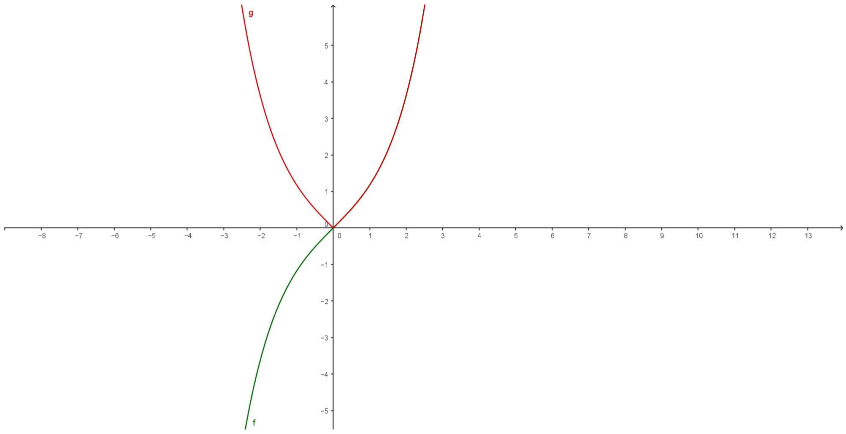
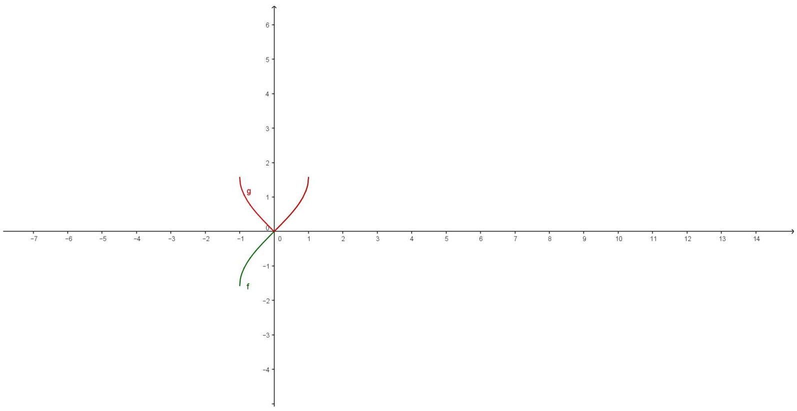
Esempio: dal grafico di $y = Sinh(x)$ dedurre il grafico di $y = Sinh(|x|)$,

schematicamente: $y = Sinh(x) \to y = Sinh(|x|)$



$y = \arcsin(x) \to y = \arcsin(|x|)$



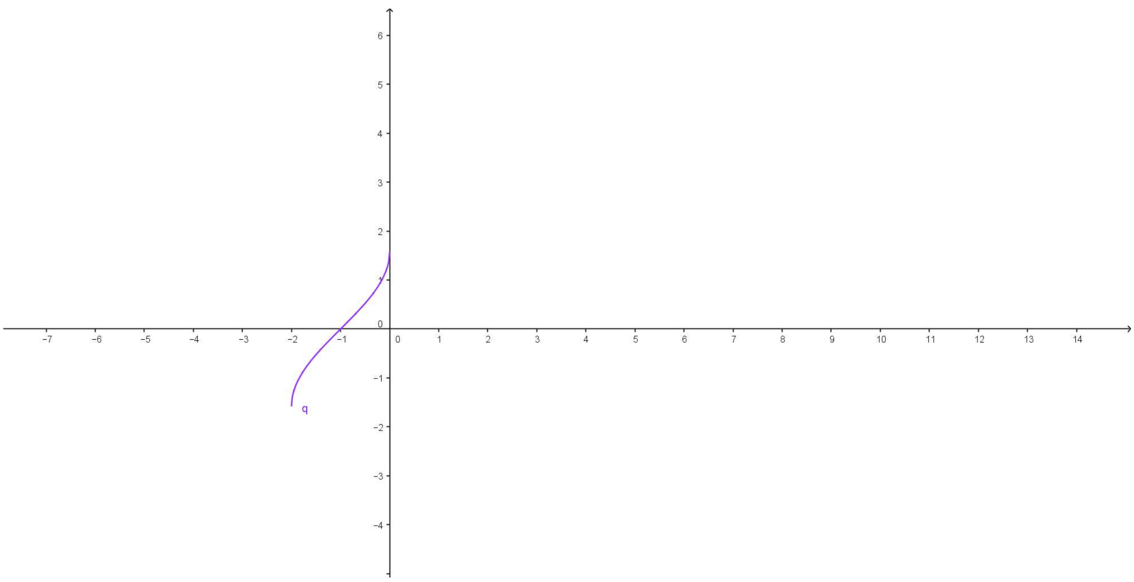
$$y = \arcsin(|x|+1) = \begin{cases} \arcsin(x+1), x \geq 0 \\ \arcsin(-x+1), x < 0 \end{cases}$$

$y = \arcsin(x+1)$

per $x \geq 0$,

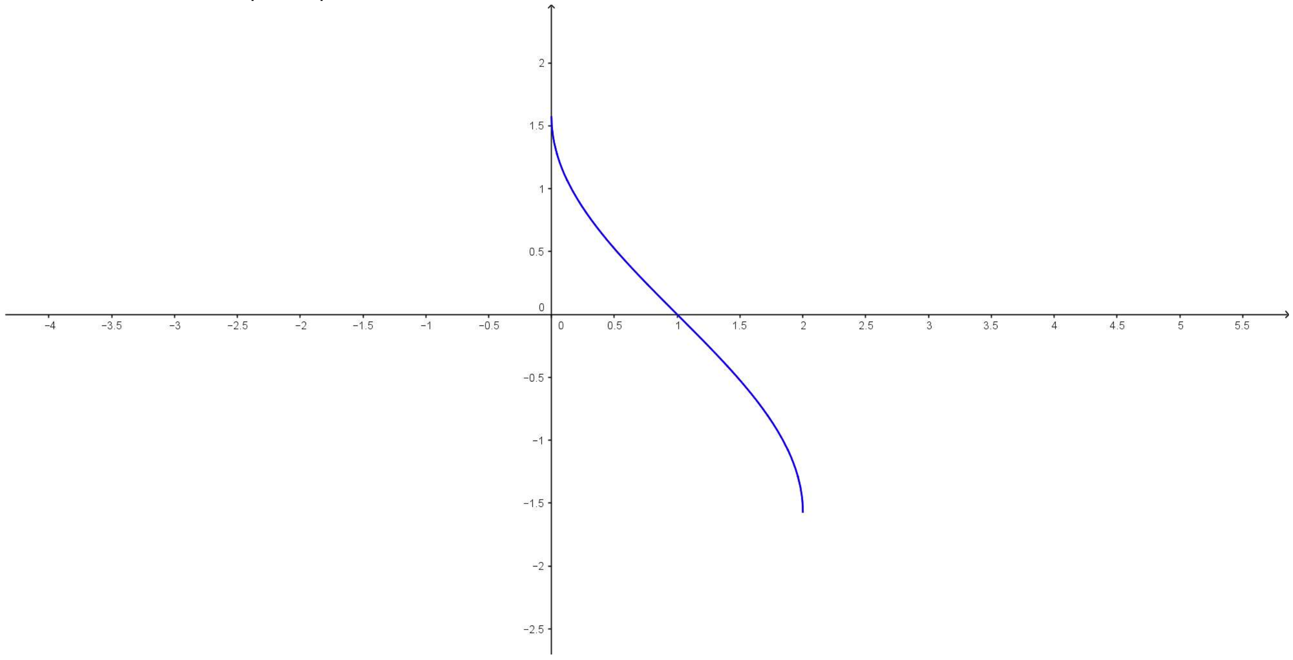
quindi il solo punto

$\left(0, \frac{\pi}{2}\right)$



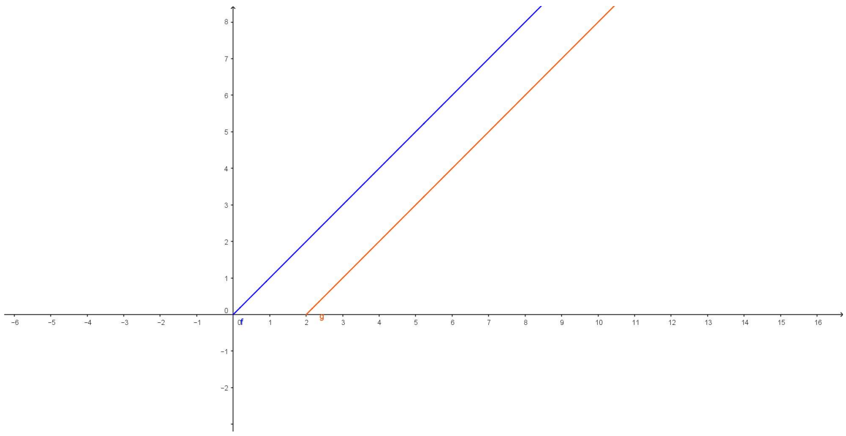
$y = \arcsin(-x+1)$, per $x < 0$

La curva non ha alcun punto per $x < 0$

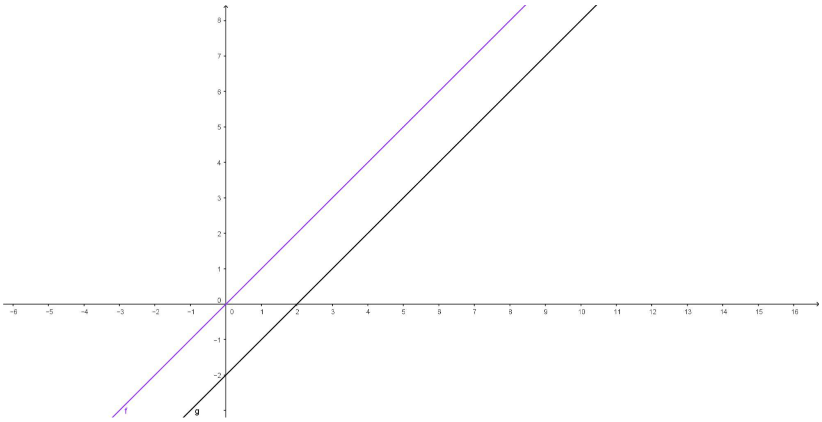


quindi il grafico della funzione si riduce al solo punto: $\left(0, \frac{\pi}{2}\right)$

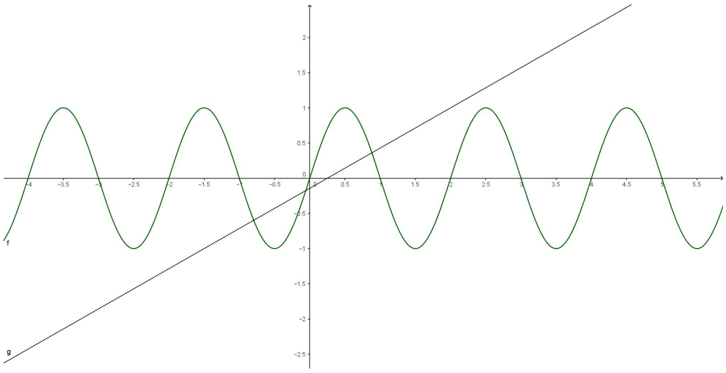
$y = e^{\log x}$ e $y = e^{\log(x-2)}$ Il dominio della prima funzione è : $x > 0$, della seconda: $x > 2$.



$y = \log e^{x}$ e $y = \log e^{x-2}$, per entrambe le funzioni il dominio è $\mathbb{R}$

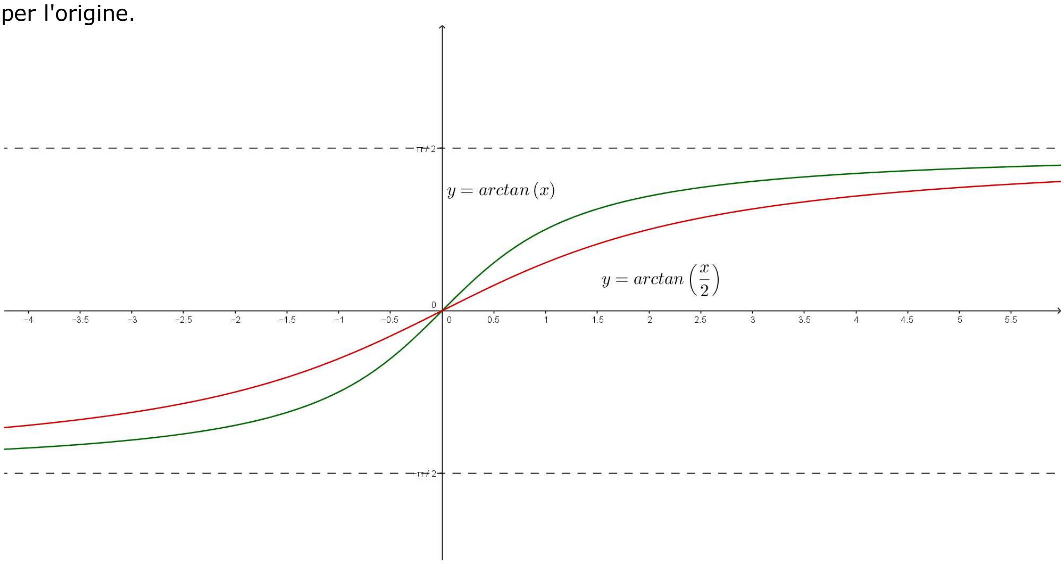


$\sin(\pi x) = \frac{4x-1}{7}$

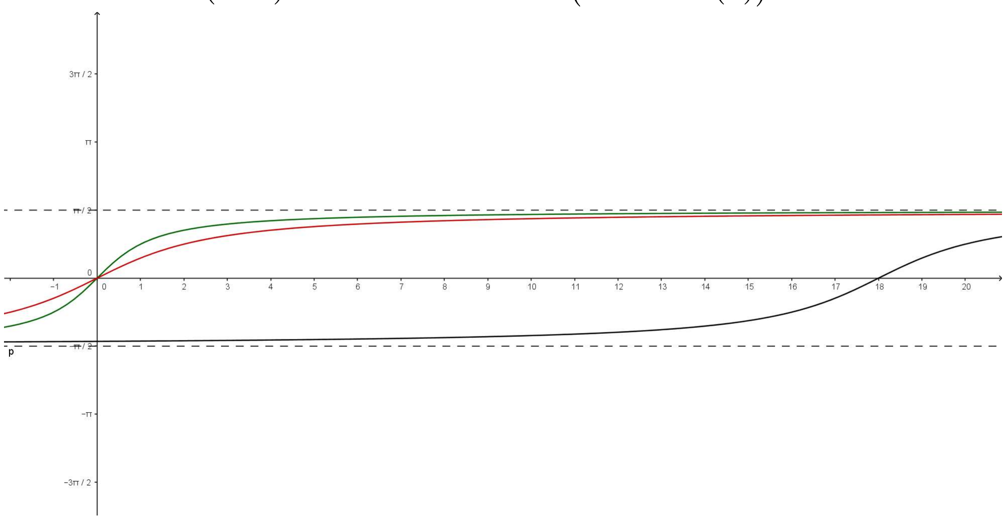


$y = \left| \arctan\left(\frac{x}{2} - 9\right) - \frac{\pi}{4} \right|$

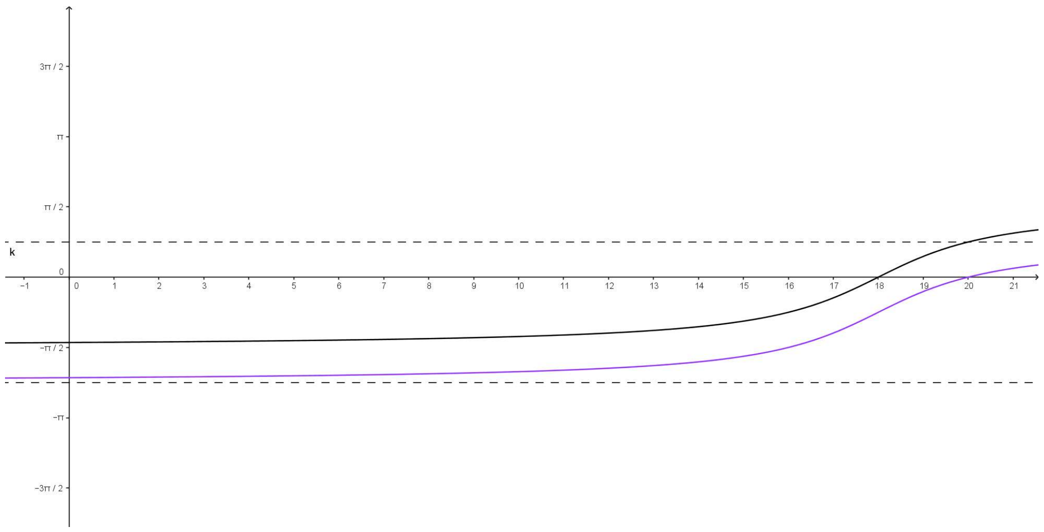
$y = arc\tan(x) \rightarrow y = arc\tan\left(\frac{x}{2}\right)$. Entrambe hanno per asintoti le rette di equazione $y = \pm\frac{\pi}{2}$ e passano entrambe per l'origine.



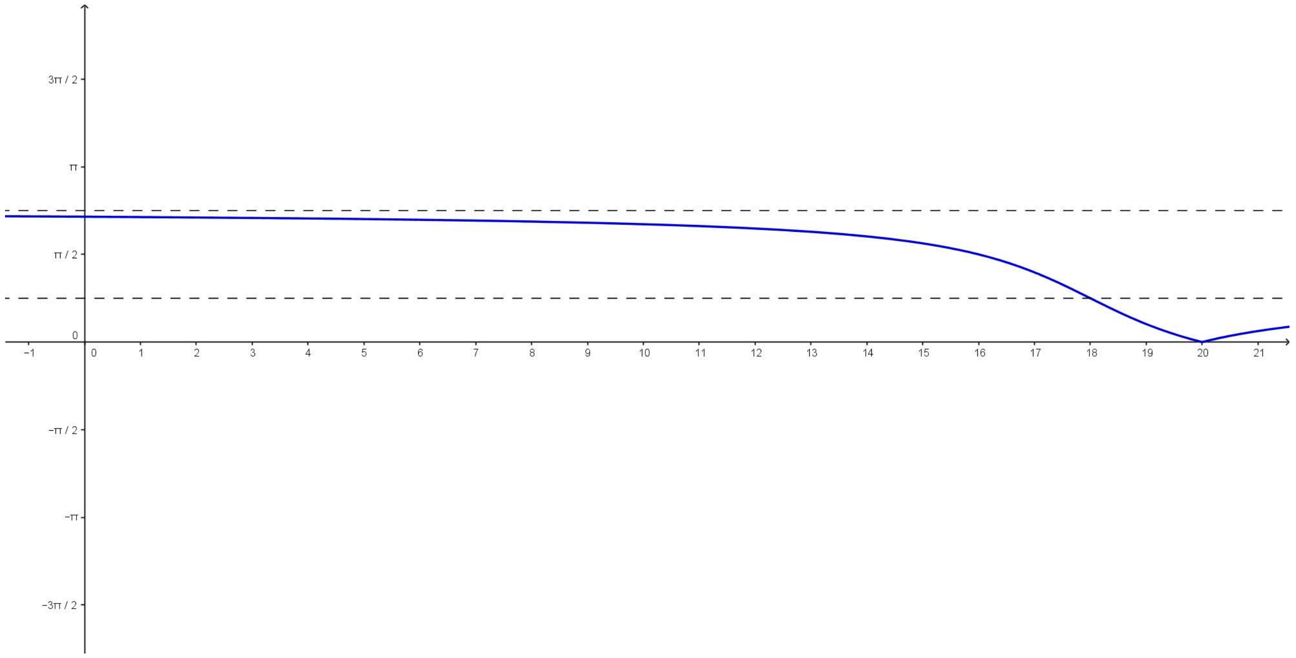
$y = arc\tan\left(\frac{x}{2}\right) \rightarrow y = arc\tan\left(\frac{x}{2} - 9\right) = \arctan\left[\frac{1}{2}(x-18)\right]$. Gli asintoti rimangono $y = \pm\frac{\pi}{2}$. La curva interseca l'asse delle ascisse in $(18, 0)$ e l'asse delle ordinate in $(0, -\arctan(9))$



$y = arc\tan\left(\frac{x}{2}-9\right) \to y = arc\tan\left(\frac{x}{2}-9\right)-\frac{\pi}{4}$. Gli asintoti diventano $y=-\frac{3}{4}\pi$ e $y=\frac{\pi}{4}$ La curva interseca l'asse delle ascisse in $(20,0)$ e l'asse delle ordinate in $\left(0,-\arctan(9)-\frac{\pi}{4}\right)$



$y = arc\tan\left(\frac{x}{2}-9\right)-\frac{\pi}{4} \to y = \left|arc\tan\left(\frac{x}{2}-9\right)-\frac{\pi}{4}\right|$. Gli asintoti diventano $y=+\frac{3}{4}\pi$ e $y=\frac{\pi}{4}$ La curva interseca l'asse delle ascisse in $(20,0)$ e l'asse delle ordinate in $\left(0,+\arctan(9)+\frac{\pi}{4}\right)$.



Calcolo di limiti:

a. $\lim_{n\to\infty} n^{\frac{1}{\log_a(n+1)}}$, con $a>1$, indeterminazione $+\infty^0$

$n^{\frac{1}{\log_a(n+1)}} = a^{\log_a n^{\frac{1}{\log_a(n+1)}}} = a^{\frac{1}{\log_a(n+1)}\log_a n} = a^{\frac{\log_a n}{\log_a(n+1)}} \sim a^{\frac{\log_a n}{\log_a n}} = a$, quindi: $\lim_{n\to\infty} n^{\frac{1}{\log_a(n+1)}} = a$

b. $\lim_{n\to\infty}\left[\log_2(n+1)-\log_2\sqrt{n^2+3}\right]$ indeterminazione $+\infty-\infty$

$\log_2(n+1)-\log_2\sqrt{n^2+3}=\log_2\frac{(n+1)}{\sqrt{n^2+3}}\sim\log_2\frac{n}{n}=\log_2 1=0$, quindi:

$\lim_{n\to\infty}\left[\log_2(n+1)-\log_2\sqrt{n^2+3}\right]=0$

c. $\lim_{n\to\infty}\frac{n+\log(n^2)-2^n}{(\log n)^3+n^2}$ Forma indeterminata $\frac{\infty}{\infty}$

$\frac{n+\log(n^2)-2^n}{(\log n)^3+n^2}\sim\frac{-2^n}{n^2}$, $\lim_{n\to\infty}\frac{n+\log(n^2)-2^n}{(\log n)^3+n^2}=\lim_{n\to\infty}\frac{-2^n}{n^2}=\lim_{n\to\infty}(-2^n)=-\infty$

d. $\lim_{n\to\infty}\left[n\left(\sqrt{n^2+1}-n\right)\right]$ Forma indeterminata $+\infty-\infty$

$\left(\sqrt{n^2+1}-n\right)\sim n-n=0$ non possiamo utilizzare l'asintotico!

$$\lim_{n\to\infty}\left[n\left(\sqrt{n^2+1}-n\right)\right]=\lim_{n\to\infty}\frac{n\left(\sqrt{n^2+1}-n\right)\left(\sqrt{n^2+1}+n\right)}{\left(\sqrt{n^2+1}+n\right)}=\lim_{n\to\infty}\frac{n}{\left(\sqrt{n^2+1}+n\right)}=\lim_{n\to\infty}\frac{n}{2n}=\frac{1}{2}$$

e. $\lim_{n\to\infty}\frac{n^2\sin(n)+\sin(n^2)}{n^2+1}$ Forma indeterminata $\frac{\infty}{\infty}$

$$\lim_{n\to\infty}\frac{n^2\sin(n)+\sin(n^2)}{n^2+1}=\lim_{n\to\infty}\left[\frac{n^2}{n^2+1}\sin(n)+\frac{1}{n^2+1}\sin(n^2)\right]$$

Il secondo limite tende a zero (infinitesimo per limitato) il primo limite non esiste in quanto: $\frac{n^2}{n^2+1}\sin(n)\sim\sin(n)$ che non ha limite per $n\to\infty$.

f. $\lim_{n\to\infty}\left(\frac{n+2}{n-1}\right)^{2n}$ Forma indeterminata $1^{+\infty}$, riconducibile al limite fondamentale $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$

$$\lim_{n\to\infty}\left(\frac{n+2}{n-1}\right)^{2n}=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n-1}{3}}\right)^{\frac{n-1}{3}\cdot\frac{6n}{n-1}}=e^6$$

g. $\lim_{n\to\infty}\left(\frac{n^2+n-1}{n^2-3n+4}\right)^n$ Forma indeterminata $1^{+\infty}$, riconducibile al limite fondamentale

$$\lim_{n\to\infty}\left(\frac{n^2+n-1}{n^2-3n+4}\right)^n=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2-3n+4}{4n-5}}\right)^{\frac{n^2-3n+4}{4n-5}\cdot\frac{n(4n-5)}{n^2-3n+4}}=e^4$$

h. $\lim_{n\to\infty}\left[\log\left(n+2^n\right).\frac{n^n}{(n+1)^{n+1}}\right]$ Indeterminata $\infty\cdot 0$

$$\log\left(n+2^n\right)\sim\log 2^n=n\cdot\log 2$$

$$\frac{n^n}{(n+1)^{n+1}}=\left(\frac{n}{n+1}\right)^n\cdot\frac{1}{n+1}\sim\frac{1}{e}\cdot\frac{1}{n}=\frac{1}{ne}$$

$$\log\left(n+2^n\right).\frac{n^n}{(n+1)^{n+1}}\sim n\cdot\log 2\cdot\frac{1}{ne}=\frac{\log 2}{e}$$

quindi: $\lim_{n\to\infty}\left[\log\left(n+2^n\right).\frac{n^n}{(n+1)^{n+1}}\right]=\frac{\log 2}{e}$

i. $\lim_{n\to\infty}\left[\frac{1}{3+\sin(2n)}\right]^n$

$0<\frac{1}{3+\sin(2n)}<1$, quindi $\lim_{n\to\infty}\left[\frac{1}{3+\sin(2n)}\right]^n=0$.

Più dettagliatamente:

$$-1\le\sin(2n)\le 1\Rightarrow 2\le 3+\sin(2n)\le 4\Rightarrow\frac{1}{4}\le\frac{1}{3+\sin(2n)}\le\frac{1}{2}$$

$$\left(\frac{1}{4}\right)^n\le\left(\frac{1}{3+\sin(2n)}\right)^n\le\left(\frac{1}{2}\right)^n$$

Le successioni $\left(\frac{1}{4}\right)^n;\left(\frac{1}{2}\right)^n$ sono successioni geometriche convergenti a zero, quindi per il teorema del confronto si ha: $\lim_{n\to\infty}\left[\frac{1}{3+\sin(2n)}\right]^n=0$.