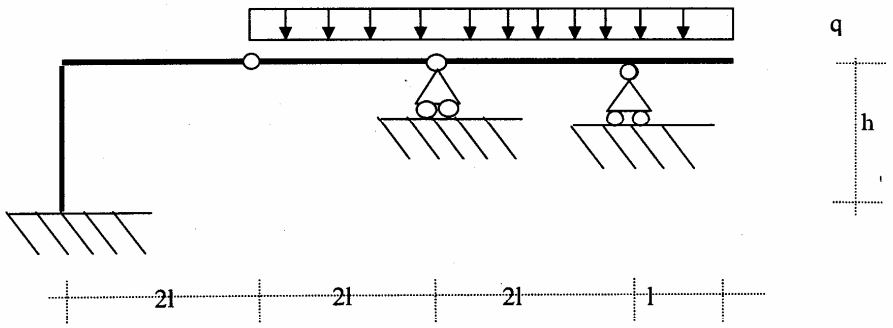
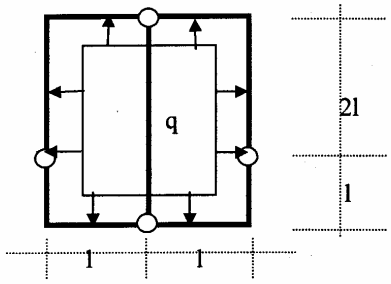
Scienza delle costruzioni I AA 2009/2010 19/03/2010 **A**

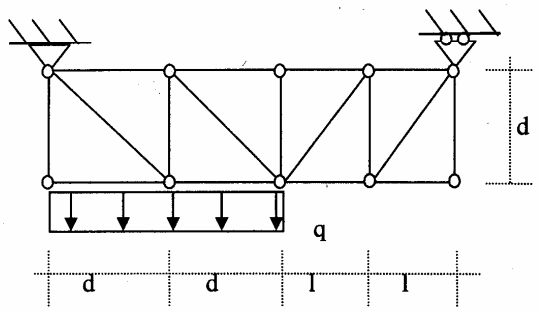
1) Risolvere e determinare i diagrammi quotati delle azioni interne N,T,M della struttura in figura dove l=2m, h=3m, q=1000Kg/m



2) Risolvere e determinare i diagrammi quotati delle azioni interne N,T,M per la struttura simmetrica in figura dove l=1m; q=500 Kg/m

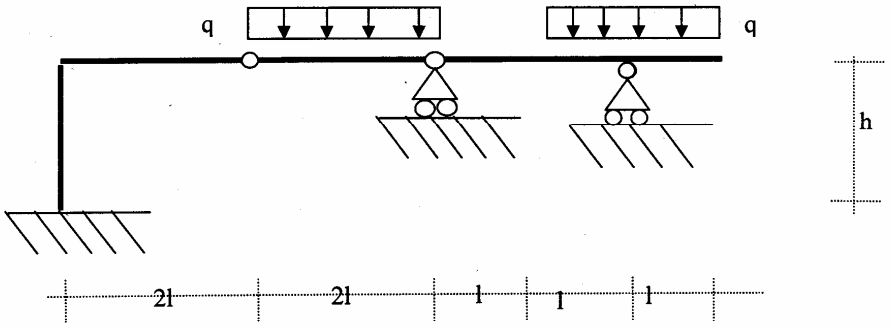


3) Determinare lo stato di sollecitazione primario e secondario della reticolare in figura dove q=100Kg/m, d=1,5 m, l=1m

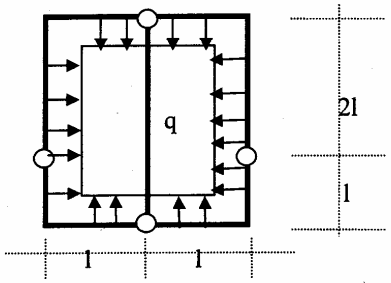


Scienza delle costruzioni I AA 2009/2010 19/03/2010 **B**

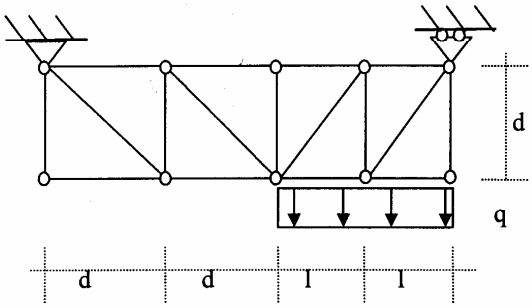
1) Risolvere e determinare i diagrammi quotati delle azioni interne N,T,M della struttura in figura dove l=2m, h=3m, q=1000Kg/m



2) Risolvere e determinare i diagrammi quotati delle azioni interne N,T,M per la struttura simmetrica in figura dove l=1m; q=500 Kg/m

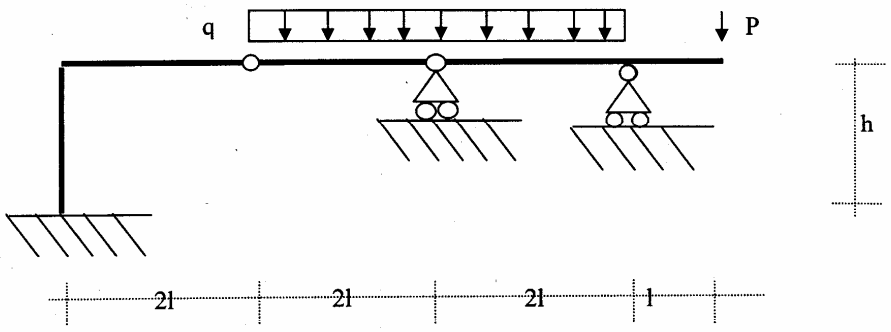


3) Determinare lo stato di sollecitazione primario e secondario della reticolare in figura dove q=100Kg/m, d=1.5 m, l=1m

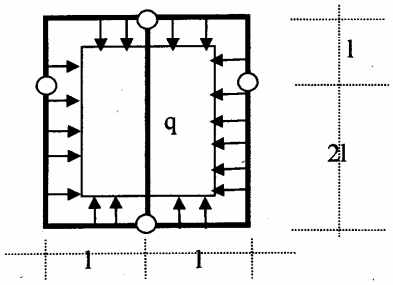


Scienza delle costruzioni I AA 2009/2010 19/03/2010 **C**

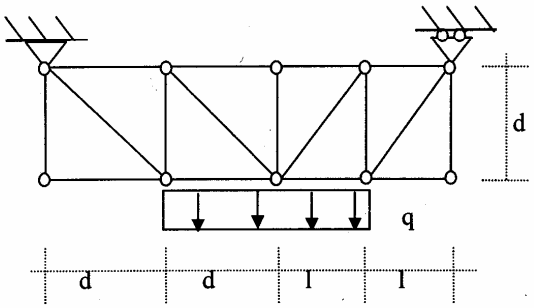
1) Risolvere e determinare i diagrammi quotati delle azioni interne N,T,M della struttura in figura dove l=2m, h=3m, q=1000Kg/m, P=ql



2) Risolvere e determinare i diagrammi quotati delle azioni interne N,T,M per la struttura simmetrica in figura dove l=1m; q=500 Kg/m

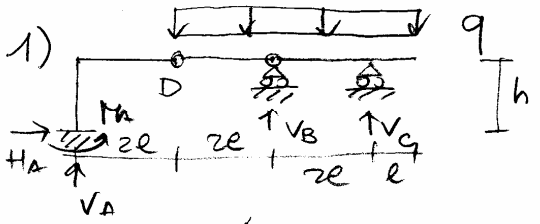


3) Determinare lo stato di sollecitazione primario e secondario della reticolare in figura dove q=100Kg/m, d=1.5 m, l=1m



SCIENZA delle COSTRUZIONI 19/03/2010 (A)

1)



$q = 1000\ kg/m$; $l = 2m$; $h = 3m$

$H_A = 0$

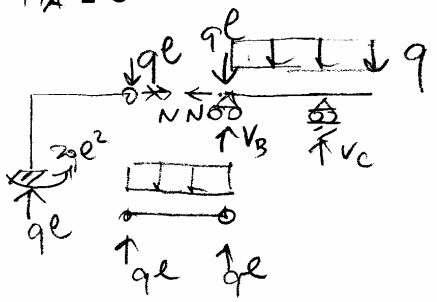
$V_C 2l = 3ql^2 \cdot \frac{3}{2} \Rightarrow V_C = \frac{9}{4}ql = 4500\,kg$

$V_C 4l + V_B 2l = 5ql^2 \cdot \frac{5}{2}$

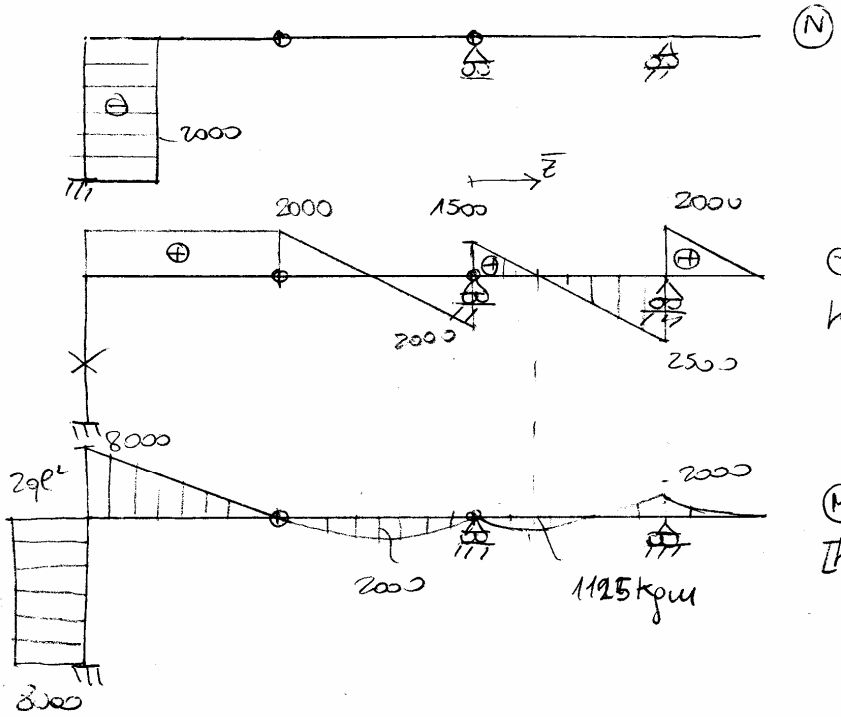
$V_B = \frac{1}{2}\left(\frac{25}{2}ql - 9ql\right) = \frac{7}{4}ql$

$V_A = 5ql - \frac{7}{4}ql - \frac{9}{4}ql = ql$

$M_A = 2ql^2$



In alternativa, modo più veloce!

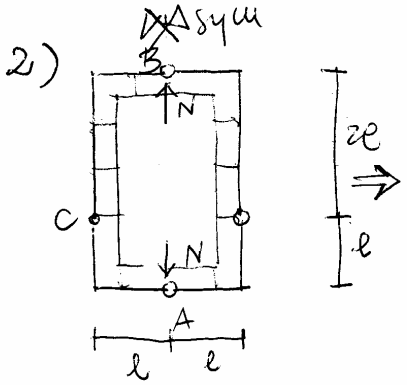


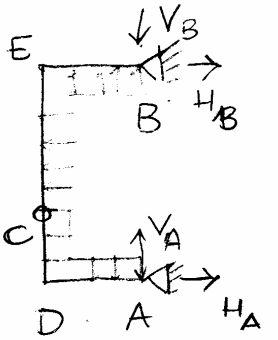
$\bar{z} = 1500 \cdot \frac{2l}{4000} = \frac{3}{4}l$

$M(\bar{z}) = 1500 \cdot \frac{3}{4}l - \frac{9}{32}ql^2$

$= 1125\ kgm$

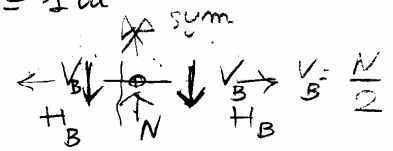
2)





$q = 500 kg/m$

$l = 1 m$



$V_A = V_B$ per equil. ↑)

$H_B + H_A = 3ql$

C↑ $H_A l + V_A l = q\frac{l^2}{2} + q\frac{l^2}{2}$

A↱ globale

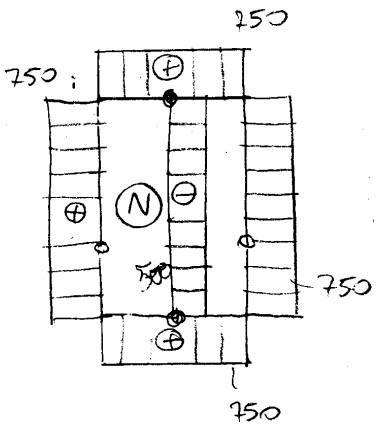
$-H_B 3l + 3ql \cdot \frac{3}{2} = 0 \Rightarrow H_B = \frac{3}{2}ql$

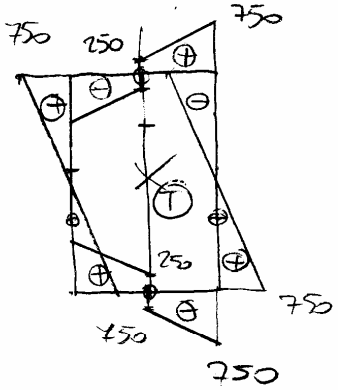
$H_A = \frac{3}{2}ql = 750 kg$

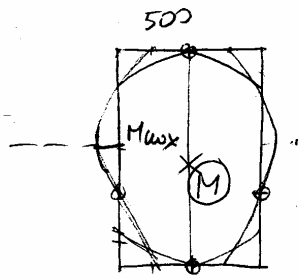
$V_A = ql - \frac{3}{2}ql = -\frac{ql}{2}$

$V_B = -\frac{ql}{2} = -250 kg$

$N = -ql = -500 kg$







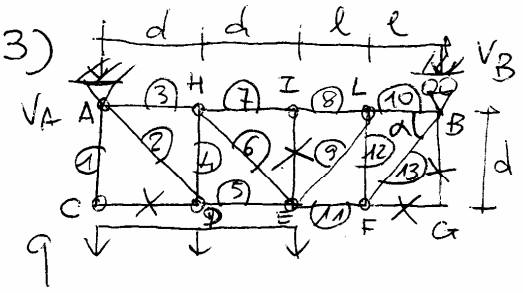
$M_{EB} = 250 + 250 = 500 kgm$

$M_{max} = -750\frac{3}{2}l + 500 + q \cdot \frac{3}{2}l \cdot \frac{3}{4}l =$

$= -1125 + 500 + 562,5 =$

$= -62,5 kgm$

3)



$d = 1{,}5\,m$; $l = 1\,m \Rightarrow d = \frac{3}{2}l$

$q = 100\,kg/m$ $\qquad l = \frac{2}{3}d$

$V_A + V_B = -2qd$

$V_A(2d+2l) + 2qd(d+2l) = 0$

$V_A = -\cancel{2}qd \cdot \frac{d+2l}{\cancel{2}d+\cancel{2}l}$

$= -q\frac{3}{2}l \frac{(\frac{3}{2}l+2l)}{\frac{3}{2}l+l} =$

$= -q\frac{3}{2}l \cdot \frac{\cancel{7}}{\cancel{2}}\cancel{l} \cdot \frac{\cancel{2}}{5\cancel{3}} = \boxed{-\frac{21}{10}ql}$

$V_B = -2q\frac{3}{2}l + \frac{21}{10}ql = \frac{ql}{10} \cdot -9$

check AT tot. $\qquad = \boxed{-\frac{9}{10}ql}$

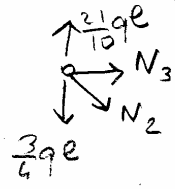
$+\frac{9}{10}ql \cdot (3l+2l) = \cancel{2}q \cdot \frac{3}{\cancel{2}}l \cdot \frac{3}{2}$

$tg\,\alpha = \frac{3}{2}$; $\sin\alpha = \frac{3/2}{\sqrt{1+9/4}} = 3/\sqrt{13}$

$\cos\alpha = 2/\sqrt{13}$

(C) $N_1 = +q \cdot \frac{3}{4}l$

(A)

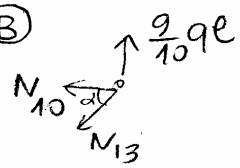


$N_2 = \sqrt{2}\left(\frac{21}{10}ql - \frac{3}{4}ql\right)$

$= \sqrt{2}ql \cdot \frac{27}{20}$

$N_3 = -\frac{27}{20}ql$

(B)

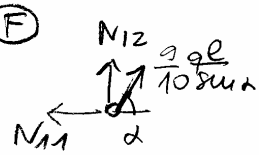


$N_{13} = \frac{9}{10}ql \cdot \frac{1}{\sin\alpha}$

$N_{10} = -\frac{\cancel{9}^{3}}{\cancel{10}_{5}}ql \cdot \frac{\cancel{2}}{\cancel{3}} =$

$= -\frac{3}{5}ql$

(F)

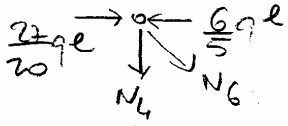


$N_{12} = -\frac{9}{10}ql$

$N_{11} = \frac{\cancel{9}^{3}}{\cancel{10}_{5}}ql \cdot \frac{\cancel{2}}{\cancel{3}}$

$= +\frac{3}{5}ql$

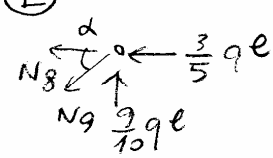
(H)



$N_6 = \sqrt{2} \cdot -\frac{3}{20}ql$

$N_4 = \frac{3}{20}ql$

(L)



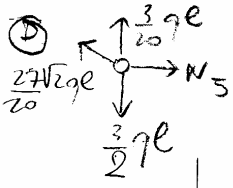
$N_9 = \frac{9ql}{10\sin\alpha}$

$N_8 = ql\left(-\frac{3}{5} - \frac{\cancel{9}^{3}}{\cancel{10}_{5}} \cdot \frac{\cancel{2}}{\cancel{3}}\right)$

$= -\frac{6}{5}ql$

$N_7 = -\frac{6}{5}ql$

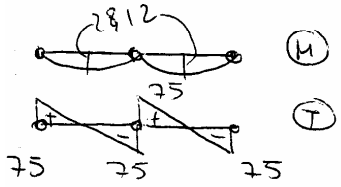
(D)



$N_5 = \frac{27}{20}ql$

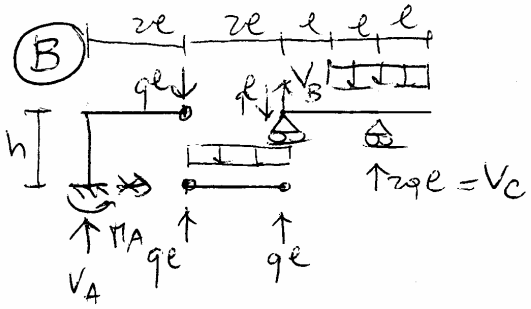
↑) ok

SECONDARIO



$N = 0$

1	75
2	+190,92
3	−135
4	15
5	135
6	−21,21
7	−120
8	−120
9	108,16
10	−60
11	+60
12	−90
13	108,16



$q = 1000\,kg/m$
$\ell = 2\,m$
$h = 3\,m$

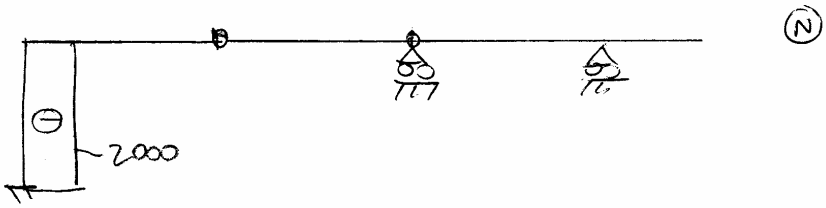
$V_A = q\ell$

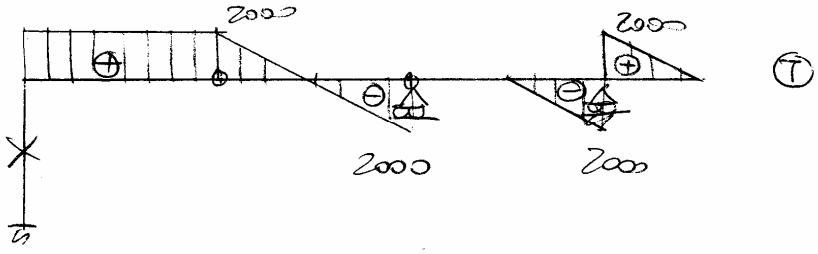
$V_C\, 2\ell - 2q\ell \cdot 2\ell = 0$

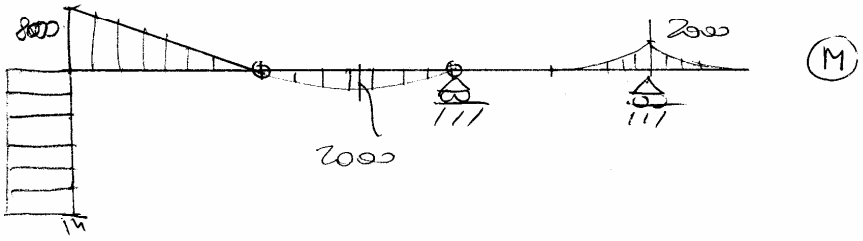
$V_C = 2q\ell$

$V_B = q\ell$

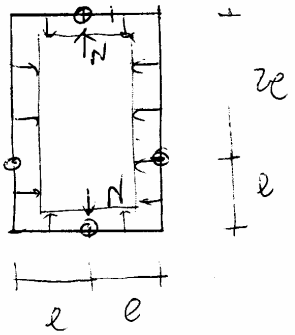
$M_A = 2q\ell^2$



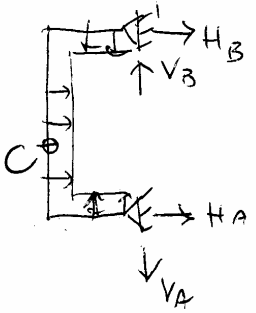


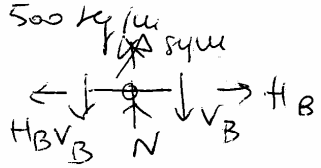


2)



$\ell = 1m \quad q = 500\ kg/m$





$$V_B = \frac{N}{2}$$

$$H_B + H_A = -3q\ell$$

B) $H_A 3\ell + 3q\ell \cdot \frac{3\ell}{2} = 0$

$$H_A = -\frac{3}{2}q\ell = -750\ kg$$

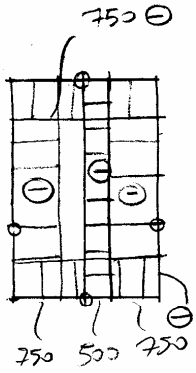
$$H_B = -\frac{3}{2}q\ell = -750\ kg$$

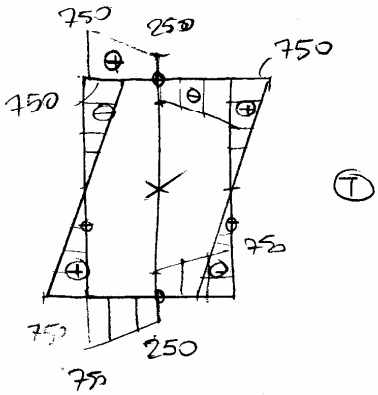
C) $-V_A \ell + H_A \ell + q\frac{\ell^2}{2} - q\frac{\ell^2}{2} = 0$

$$V_A = -\frac{q\ell}{2} = -250\ kg$$

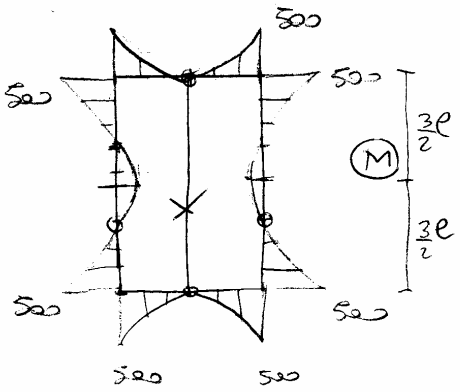
$$V_B = -\frac{q\ell}{2} = -250\ kg$$

$$N = -q\ell = -500\ kg$$



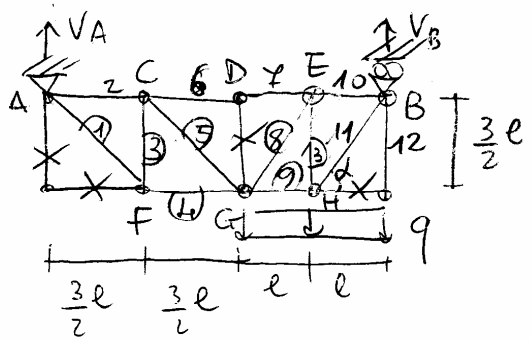


T



$$M_B = -250 - 250 = -500\ kg\,m$$

3)



$l = 1\,m$

$q = 100\,kg/m$

$tg\,\alpha = \frac{3}{2}$; $\sin\alpha = \frac{3/2}{\sqrt{1+9/4}} = 3/\sqrt{13}$

$\cos\alpha = 2/\sqrt{13}$

$A\circlearrowleft\ V_B\, 5l = 2ql \cdot 4l$

$V_B = \frac{8}{5} ql$

$\uparrow)\ V_A = -\frac{8}{5} ql + 2ql$

$= +\frac{2}{5} ql$

check

$B\circlearrowleft\ -V_A 5l + 2ql^2 = 0$

(A) $\uparrow \frac{2}{5} ql$, $\rightarrow N_2$, $\searrow N_1$

$N_1 = \boxed{\frac{2}{5} ql\sqrt{2}}$

$N_2 = \boxed{-\frac{2}{5} ql}$

$N_{12} = \frac{1}{2} ql = 50$

(B) $\uparrow \frac{8}{5} ql$, N_{10}, N_{11}, $\frac{ql}{2}$

$N_{11} = \boxed{\frac{11}{10} \frac{ql}{\sin\alpha}}$

$N_{10} = -\frac{11}{\cancel{10}_{5}} \frac{ql}{3}\,\cancel{2}$

(F) $\frac{2}{5} ql\sqrt{2}$, N_3, N_4

$N_3 = \boxed{-\frac{2}{5} ql}$

$N_4 = \boxed{\frac{2}{5} ql}$

(C) $\frac{2}{5} ql$, N_6, N_5, $\frac{2}{5} ql$

$N_5 = \boxed{\frac{2}{5} ql\sqrt{2}}$

$N_6 = -\frac{2}{5} ql - \frac{2}{5} ql$

$= \boxed{-\frac{4}{5} ql}$

(D) $N_7 = \boxed{-\frac{4}{5} ql}$

(E) $\frac{4}{5} ql$, α, $\frac{11}{15} ql$, N_8, N_{13}

$N_8 = +\frac{ql}{15} \frac{1}{\cos\alpha}$

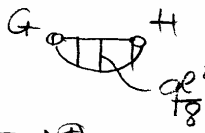
$N_{13} = \boxed{-ql/10}$

(G) $\frac{2}{5} ql\sqrt{2}$, $\frac{ql}{15} \frac{1}{\cos\alpha}$, N_9, $\frac{2}{5} ql$, $\frac{ql}{2}$

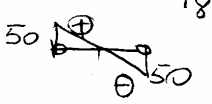
$\uparrow)$ OK

$\rightarrow)\ N_9 = \boxed{+\frac{11}{15} ql}$

SECONDARIO



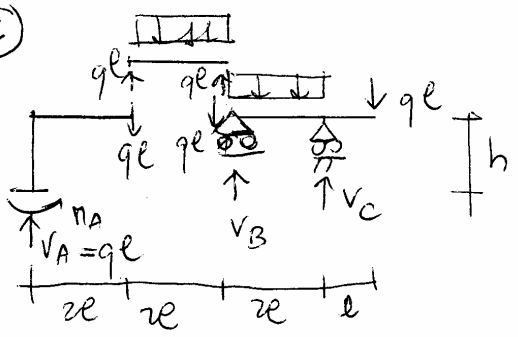
(M) $\frac{ql^2}{8} = 12{,}5\,kg\,m$



(T) tg

1	56,56	7	−80
2	−40	8	12,02
3	−40	9	73,33
4	40	10	−73,33
5	56,56	11	132,20
6	−80	12	50
		13	−10

(c)



$V_C 2\ell = 3q\ell^2 + 2q\ell^2$

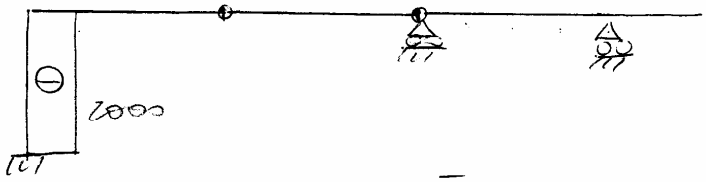
$V_C = \frac{5}{2}q\ell$

$V_B = 3q\ell + q\ell - \frac{5}{2}q\ell = \frac{3}{2}q\ell$

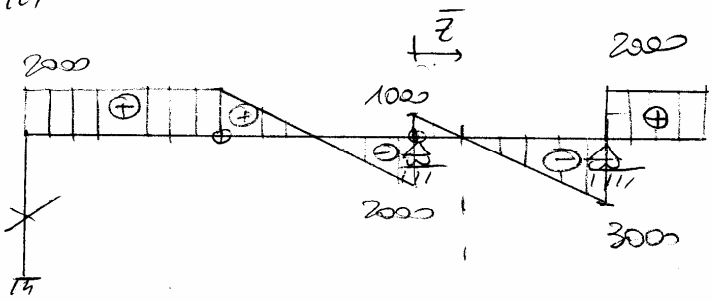
$V_A = q\ell$

$M_A = 2q\ell^2$

$\ell = 2m \quad h = 3m \quad q = 1000 kg/m$



(N)



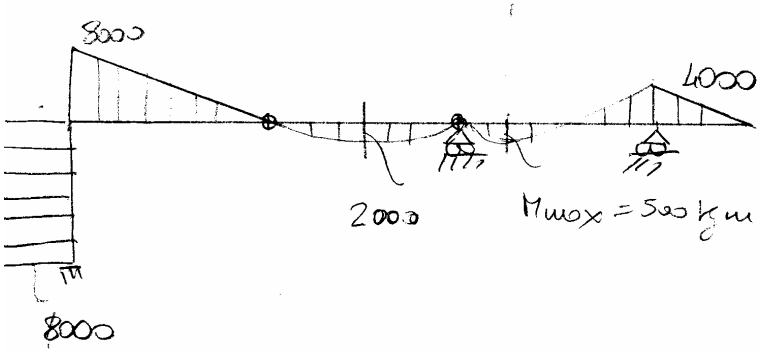
(T)

$\bar{z} = 1000 \frac{kg}{400}$

$\bar{z} = 1 m = \frac{\ell}{2}$

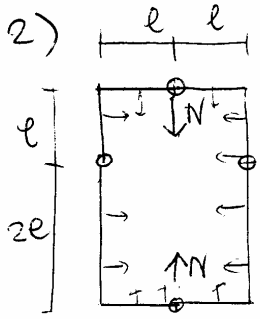
$M_{max} = 1000 \cdot 1 - q\frac{1}{2}$

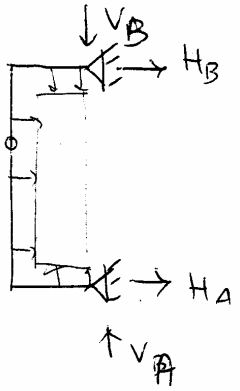
$= 500 kgm$

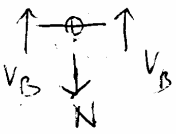


(M)

2)







$N = 2V_B$

$q = 500\,kg/m$

$l = 1\,m$

B↺ $H_A 3l + 3ql\frac{3}{2}l = 0$

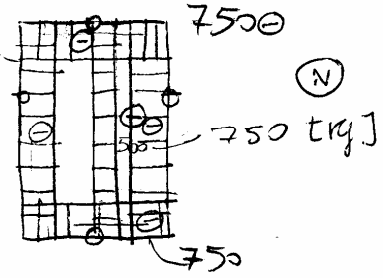
$H_A = -\frac{3}{2}ql = H_B = -750\,kg$

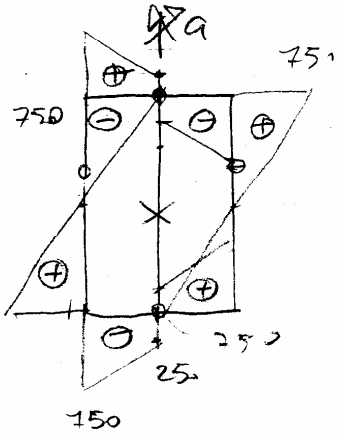
C↺ $-V_B l - H_B^2 l - 2q\frac{l^2}{2} = 0$

$V_B = -H_B - ql = \frac{ql}{2} = 250\,kg$

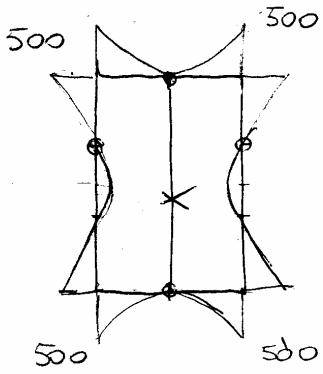
$V_A = \frac{ql}{2}$

$N = ql = 500\,kg$



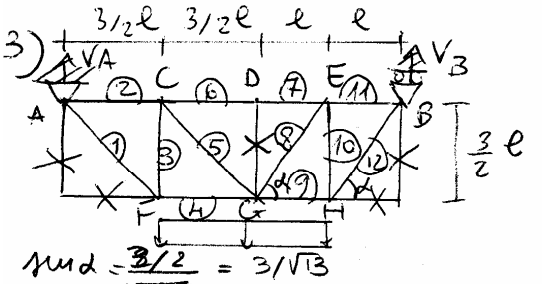


[kg]



M [kgm]

3)



$\sin\alpha = \frac{3/2}{\sqrt{1+9/4}} = 3/\sqrt{13}$

$\ell = 1\,m,\ q = 100\ kg/m$

$V_A + V_B = q\frac{5}{2}\ell$

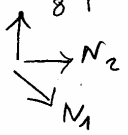
$A\uparrow \quad V_B\,5\ell = q\frac{5}{2}\ell\left(\frac{3}{2}\ell + \frac{5}{4}\ell\right) =$

$V_B = \frac{1}{2}q\ell\cdot\frac{6+5}{4} = \frac{11}{8}q\ell$

$V_A = -\frac{11}{8}q\ell + \frac{5}{2}q\ell = \frac{9}{8}q\ell$

$-\frac{9}{8}q\ell\cdot 5\ell + \frac{5}{2}q\ell\cdot\left(\ell+\frac{5}{2}\ell\right) = 0$

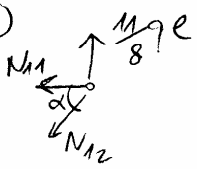
(A)



$N_1 = \frac{9}{8}q\ell\sqrt{2}$

$N_2 = -\frac{9}{8}q\ell$

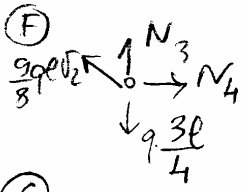
(B)



$N_{12} = \frac{\frac{11}{8}q\ell}{\sin\alpha}$

$N_{11} = -\frac{11}{8}q\ell\frac{3}{4} = -\frac{11}{12}q\ell$

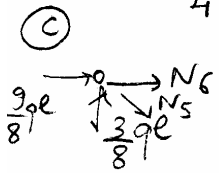
(F)



$N_3 = -\frac{3}{8}q\ell$

$N_4 = \frac{9}{8}q\ell$

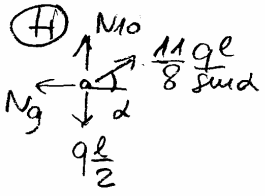
(C)



$N_5 = \frac{3}{8}q\ell\sqrt{2}$

$N_6 = -\frac{9}{8}q\ell - \frac{3}{8}q\ell = -\frac{3}{2}q\ell$

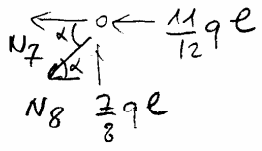
(H)



$N_9 = \frac{11}{8}q\ell\frac{3}{4} = \frac{11}{12}q\ell$

$N_{10} = q\frac{\ell}{2} - \frac{11}{8}q\ell = -\frac{7}{8}q\ell$

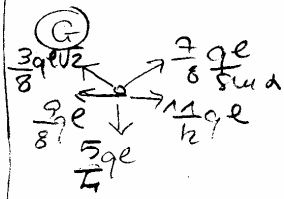
(E)



$N_8 = \frac{\frac{7}{8}q\ell}{\sin\alpha}$

$N_7 = -\frac{7}{8}q\ell\frac{3}{4} - \frac{11}{12}q\ell = -\frac{18}{12}q\ell = -\frac{3}{2}q\ell$

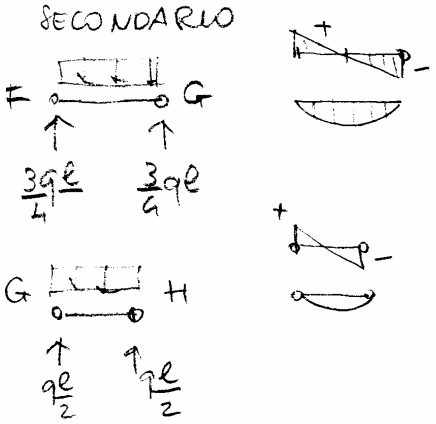
(G)



$\rightarrow) -\frac{9}{8} + \frac{11}{12} + \frac{7}{8}\cdot\frac{3}{4} - \frac{3}{2} = 0$ OK

$\uparrow) \left(-\frac{5}{4} + \frac{3}{8} + \frac{7}{8}\right)q\ell = 0$ OK

SECONDARIO



1	159,09
2	−112,5
3	−37,5
4	112,5
5	53,03
6	−150
7	−150
8	105,16
9	91,66
10	−87,5
11	−91,66
12	165,25