University of Ferrara

DIPARTIMENTO DI ECONOMIA E MANAGEMENT

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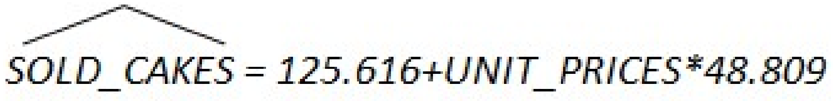
Simple Linear Regression Analysis: What about the inference about the correlation coefficient?

Lecture 6
2019, Feb 22nd



$$\widehat{SOLD\_CAKES} = 125.616 + UNIT\_PRICES * 48.809$$

The state of the art:

-We are studying **the linear relationship** between unit-price (x) and number of total cakes sold per week (y).

-The **goodness of fit (R^2)** is 0.4588: thus the 45.88% of total variance in y is explained by our model (on the other side, the 54.12% of that variance is still unexplained!)

- We **graphically tested the 4 main linear regression conditions** (plot: error terms and its relationship with the explanatory variable)

- We tested the linear relationship between x and y within the reality as a whole (and we make inference)

Our final aim is to understand whether the correlation (with that given strength) between x and y
does exist within the population as a whole

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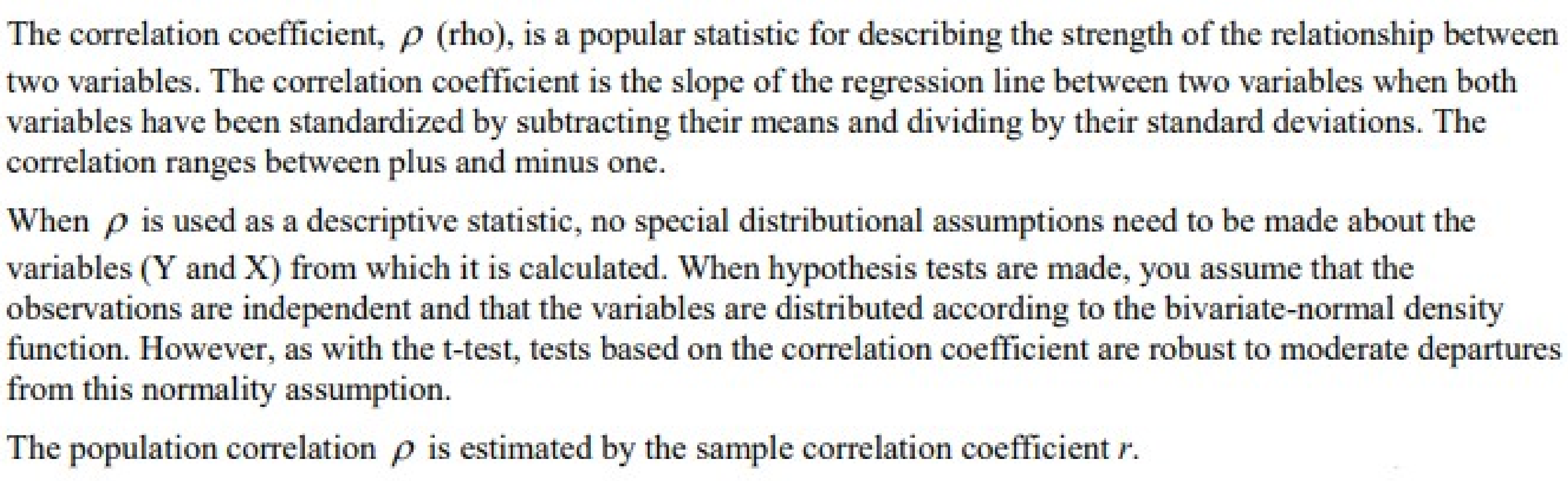
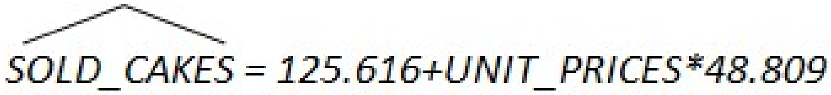
We use the Pearson's correlation test



The correlation coefficient, ρ (rho), is a popular statistic for describing the strength of the relationship between two variables. The correlation coefficient is the slope of the regression line between two variables when both variables have been standardized by subtracting their means and dividing by their standard deviations. The correlation ranges between plus and minus one.

When ρ is used as a descriptive statistic, no special distributional assumptions need to be made about the variables (Y and X) from which it is calculated. When hypothesis tests are made, you assume that the observations are independent and that the variables are distributed according to the bivariate-normal density function. However, as with the t-test, tests based on the correlation coefficient are robust to moderate departures from this normality assumption.

The population correlation ρ is estimated by the sample correlation coefficient r.



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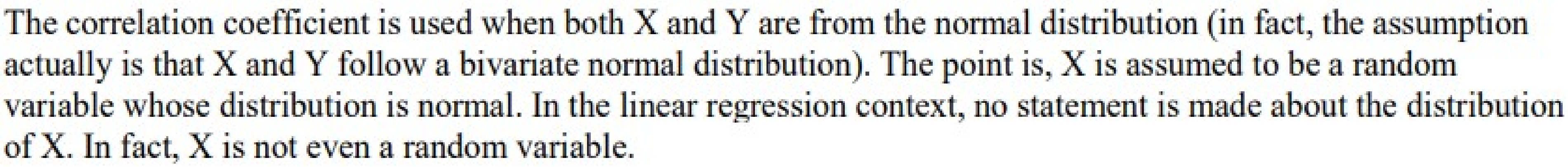
Note: difference between linear relation and correlation



The correlation coefficient is used when both X and Y are from the normal distribution (in fact, the assumption actually is that X and Y follow a bivariate normal distribution). The point is, X is assumed to be a random variable whose distribution is normal. In the linear regression context, no statement is made about the distribution of X. In fact, X is not even a random variable.

Always remember:
correlation doesn’t imply causal relationship

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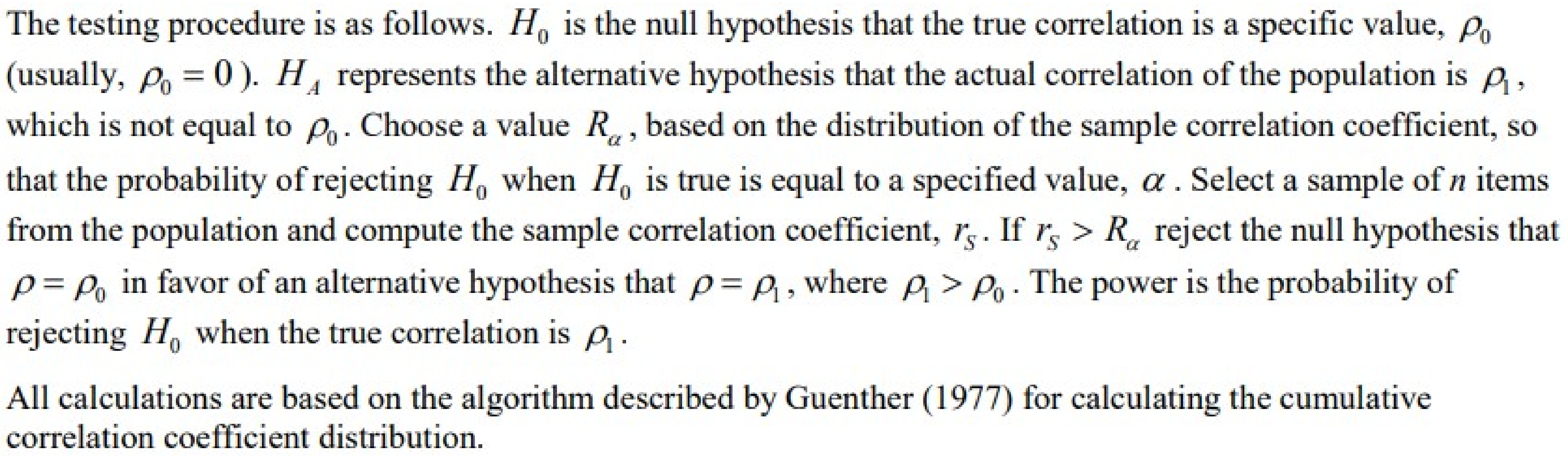
Test Procedure



The testing procedure is as follows. H_0 is the null hypothesis that the true correlation is a specific value, ρ_0 (usually, $\rho_0 = 0$). H_A represents the alternative hypothesis that the actual correlation of the population is ρ_1, which is not equal to ρ_0. Choose a value R_α, based on the distribution of the sample correlation coefficient, so that the probability of rejecting H_0 when H_0 is true is equal to a specified value, α . Select a sample of n items from the population and compute the sample correlation coefficient, r_S. If $r_S > R_\alpha$ reject the null hypothesis that $\rho = \rho_0$ in favor of an alternative hypothesis that $\rho = \rho_1$, where $\rho_1 > \rho_0$. The power is the probability of rejecting H_0 when the true correlation is ρ_1.

All calculations are based on the algorithm described by Guenther (1977) for calculating the cumulative correlation coefficient distribution.

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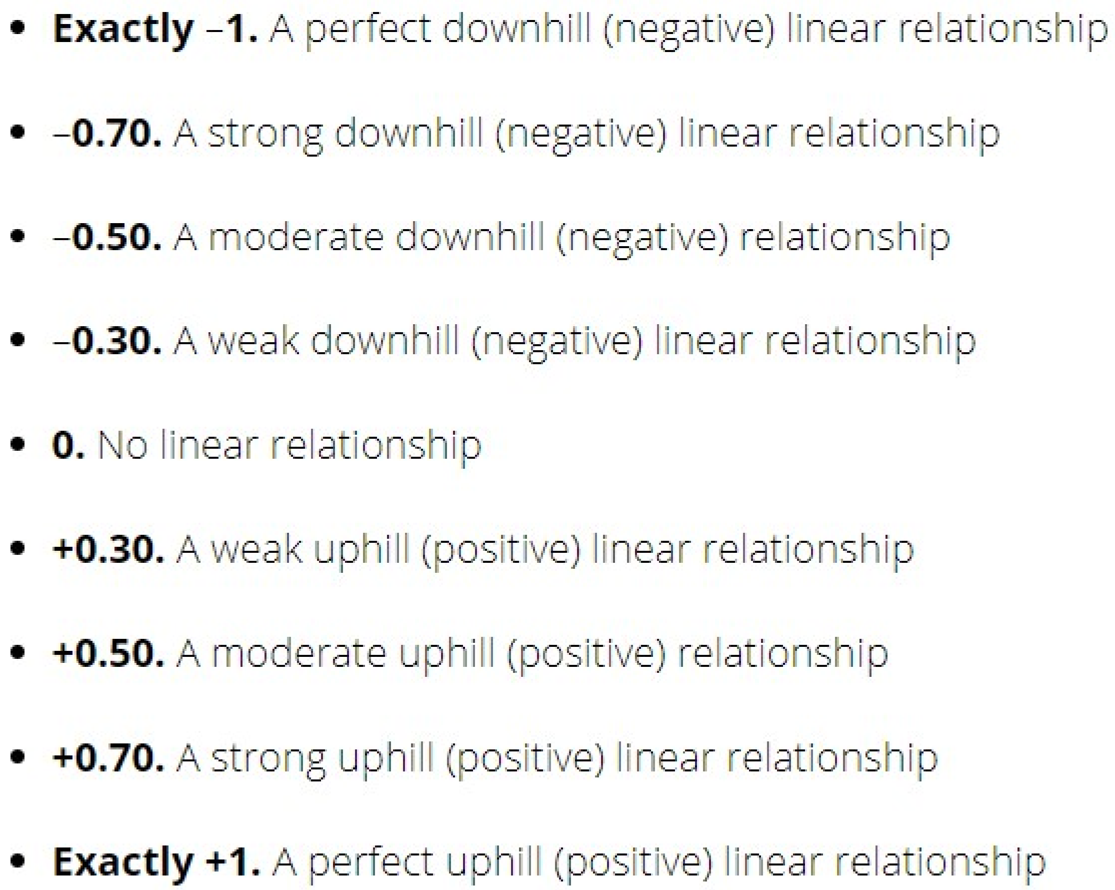
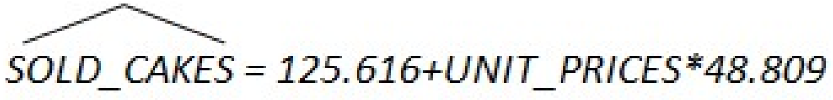
Test Procedure

We use the notation *r* to describe the correlation coefficient in our sample

Its value is always between -1 an 1, i.e.:



- **Exactly –1.** A perfect downhill (negative) linear relationship
- **–0.70.** A strong downhill (negative) linear relationship
- **–0.50.** A moderate downhill (negative) relationship
- **–0.30.** A weak downhill (negative) linear relationship
- **0.** No linear relationship
- **+0.30.** A weak uphill (positive) linear relationship
- **+0.50.** A moderate uphill (positive) relationship
- **+0.70.** A strong uphill (positive) linear relationship
- **Exactly +1.** A perfect uphill (positive) linear relationship



$$\widehat{SOLD\_CAKES} = 125.616+UNIT\_PRICES*48.809$$

Test Procedure

We test the null/alternative hypothesis comparing the t-statistic.r and a critical value (c.v.)

Decision rule:

T-stat > $t_{\alpha/2}$ → we reject the null hypothesis

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Testing the null/alternative hypothesis : test rho for a population correlation

Central question:
Is there a correlation –whit that strength - between unit_price (X) and the number of cakes sold in a week (Y) in the general population?

Null and alternative hypotheses:

$\mathrm{H}_0\colon\ \rho = 0$ → no correlation
$\mathrm{H}_A\colon\ \rho \neq 0$ →correlation

NOW WE NEED TO COMPUTE THE $t_{\alpha/2}$ value:

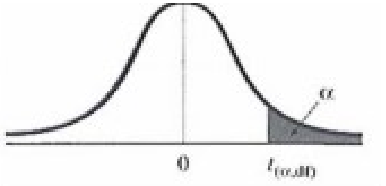
i) The T-statistic used to test H0:rho=0 is the same as the t-stat for testing $ß_1=0$

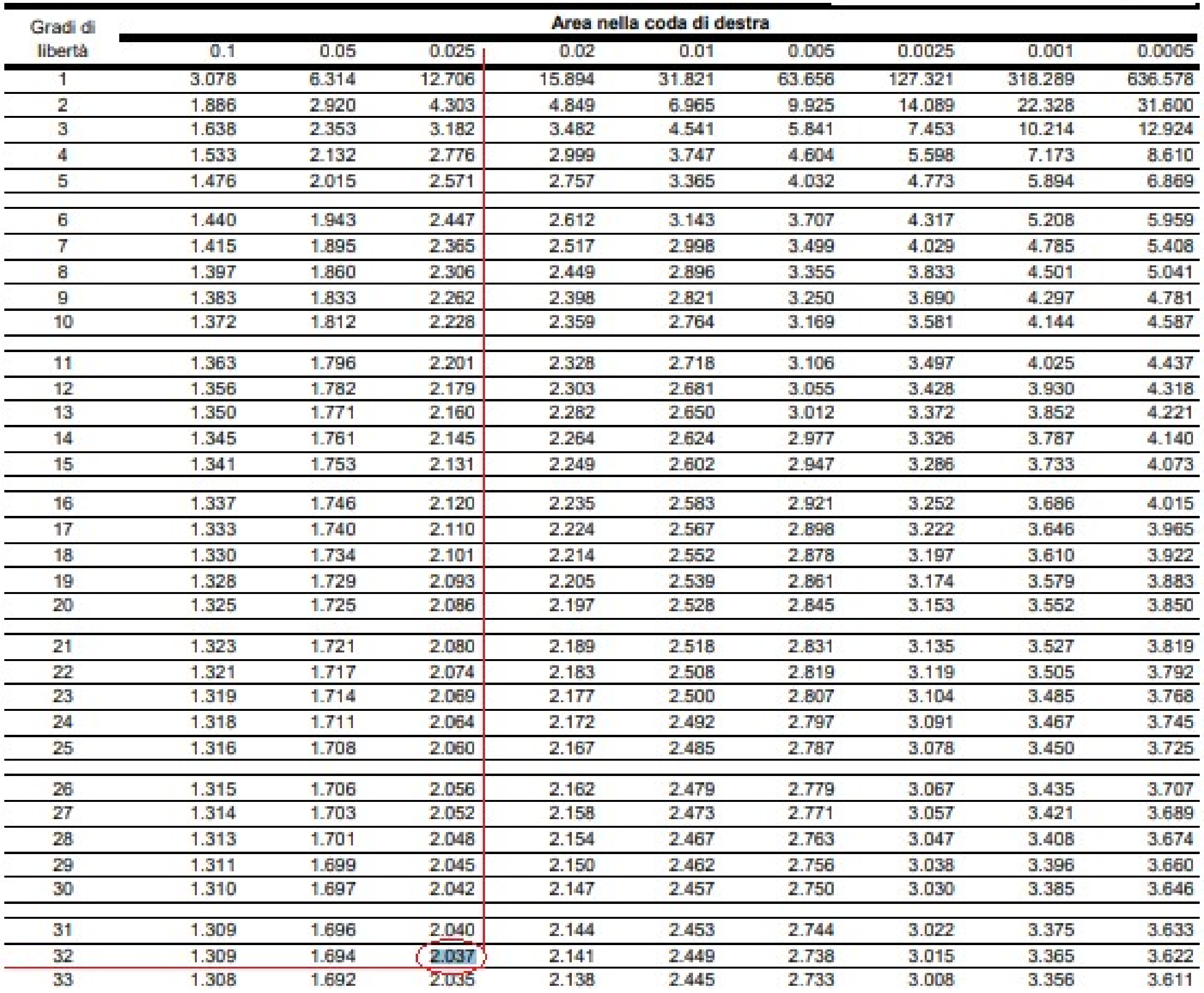
ii) The significance level (α) is defined a priori (considering the value of our research) : let's imagine we want a **confidence level of 95%,** so our **significance level is (1-95** = 0.05) → α = 0.05 → α /2 = 0.05 /2 = 0.025

ii) The degree of freedom (d.f.) for linear regression is n-2: thus in our case d.f. = 34-2 =32

iii) Using those information, let's check on a **T-student** table to discover the $t_{a/2}$ value: 2.037

Tavola della distribuzione T di Student





Gradi di libertà	Area nella coda di destra								
	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	3.078	6.314	12.706	15.894	31.821	63.656	127.321	318.289	636.578
2	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.328	31.600
3	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.214	12.924
4	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.894	6.869
6	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.689
28	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.660
30	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	1.309	1.696	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	1.308	1.692	2.035	2.138	2.445	2.733	3.008	3.356	3.611

Value of t-statistic:

$$r = \sqrt{R} = \sqrt{\frac{SSR}{SST}}$$

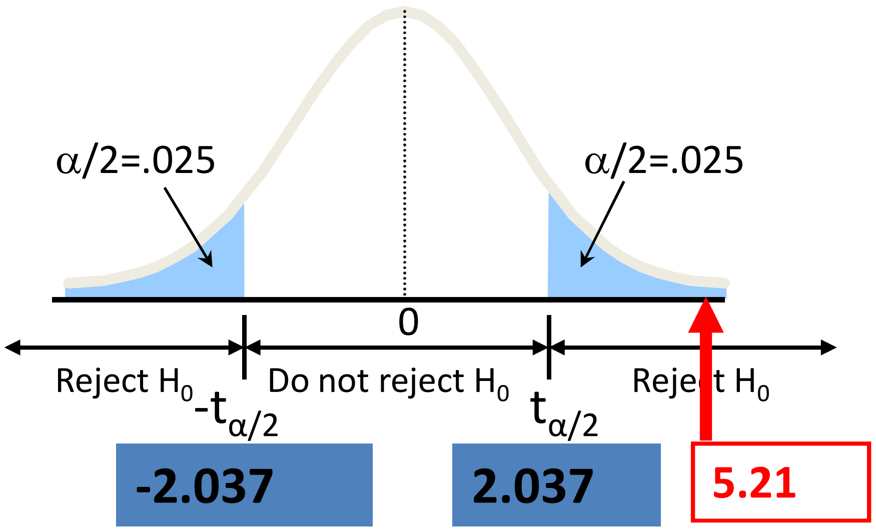
$$T - stat\, r = \frac{r - \rho}{\sqrt{\frac{1 - r^2}{n - 2}}}$$

T-stat r = (0.6773 – 0)/0.1300=5.21

Last step: the comparison of computed values

T-stat r = 5.21
tα/2 = 2.037

T-stat > tα/2 → the statistic falls within the rejection area!



We reject the null hypothesis (H0) , thus:

there is sufficient evidence that (at a 95% of confidence level) within the population the unit-price and the number of cakes are correlated with a strength identify by the analysis