6% and the inflation rate is 5%.

a. Calculate the three- and two-year *level nominal* annuities which have present values of 28.37 and 21.00. Explain why these annuities are *not* realistic estimates of equivalent annual costs. (*Hint:* In real life machinery rentals increase with inflation.)

b. Suppose the inflation rate increases to 25%. The real interest rate stays at 6%. Recalculate the level nominal annuities. Note that the *ranking* of machines A and B appears to change. Why?

33. In December 2005 Mid-American Energy brought online one of the largest wind farms in the world. It cost an estimated \$386 million and the 257 turbines have a total capacity of 360.5 megawatts (mW). Wind speeds fluctuate and most wind farms are expected to operate at an average of only 35% of their rated capacity. In this case, at an electricity price of \$55 per megawatt-hour (mWh), the project will produce revenues in the first year of \$60.8 million (i.e., .35 × 8,760 hours × 360.5 mW × \$55 per mWh). A reasonable estimate of maintenance and other costs is about \$18.9 million in the first year of operation. Thereafter, revenues and costs should increase with inflation by around 3% a year.

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Conventional power stations can be depreciated using 20-year MACRS, and their profits are taxed at 35%. Suppose that the project will last 20 years and the cost of capital is 12%. To encourage renewable energy sources, the government offers several tax breaks for wind farms.

a. How large a tax break (if any) was needed to make Mid-American's investment a positive-NPV venture?

b. Some wind farm operators assume a capacity factor of 30% rather than 35%. How would this lower capacity factor alter project NPV?

MINI-CASE

New Economy Transport (A)

The New Economy Transport Company (NETCO) was formed in 1955 to carry cargo and passengers between ports in the Pacific Northwest and Alaska. By 2008 its fleet had grown to four vessels, including a small dry-cargo vessel, the *Vital Spark.*

The *Vital Spark* is 25 years old and badly in need of an overhaul. Peter Handy, the finance director, has just been presented with a proposal that would require the following expenditures:

Overhaul engine and generators	\$340,000
Replace radar and other electronic equipment	75,000
Repairs to hull and superstructure	310,000
Painting and other repairs	95,000
	\$820,000

Mr. Handy believes that all these outlays could be depreciated for tax purposes in the seven-year MACRS class.

NETCO's chief engineer, McPhail, estimates the postoverhaul operating costs as follows:

Fuel	$ 450,000
Labor and benefits	480,000
Maintenance	141,000
Other	110,000
	$1,181,000

These costs generally increase with inflation, which is forecasted at 2.5% a year.

The *Vital Spark* is carried on NETCO's books at a net depreciated value of only $100,000, but could probably be sold "as is," along with an extensive inventory of spare parts, for $200,000. The book value of the spare parts inventory is $40,000. Sale of the *Vital Spark* would generate an immediate tax liability on the difference between sale price and book value.

The chief engineer also suggests installation of a brand-new engine and control system, which would cost an extra $600,000.[15] This additional equipment would not substantially improve the *Vital Spark*'s performance, but would result in the following reduced annual fuel, labor, and maintenance costs:

Fuel	$ 400,000
Labor and benefits	405,000
Maintenance	105,000
Other	110,000
	$1,020,000

Overhaul of the *Vital Spark* would take it out of service for several months. The overhauled vessel would resume commercial service next year. Based on past experience, Mr. Handy believes that it would generate revenues of about $1.4 million next year, increasing with inflation thereafter.

But the *Vital Spark* cannot continue forever. Even if overhauled, its useful life is probably no more than 10 years, 12 years at the most. Its salvage value when finally taken out of service will be trivial.

NETCO is a conservatively financed firm in a mature business. It normally evaluates capital investments using an 11% cost of capital. This is a nominal, not a real, rate. NETCO's tax rate is 35%.

QUESTION

1. Calculate the NPV of the proposed overhaul of the *Vital Spark*, with and without the new engine and control system. To do the calculation, you will have to prepare a spreadsheet table showing all costs after taxes over the vessel's remaining economic life. Take special care with your assumptions about depreciation tax shields and inflation.

New Economy Transport (B)

There is no question that the *Vital Spark* needs an overhaul soon. However, Mr. Handy feels it unwise to proceed without also considering the purchase of a new vessel. Cohn and Doyle, Inc., a Wisconsin shipyard, has approached NETCO with a design incorporating a Kort nozzle, extensively automated navigation and power control systems, and much more

[15] This additional outlay would also qualify for tax depreciation in the seven-year MACRS class.

comfortable accommodations for the crew. Estimated annual operating costs of the new vessel are:

Fuel	$380,000
Labor and benefits	330,000
Maintenance	70,000
Other	105,000
	$885,000

The crew would require additional training to handle the new vessel's more complex and sophisticated equipment. Training would probably cost $50,000 next year.

The estimated operating costs for the new vessel assume that it would be operated in the same way as the *Vital Spark*. However, the new vessel should be able to handle a larger load on some routes, which could generate additional revenues, net of additional out-of-pocket costs, of as much as $100,000 per year. Moreover, a new vessel would have a useful service life of 20 years or more.

Cohn and Doyle offered the new vessel for a fixed price of $3,000,000, payable half immediately and half on delivery next year.

Mr. Handy stepped out on the foredeck of the *Vital Spark* as she chugged down the Cook Inlet. "A rusty old tub," he muttered, "but she's never let us down. I'll bet we could keep her going until next year while Cohn and Doyle are building her replacement. We could use up the spare parts to keep her going. We might even be able to sell or scrap her for book value when her replacement arrives.

"But how do I compare the NPV of a new ship with the old *Vital Spark?* Sure, I could run a 20-year NPV spreadsheet, but I don't have a clue how the replacement will be used in 2023 or 2028. Maybe I could compare the overall *cost* of overhauling and operating the *Vital Spark* to the cost of buying and operating the proposed replacement."

QUESTIONS

1. Calculate and compare the equivalent annual costs of (a) overhauling and operating the *Vital Spark* for 12 more years, and (b) buying and operating the proposed replacement vessel for 20 years. What should Mr. Handy do if the replacement's annual costs are the same or lower?
2. Suppose the replacement's equivalent annual costs are higher than the *Vital Spark*'s. What additional information should Mr. Handy seek in this case?