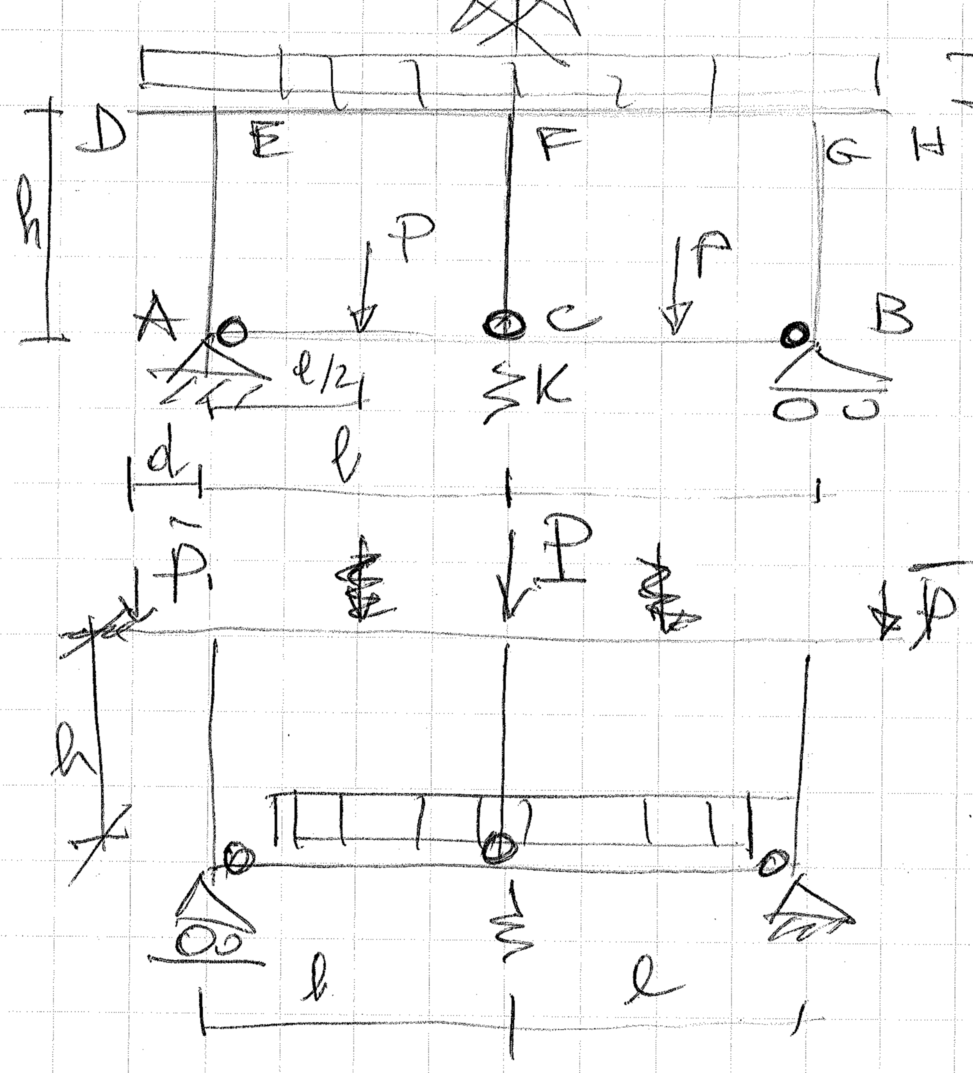


Tubi

$l = 4\,m$

$d = 1\,m$

$h = 3\,m$

$q = 2000\ Kg/m$

$P = 5\ ton = 10\ KN$

Tubi rettangolari

$l = 5\,m$

$h = 4\,m$

$d = 1\,m$

$q = 2000\ Kg/m$

$P = 8\ tn$

$\bar{P} = 2\ ton$

A) Risolvere la struttura in Figura (in questa fase è corretto trascurare le deformazioni assiali delle aste pendolari, la rigidezza $K = 0$)

B) Disegnare i diagrammi NMT

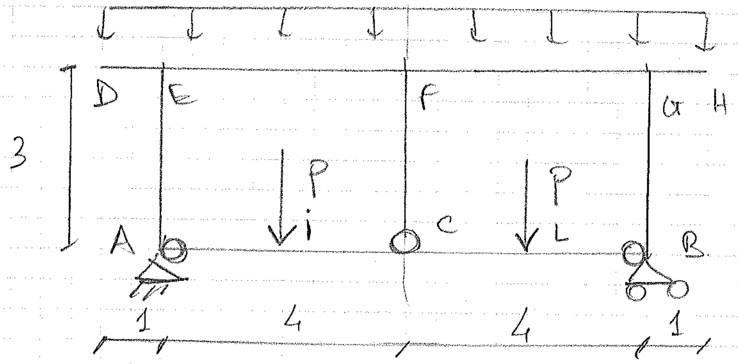
C) Progettare la struttura con Tubi ...

D) Verificare l'incremento dell'asta [illegible] nella sezione maggiormente sollecitata a flessione

E) Mettere in conto la rigidezza della molla $K = 1\ KN/cm$ (e di seguire i diagrammi delle sollecitaz.)

F) calcolare l'abbassamento del punto C e la rotazione del nodo A equivalente al pilastro

PROVA A 12/07/10

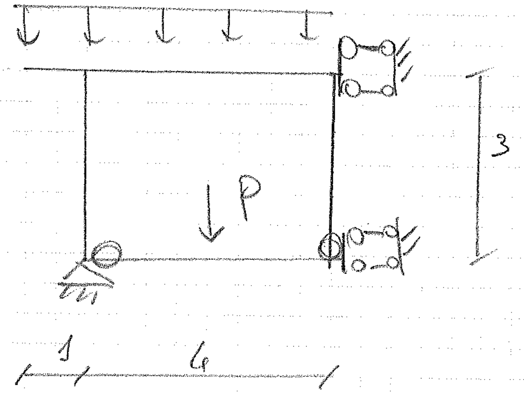


$q = 2500$ kp/m

(1)

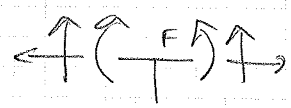
P = 5 ton
= 5000 kp

Sfrutto la simmetria.

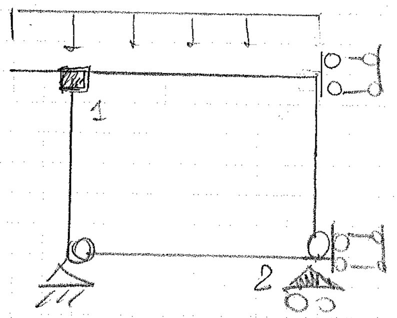


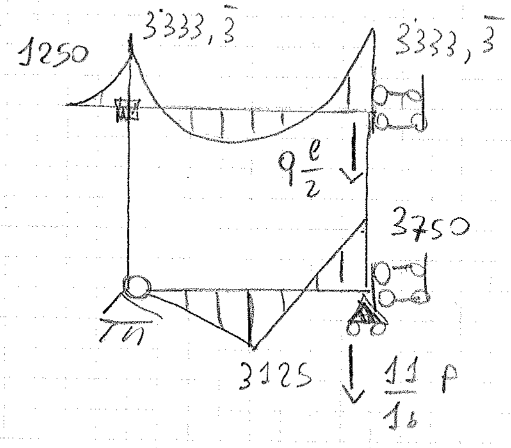
2 spostamenti

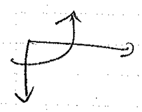
Oss. la rotazione in F è nulla poichè la struttura è simmetrica



Sistema 0



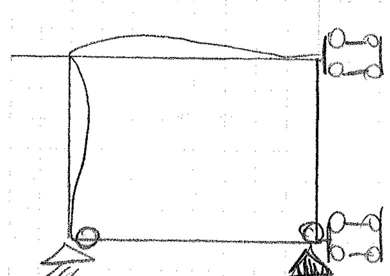


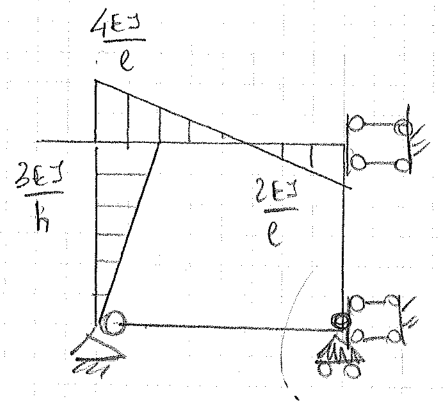


$$K_{10} = 3333,\bar{3} - 1250 = 2083,33$$

$$K_{20} = -8437,5$$

Sistema 1



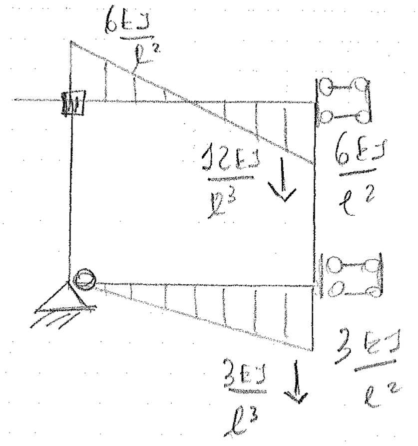


$$K_{11} = EJ\left(\frac{4}{l} + \frac{3}{h}\right)$$

$$= 2EJ$$

$$K_{12} = \frac{6EJ}{l^2} = \frac{3}{8}EJ$$

Sistema 2



$$K_{21} = \frac{6EJ}{l^2} = \frac{3}{8} EJ$$

$$K_{22} = EJ\left(\frac{12}{l^3} + \frac{3}{l^3}\right) = \frac{15}{64} EJ$$

$$EJ \begin{pmatrix} 2 & 3/8 \\ 3/8 & 15/64 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -2083,33 \\ 8437,5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \varphi_1 \\ u_2 \end{pmatrix} = \frac{1}{EJ} \begin{pmatrix} -11'131 \\ 53'809,5 \end{pmatrix}$$

$$M_{EF} = 3'333,\bar{3} + EJ\left(-\frac{11'131}{EJ}\right) + \frac{3}{8} \cancel{EJ}\left(\frac{53'809,5}{\cancel{EJ}}\right) = 12'380,9$$

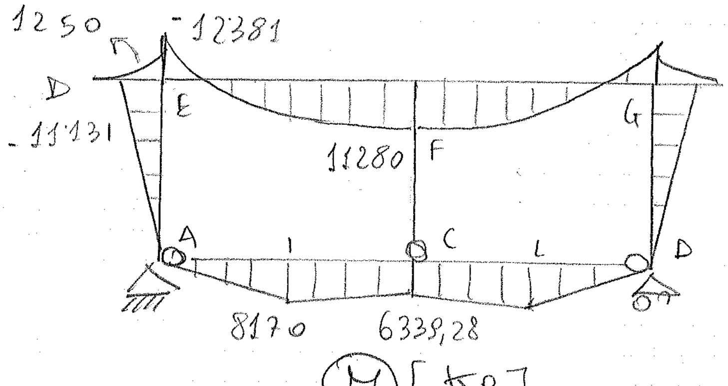
$$M_F = -3'333,\bar{3} + \frac{EJ}{2}\left(-\frac{11'131}{EJ}\right) + \frac{3}{8} EJ\left(53'809,5\right) = 11'279,7$$

$$M_{EA} = EJ\left(-\frac{11'131}{EJ}\right) = -\frac{11'131}{EJ}$$

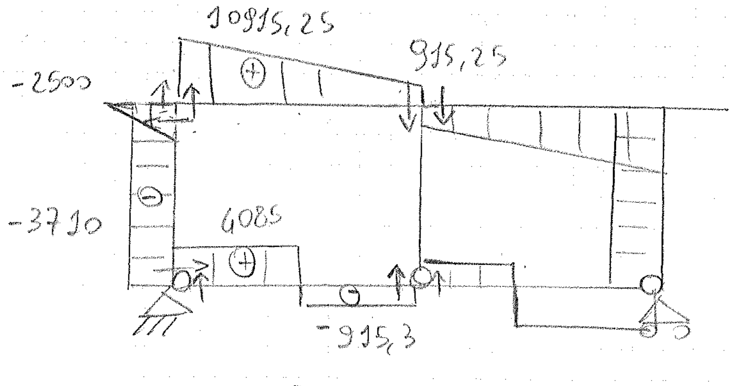
$$M_i = 3125 + \frac{3}{32} EJ\left(53'809,5\right) = 8170$$

$$M_c = -3750 + \frac{3}{16} EJ\left(53'809,5\right) = 6339,28$$

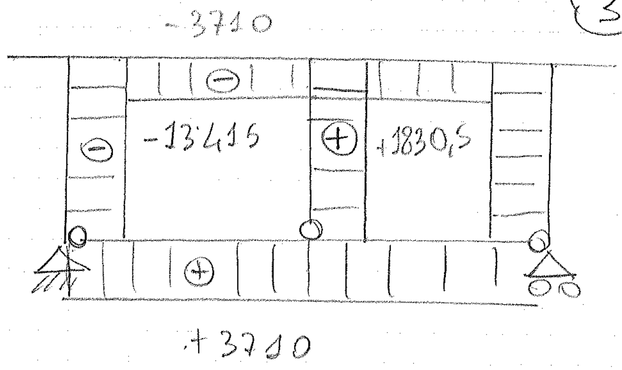
B



M [kp]



(T) [kp]



(N) [kp]

c) $M_{max} = 12'381$ kg.m
$T = 10915,25$ kp
$N = -3710$ kp

TUBI CIRCOLARI

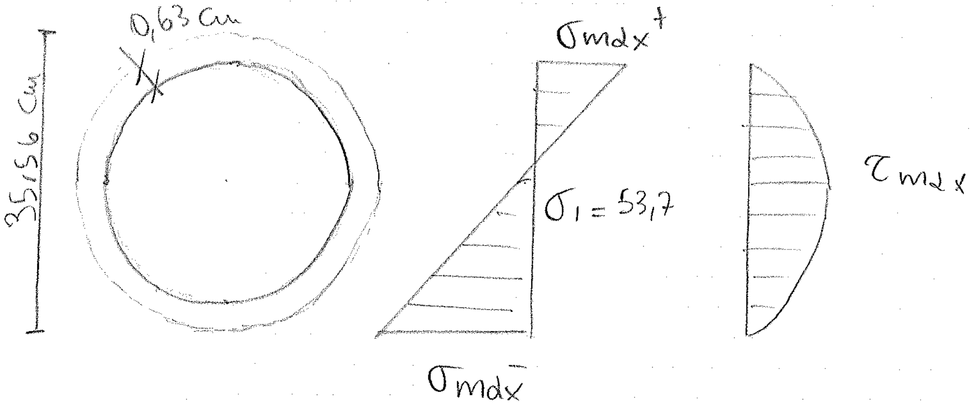
$$W_{min} = \frac{12'381}{2600} \cdot 10^2 = 476,1 \text{ cm}^3$$

$\Rightarrow$ TUBO ϕ 355,6 :
$s = 6,3$ mm
$A = 69,1$ cm²
$W = 593$ cm³
$J = 10'547$ cm⁴

Verifico di pressoflessione:

$$\sigma = \frac{N}{A} \pm \frac{M}{W} = \frac{-3710}{69,1} \pm \frac{12'38100}{593} = \begin{cases} \sigma_{max}^{+} = 2034,17 \\ \sigma_{max}^{-} = -2141,55 \end{cases}$$

< 2600 kg/cm² $\Rightarrow$ OK



$$\tau_{max} = \frac{T}{\pi \cdot R \cdot s} = \frac{10915,25}{\pi \cdot \frac{35,56}{2} \cdot 0,63} = 310,18 \text{ kg/cm}^2 < \sigma_{adm}/\sqrt{3}$$

$$\sigma_{id} = \sqrt{\sigma_1^2 + 3 \cdot \tau_{max}^2} = 539,92 \text{ kg/cm}^2 < 2600 \frac{\text{kg}}{\text{cm}^2}$$

E)

$$k = 1 \text{ kN/cm} = 100 \text{ kg/cm} = 10000 \text{ kg/m}$$

essendo le molle sull'asse di simmetria considero metà rigidezza.
Il contributo di rigidezza delle molle va aggiunto al termine K_{22}
Pertanto il nuovo K_{22} diventa:

$$K_{22} = \frac{15}{64} EJ + \frac{k}{2}$$

$$\Rightarrow EJ \begin{pmatrix} 2 & 3/8 \\ 3/8 & 15/64 + \frac{5000}{EJ} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -2083,33 \\ 8437,5 \end{pmatrix}$$

$$\frac{5000}{2,1 \cdot 10^{10} \cdot 10547 \cdot 10^{-8}} = 0,002257$$

$$\Rightarrow \begin{pmatrix} \varphi_1 \\ u_2 \end{pmatrix} = \frac{1}{EJ} \begin{pmatrix} -10994 \\ 53079,3 \end{pmatrix}$$

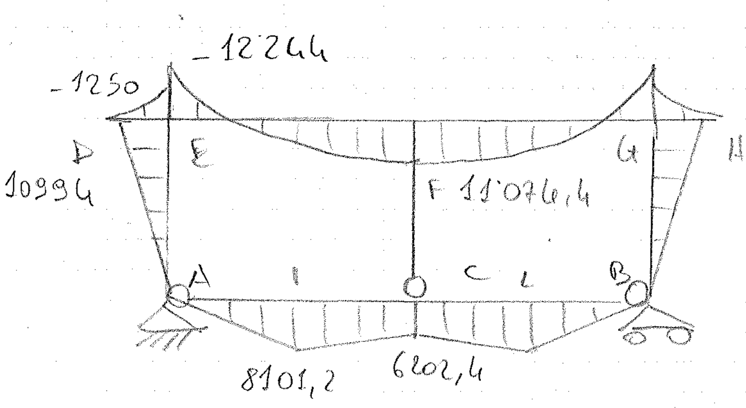
$$M_C = -3750 + \frac{3}{16} EJ \left(\frac{53079,3}{EJ} \right) = 6202,37$$

$$M_{EF} = 3333,\bar{3} + EJ \left(-\frac{10994}{EJ} \right) + \frac{3}{8} EJ \left(\frac{53079,3}{EJ} \right) = 12244$$

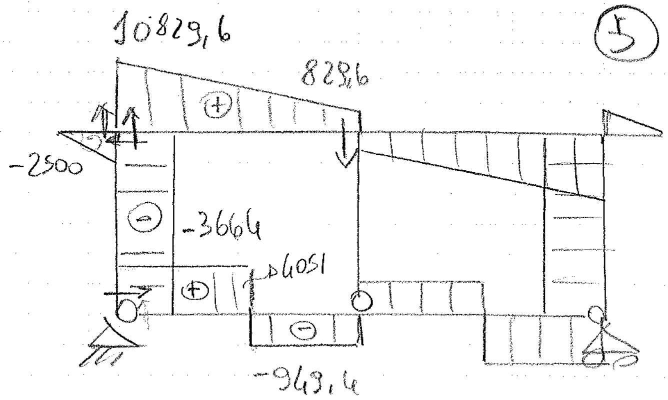
$$M_F = -3333,\bar{3} + \frac{EJ}{2} \left(-\frac{10994}{EJ} \right) + \frac{3}{8} EJ \left(\frac{53079,3}{EJ} \right) = 11074,4$$

$$M_{EA} = EJ \left(-\frac{10994}{EJ} \right) = -10994$$

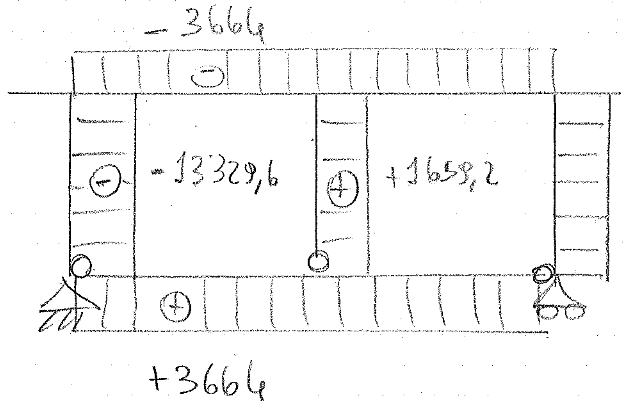
$$M_i = 3125 + \frac{3}{32} EJ (53079,3) = 8101,2$$



(M) [kp.m]



(T) [kp]



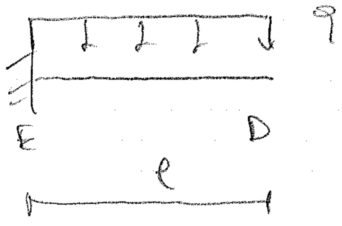
(N) [kp]

F) Abbassamento del punto D

(la rotazione in A è stata calcolata al punto A)

$$\varphi_A = -\frac{11131}{EI} = \frac{-11131}{2{,}1 \cdot 10^{10} \cdot 10547 \cdot 10^{-8}} = -0{,}005026$$

L'abbassamento dell'estremo della mensola nel caso in cui il nodo nell'incastro non ruoti è pari a:



$$v_D = \frac{q l^4}{8EI} = \frac{2500 \cdot 1}{8 \cdot 2{,}1 \cdot 10^{10} \cdot 10547 \cdot 10^{-8}}$$

$= 0{,}000141$ (diretto verso il basso)

L'abbassamento dell'estremo D dovuto alla rotazione in A è invece diretto verso l'alto e pari a:

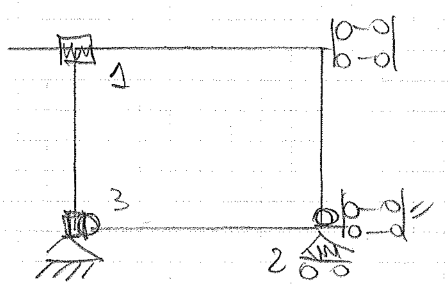
$$v_{D,\varphi_A} = -0{,}005026 \cdot 1\,m = -0{,}005026\ m$$

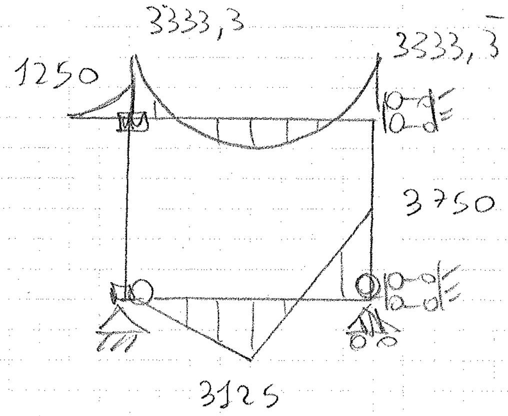
$$\Rightarrow v_D = -0{,}005026 + 0{,}000141 = -0{,}00488\ m = 4{,}88\ mm$$

Rotazione del nodo A

⇒ aggiungo uno spostamento incognito ovvero la rotazione dell'estremo A dell'asta EA

SISTEMA 0



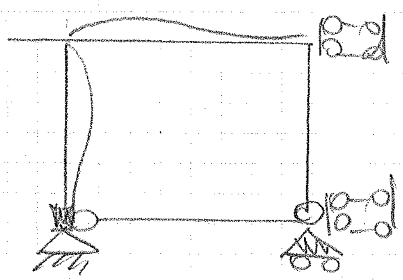


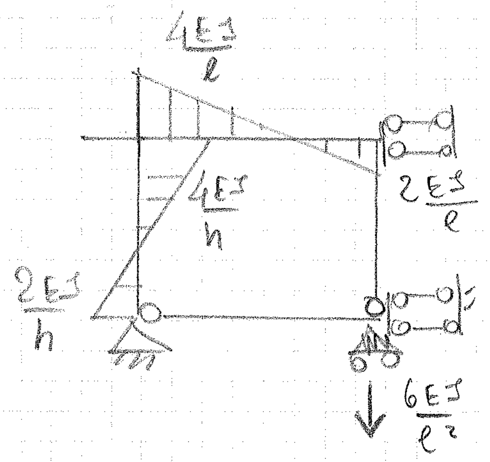
$K_{10} = 2083,33$

$K_{20} = -8437,5$

$K_{30} = 0$

SISTEMA 1



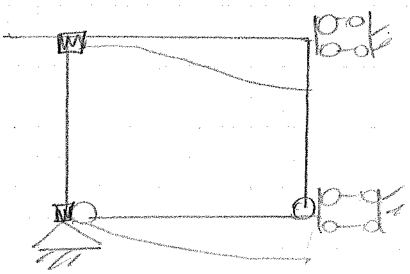


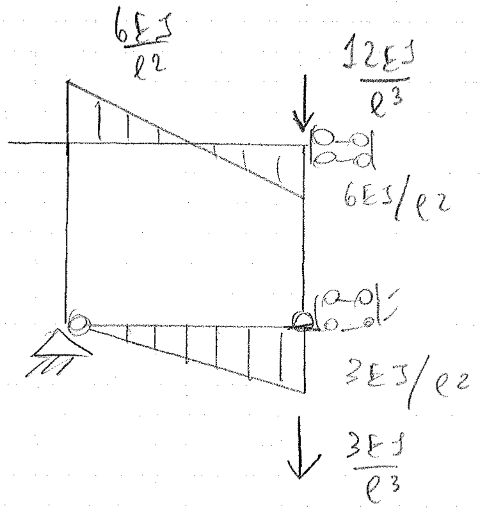
$$K_{11} = EJ\left(\frac{4}{l} + \frac{4}{h}\right) = \frac{7}{3} EJ$$

$$K_{12} = \frac{6EJ}{l^2} = \frac{3}{8} EJ$$

$$K_{13} = \frac{2EJ}{h} = \frac{2}{3} EJ$$

Sistema 2



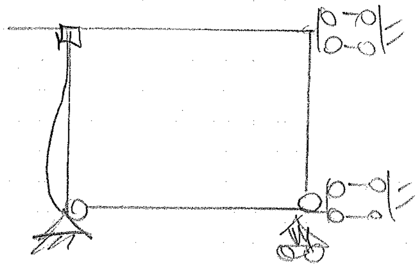


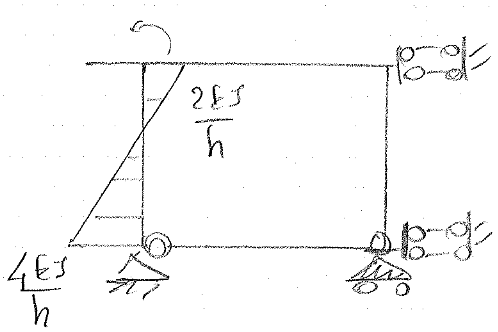
$$K_{21} = \frac{6EJ}{l^2} = \frac{3}{8} EJ$$

$$K_{22} = \frac{15EJ}{l^3} = \frac{15}{64} EJ$$

$$K_{23} = 0$$

Sistema 3





$$K_{31} = \frac{2EJ}{h} = \frac{2}{3} EJ$$

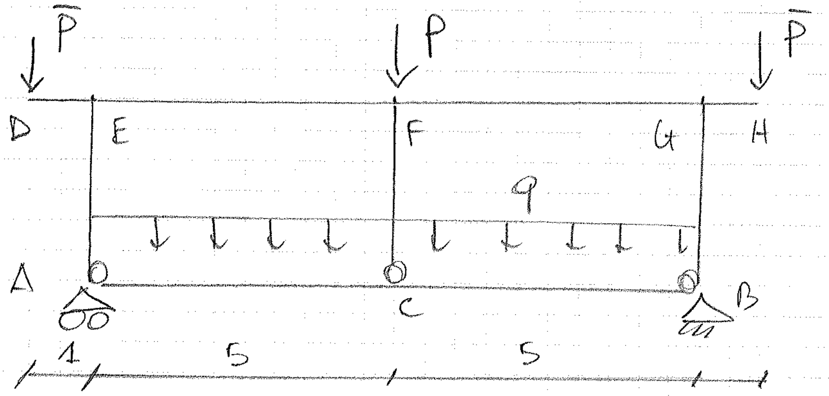
$$K_{32} = 0$$

$$K_{33} = \frac{4EJ}{h} = \frac{4}{3} EJ$$

$$EJ \begin{pmatrix} 7/3 & 3/8 & 2/3 \\ 3/8 & 15/64 & 0 \\ 2/3 & 0 & \frac{4}{3} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ u_2 \\ \varphi_3 \end{pmatrix} = \begin{pmatrix} -2083,33 \\ 8437,5 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \varphi_1 \\ u_2 \\ \varphi_3 \end{pmatrix} = \frac{1}{EJ} \begin{pmatrix} -11131 \\ 53809,5 \\ 5565,5 \end{pmatrix} \Rightarrow \varphi_3 = \frac{5565,5}{2,1 \cdot 10^{10} \cdot 10547 \cdot 10^{-8}} = 0,0025$$

PROVA B 12/07/2010



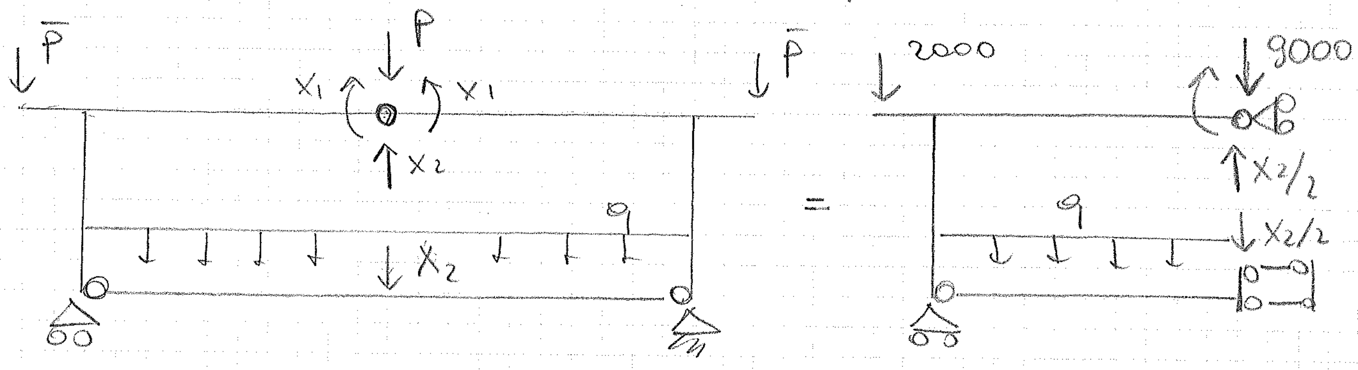
$q = 2000$ kg/m

$P = 18$ ton

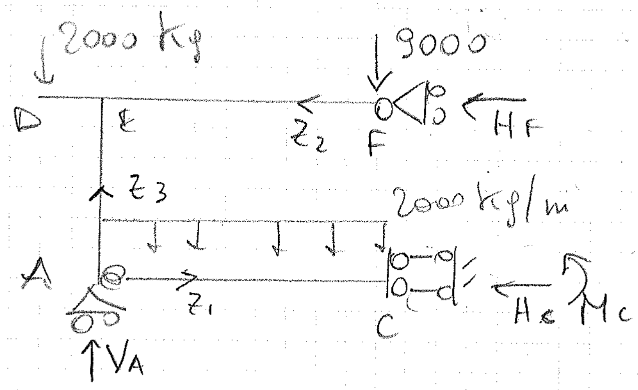
$\bar{P} = 2$ ton

Risolvo con il metodo delle forze

Scelgo la seguente isostatica equivalente.



Sistema 0



$M_{AC}(z_1) = 10000 \cdot z_1 - 1000 \cdot z_1^2$

$M_{FE}(z_2) = -9000 \cdot z_2$

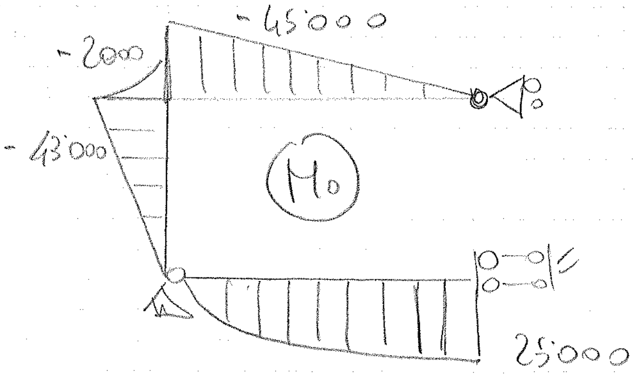
$M_{AE}(z_3) = -10750 \cdot z_3$

$\uparrow)$ $V_A = 21000$ Kg

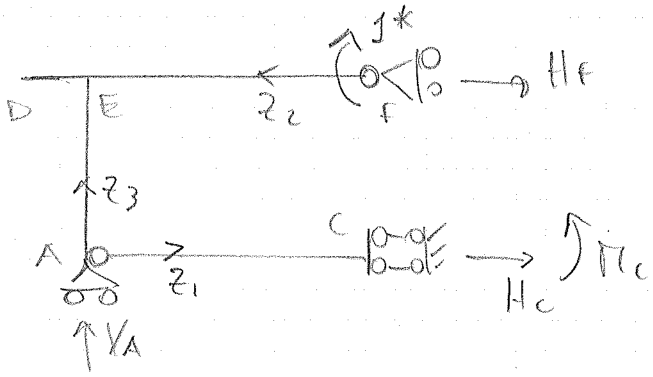
$A \circlearrowright_{AC}$ $-2000 \cdot 5 \cdot 2{,}5 + M_C = 0 \Rightarrow M_C = 25000$ kg.m

$A \circlearrowright_{AEF}$ $2000 \cdot 1 - 9000 \cdot 5 + H_F \cdot 4 = 0 \Rightarrow H_F = 10750$ Kg

$\leftarrow)$ $H_F + H_C = 0 \Rightarrow H_C = -H_F = -10750$



Sistema 1



$M_{AC}(z_1) = 0$

$M_{FE}(z_2) = -1$

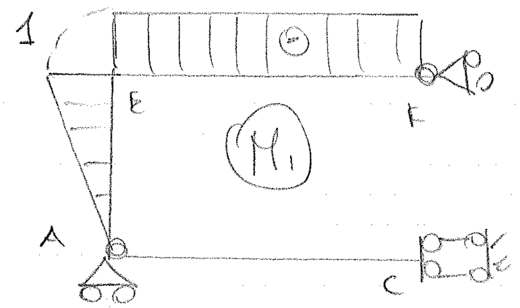
$M_{AE}(z_3) = -\frac{1}{4} \cdot z_3$

$A \curvearrowright_{AC} \quad M_C = 0$

$A \curvearrowright_{AEF} \quad -1 - H_F \cdot 4 = 0 \Rightarrow H_F = -\frac{1}{4}$

$\rightarrow) \; H_F + H_C = 0 \Rightarrow H_C = -H_F = \frac{1}{4}$

$\uparrow) \; V_A = 0$

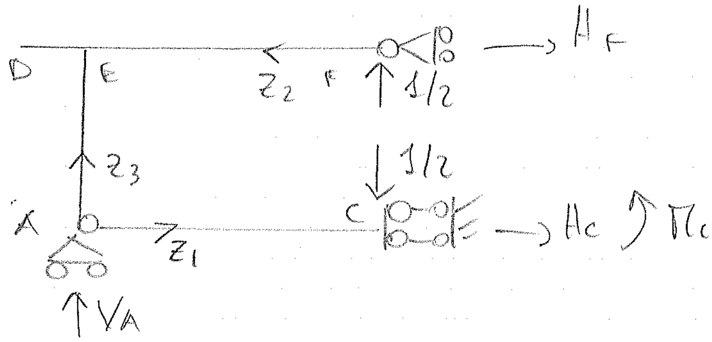


Sistema 2

$M_{AC}(z_1) = \frac{1}{2} \cdot z_1$

$M_{FE}(z_2) = \frac{1}{2} \cdot z_2$

$M_{AE}(z_3) = 0{,}625 \cdot z_3$

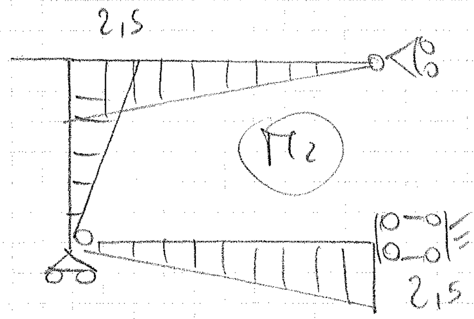


$\uparrow)\ V_A = 0$

$A \circlearrowleft_{AC} \quad M_C - \frac{1}{2} \cdot 5 = 0 \Rightarrow M_C = 2{,}5$

$A \circlearrowleft_{AEF} \quad \frac{1}{2} \cdot 5 - H_F \cdot 4 = 0 \Rightarrow H_F = 0{,}625$

$\rightarrow)\ H_F + H_C = 0 \Rightarrow H_C = -H_F = -0{,}625$



$$\eta_{10} = \frac{1}{EJ} \left\{ \int_0^5 (-9000 \cdot z)\,-1\,dz + \int_0^4 (-10750 \cdot z)(-0{,}25 \cdot z)\,dz \right\}$$

$$= \frac{1}{EJ} \left\{ 112'500 + 57'333{,}3 \right\} = \frac{169833}{EJ}$$

$$\eta_{20} = \frac{1}{EJ} \left\{ \int_0^5 (10'000 \cdot z - 1'000\, z^2)(0{,}5 \cdot z)\,dz + \int_0^5 (-9000 \cdot z)(0{,}5 \cdot z)\,dz \right.$$

$$\left. + \int_0^4 (-10750 \cdot z)(0{,}625 \cdot z)\,dz \right\}$$

$$= \frac{1}{EJ} \left\{ 130'208 - 187'500 - 143'333 \right\} = -\frac{200'625}{EJ}$$

$$\eta_{11} = \frac{1}{EJ} \left\{ \int_0^5 1\,dz + \int_0^4 0{,}0625 \cdot z^2\,dz \right\}$$

$$= \frac{1}{EJ} \left\{ 5 + 1{,}333 \right\} = \frac{6{,}333}{EJ}$$

$$\eta_{22} = \frac{1}{EJ} \left\{ \int_0^5 0{,}25 \cdot z^2\,dz + \int_0^5 0{,}25 \cdot z^2\,dz + \int_0^4 0{,}625^2 \cdot z^2\,dz \right.$$

$$\eta_{22} = \frac{1}{EJ}\left\{ 20,83 + 8,33 \right\} = \frac{29,166}{EJ}$$

$$\eta_{12} = \frac{1}{EJ}\left\{ \int_0^5 -0,5.z\,dz + \int_0^4 (-0,25.z)(0,625.z)\,dz \right\}$$

$$= \frac{1}{EJ}\left\{ -6,25 - 3,333 \right\} = -9,583$$

$$\cancel{\frac{1}{EJ}}\begin{pmatrix} 6,333 & -9,583 \\ -9,583 & 29,166 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -169833 \\ 200625 \end{pmatrix}\cdot \cancel{\frac{1}{EJ}}$$

$$\Rightarrow \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -32632,9 \\ -3843,38 \end{pmatrix}$$

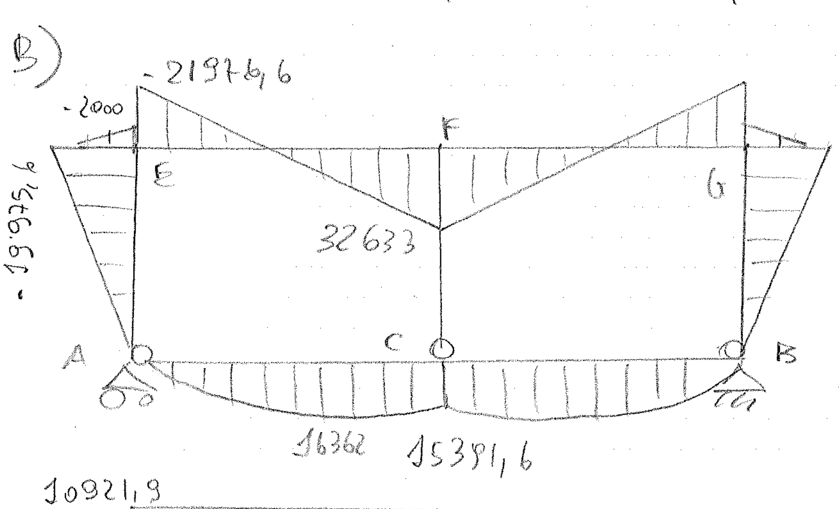
$M_C = 25000 + 2,5.(-3843,38) = 15391,6$ Kp.m

$M_{EA} = -43000 - 1(-32632,9) + 2,5(-3843,38) = -19975,6$ "

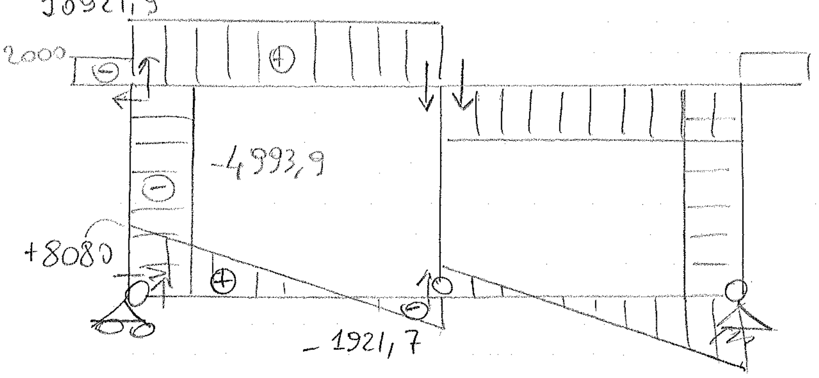
$M_{EF} = -45000 - 1.(-32632,9) + 2,5(-3843,38) = -21975,6$ "

$M_F = -1(-32632,9) = 32632,9$ "

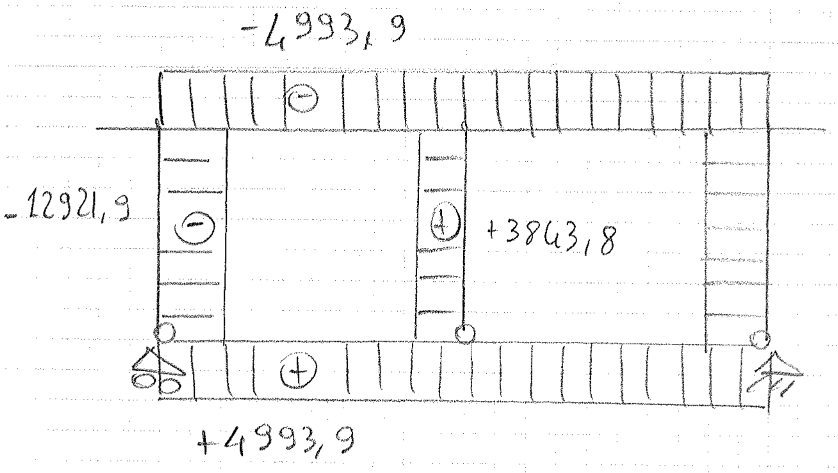
B)



(M) [Kp.m]



(T) [Kp]



N [kg]

c) $M_{max} = 32633$ kg.m

$T = 10921,9$ kg

$N = -4993,9$ kg

TUBI RETTANGOLARI

$$W_{min} = \frac{3263300}{2600} = 1255,12$$

$\Rightarrow$ TUBI 380 x 266 : $W_x = 1368\ cm^3$

$J_x = 25988\ cm^4$

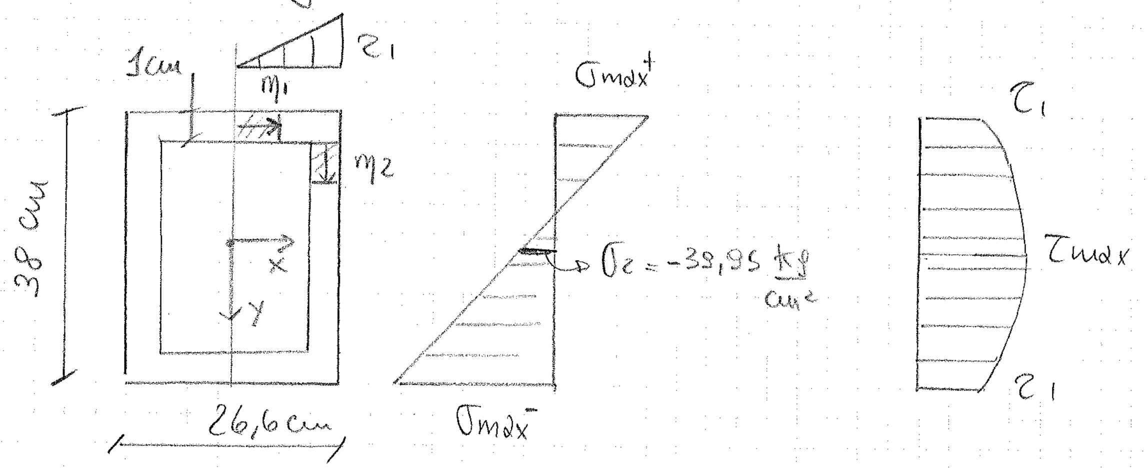
$s = 10$ mm

$A = 125\ cm^2$

D)

$$\sigma = \frac{N}{A} \pm \frac{M}{W} = \underbrace{-\frac{4993,9}{125}}_{-39,95} \pm \frac{3263300}{1368} \begin{cases} \sigma_{max}^{+} = 2365,5\ kg/cm^2 \\ \sigma_{max}^{-} = -2425,4\ " \end{cases}$$

$< 2600\ kg/cm^2$



$S_1 = -\eta_1 \cdot s \left(\frac{h}{2} - \frac{s}{2}\right)$; $S_{1\,max}\left(\eta_1 = \frac{b}{2}\right) = -246,1\ cm^3$

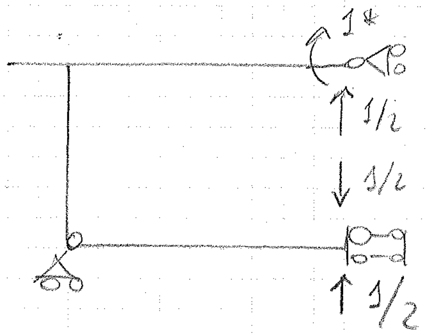
$$\tau_1 = \frac{10921,9 \cdot 246,1}{1 \cdot 25988} = 103,43\ \frac{kg}{cm^2}$$

$S_2 = -\eta_2 \cdot s \cdot \left(\frac{h}{2} - s - \frac{\eta_2}{2}\right)$; $S_{2\,max}\left(\eta_2 = \frac{h}{2} - s\right) = -162\ cm^3$

$$\tau_{max} = \frac{10921,9 \cdot 162}{1 \cdot 25988} + \tau_1 = 171,51\ kg/cm^2$$

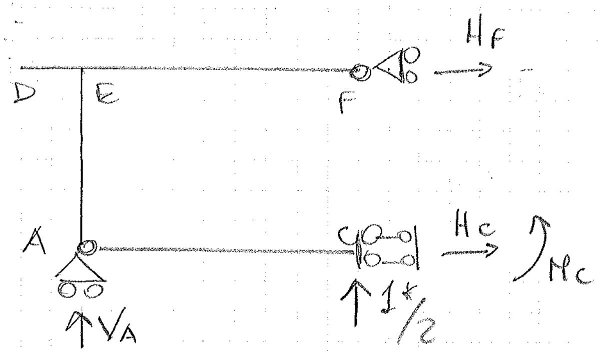
$$\left.\begin{array}{l} \sigma_{id}^{(1)} = \sqrt{\sigma_{max}^2 + 3 \cdot \tau_1^2} = 2432,01\ kg/cm^2 \\ \sigma_{id}^{(2)} = \sqrt{\sigma_2^2 + 3 \cdot \tau_{max}^2} = 299,73\ kg/cm^2 \end{array}\right\} < 2600\ kg/cm^2$$

E) Considero un'ulteriore incognita iperstatica ovvero le forze che nasce nella molla



Aggiungo quindi un sistema 3

SISTEMA 3



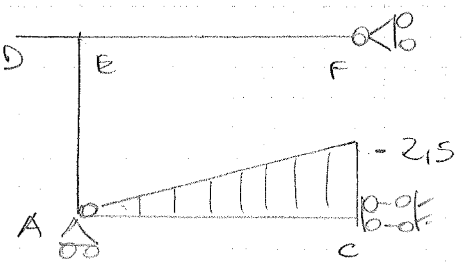
$\uparrow)\ V_A + 1 = 0 \Rightarrow V_A = -1$

$\rightarrow)\ H_F + H_C = 0$

$A \circlearrowright_{AC}\ 1.5 + M_C = 0 \Rightarrow M_C = -5$

$A \circlearrowright_{AEF}\ -H_F \cdot 4 = 0 \Rightarrow H_F = 0$

$\Rightarrow H_C = 0$



$$M_{AC}(z_1) = -\frac{1 \cdot z_1}{2}$$

Aggiungo quindi i seguenti η_{ij}

$$\eta_{30} = \frac{1}{EJ}\left\{\int_0^5 (10'000 \cdot z - 1000 \cdot z^2)\left(\frac{-z}{2}\right) dz\right\} = \frac{-130'209}{EJ}$$

$$\eta_{31} = 0$$

$$\eta_{32} = \frac{1}{EJ}\int_0^5 (0{,}5 \cdot z)\left(\frac{-z}{2}\right) dz = -\frac{10{,}417}{EJ}$$

$$\eta_{33} = \frac{1}{EJ}\int_0^5 \frac{z^2}{4}\, dz + \left(\frac{1^*}{2} \cdot \frac{1^*}{2} \cdot \frac{1}{K/2}\right)$$

$$= \frac{10{,}417}{EJ} + 5 \cdot 10^{-5}$$

$$\frac{1}{EJ}\begin{pmatrix} 6{,}333 & -9{,}583 & 0 \\ -9{,}583 & 29{,}166 & -10{,}417 \\ 0 & -10{,}417 & 283{,}28 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \frac{1}{EJ}\begin{pmatrix} -169833 \\ 200'625 \\ 130'209 \end{pmatrix}$$

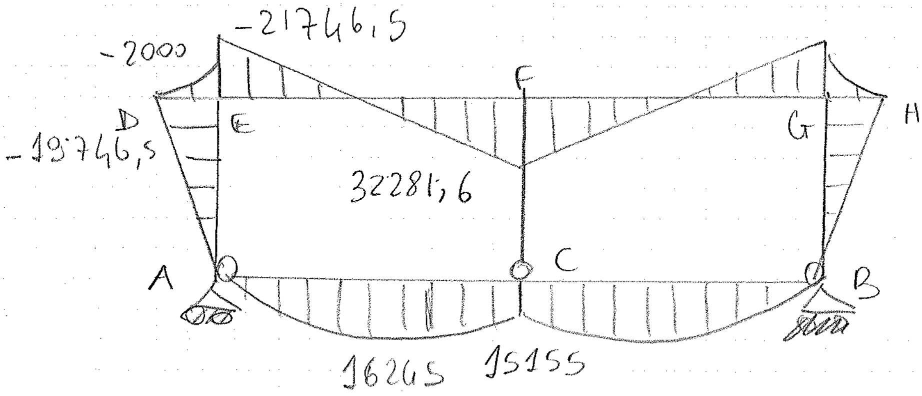
$$\Rightarrow \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} -32'281{,}6 \\ -3611{,}22 \\ 326{,}84 \end{pmatrix}$$

$$M_C = 25'000 + 2{,}5(-3611{,}22) - 2{,}5(326{,}84) = 15'155 \ \text{Kg} \cdot \text{m}$$

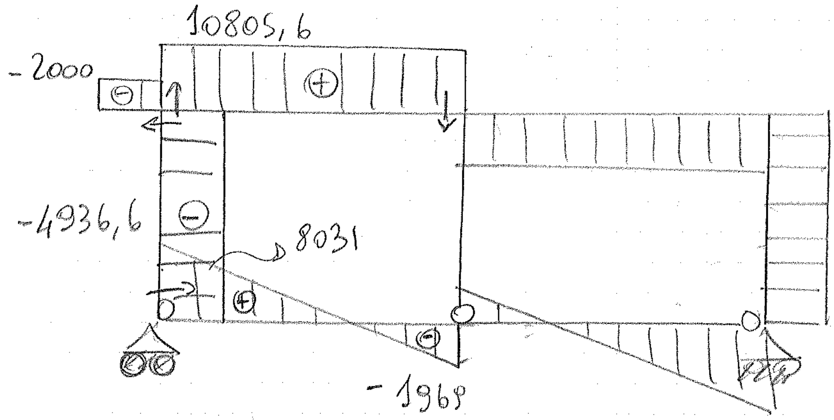
$$M_{EA} = -43'000 - 1(-32'281{,}6) + 2{,}5(-3611{,}22) = -19746{,}5 \ \text{"}$$

$$M_{EF} = -45'000 - 1(-32281{,}6) + 2{,}5(-3611{,}22) = -21746{,}5 \ \text{"}$$

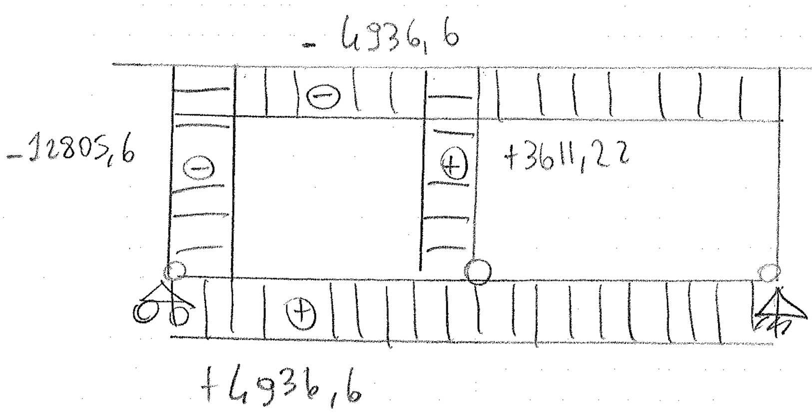
$$M_F = -1(-32281{,}6) = 32'281{,}6 \ \text{"}$$



(M) [Kp m]

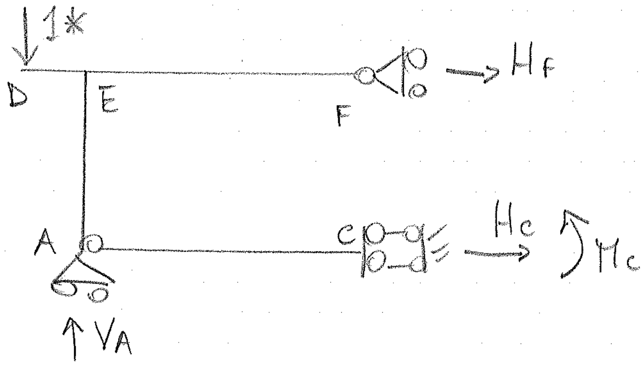


(T) [Kp]



(N) [Kp]

F) Abbassamento del punto D



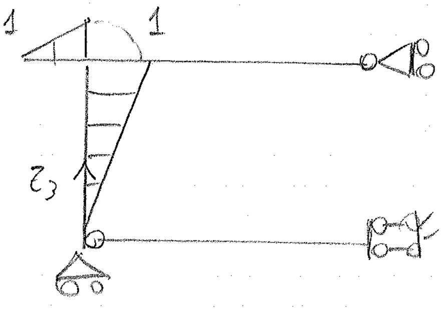
$\uparrow)\ V_A = 1$

$\rightarrow)\ H_C + H_F = 0$

$A \circlearrowleft_{AC} \quad M_C = 0$

$A \circlearrowleft_{AEF} \quad -H_F \cdot 4 + 1 = 0$

$\Rightarrow H_F = \frac{1}{4} \Rightarrow H_C = -\frac{1}{4}$

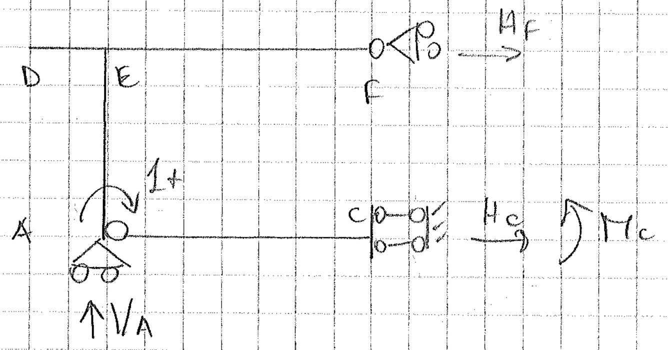


$M_{AE}(z_3) = \frac{1}{4} \cdot z$

$$1^* \cdot v_D = \frac{1}{EJ}\left\{\int_0^4 (-4936,6 \cdot z)\left(\frac{z}{4}\right)dz\right\} = -\frac{26328,5}{EJ}$$

$$\Rightarrow v_D = -0,0048 \text{ m} = -4,8 \text{ mm}$$

Per trovare la rotazione in A, applico una coppia unitaria in A nel sistema isostatico



↑) $V_A = 0$

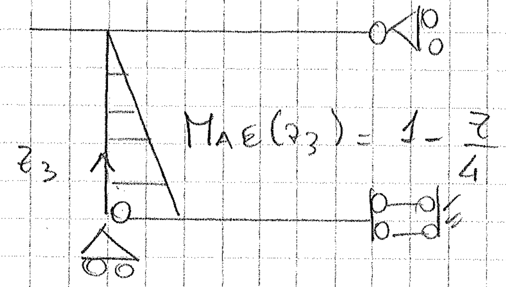
→) $H_F + H_C = 0$

$A\circlearrowleft_{AC}$ $M_C = 0$

$A\circlearrowleft_{AEF}$ $-1 - H_F \cdot 4 = 0$

$\Rightarrow H_F = -\frac{1}{4}$

$\Rightarrow H_C = \frac{1}{4}$



$$1^* \cdot \varphi_A = \frac{1}{EJ}\int_0^4 (-4936,6 \cdot z)\left(1 - \frac{z}{4}\right)dz = -\frac{13164,3}{EJ}$$

$$\Rightarrow \varphi_A = -0,002412$$