Remarks


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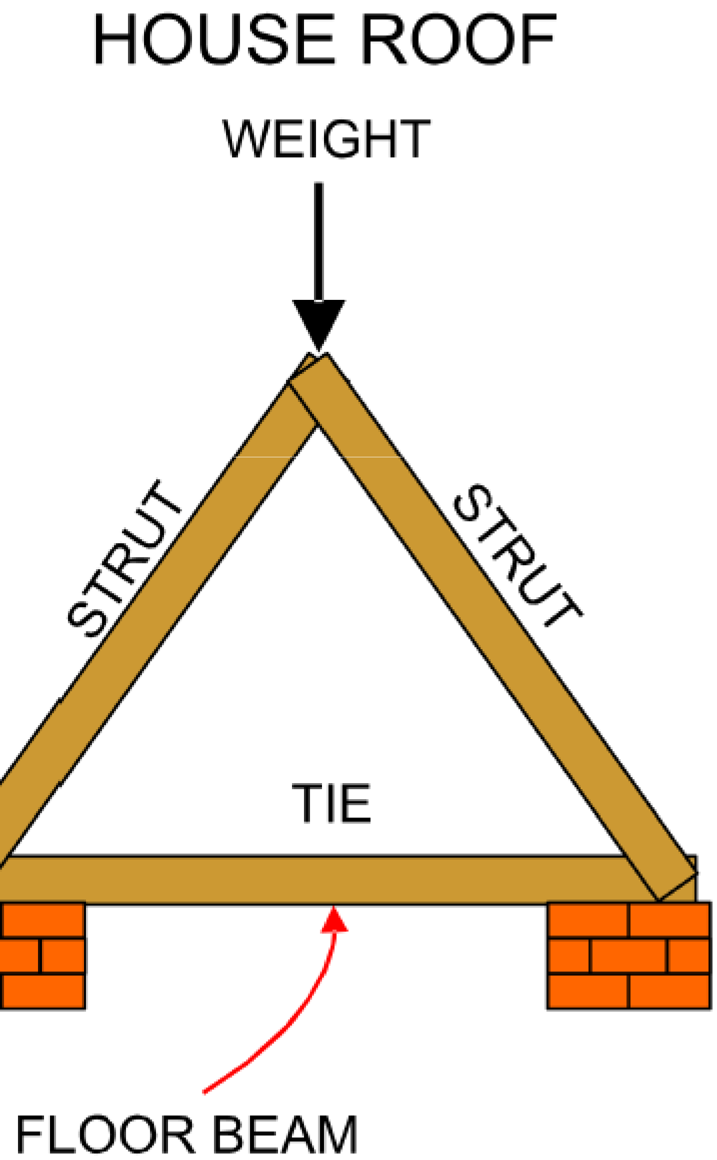
Solid Mechanics
and its Applications
Plates and FEM
Surprises and Pitfalls
Springer

Strut and tie

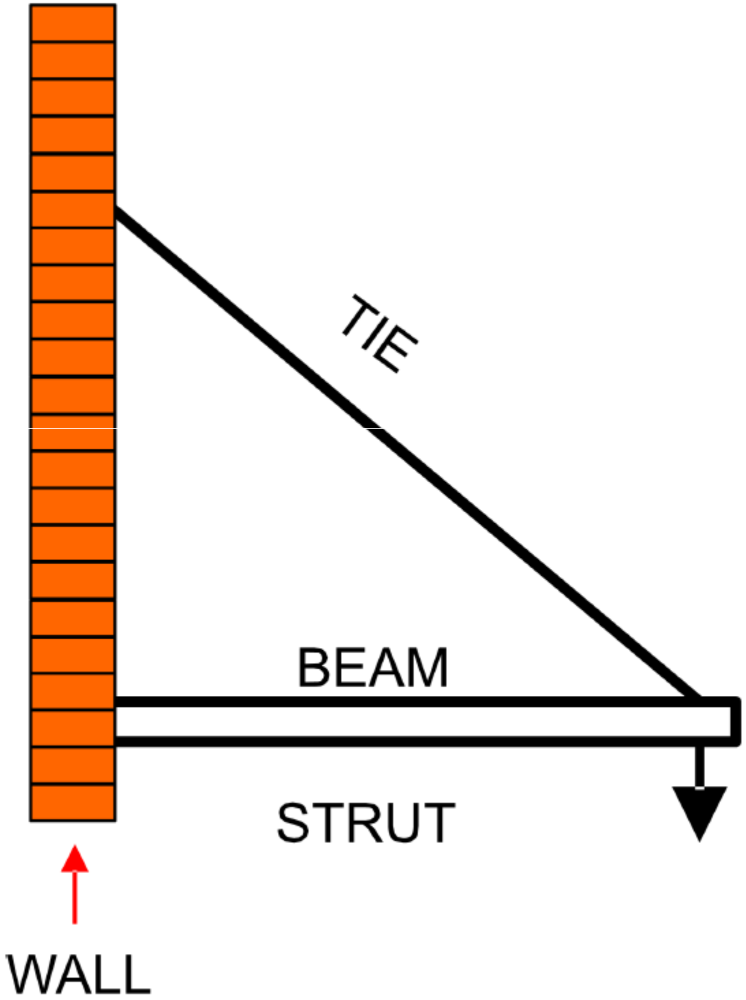
From the computed stress state we can compute two principal stresses and their direction

Trajectories are an instructive and insight-providing aid to structural designers.

Strut and tie : tirante-puntone

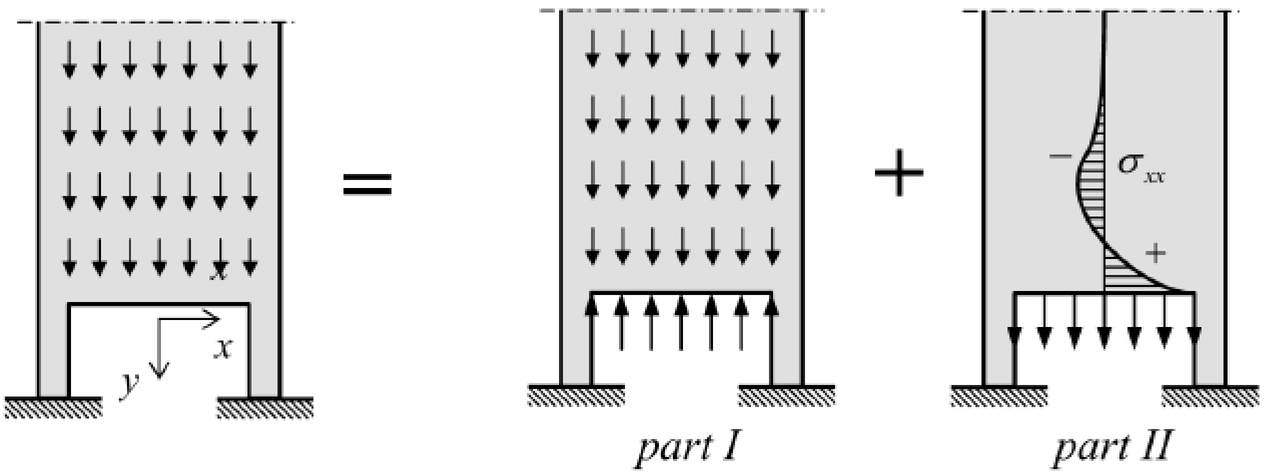


Strut and tie: tirante-puntone



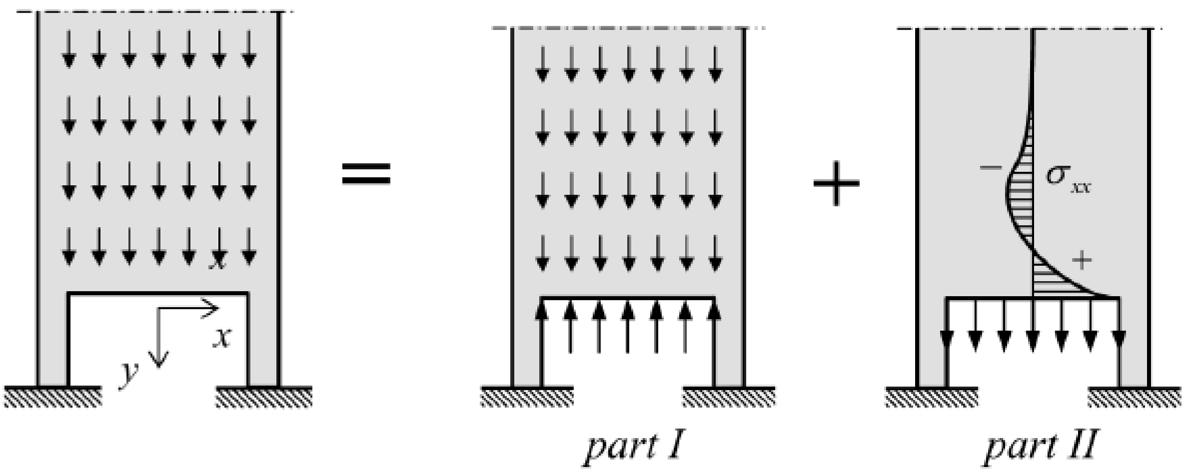
Some remarks

The discussed case of a high wall (d/l>>1) can be used to estimate the stress distribution in practical structures. An example of this is a silo wall on columns, loaded by a uniformly distributed load. This may be its own weight, and wall friction forces due to the bulk material in the silo. To estimate the horizontal stress σxx in the wall halfway between the columns, we adopt the following approach. The load can be split up into two parts



Some remarks

Part one is a simple stress state in which only vertical stresses σyy are present and no stresses σxx occur. We are not interested in this part.
The second part is the load case in which the solution for the high wall can be applied. Structural engineers who must design reinforced concrete walls often apply truss models for the determination of the reinforcement. For the silo wall they may concentrate the total distributed load in two forces F as shown below. Each support reaction R is equal to F. The green lines carry compressive forces and the red line the tensile force. The structural engineer wants to know where to place the horizontal compressive strut and the tensile tie, because the distance between them influences the magnitude of the forces in the strut and tie. Knowledge about the elastic solution will be a great help.



Some remarks

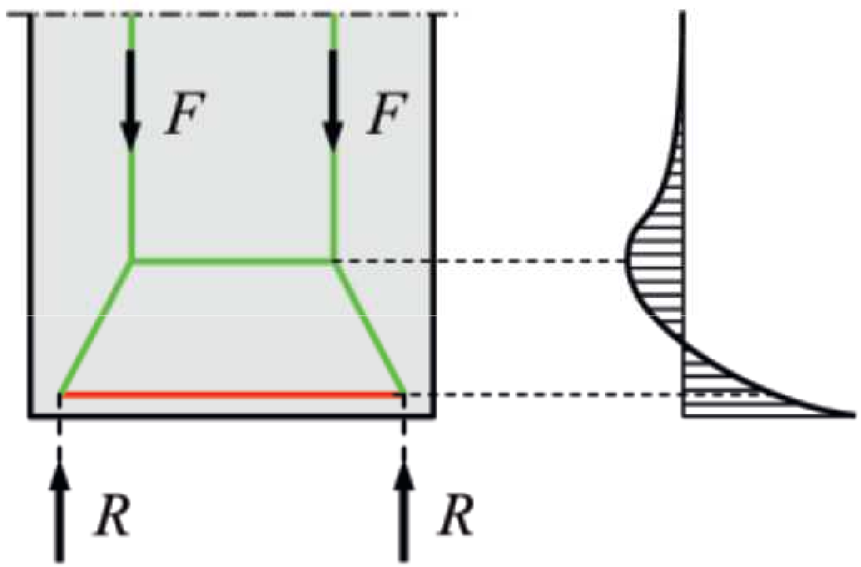


Figure 2.15 Strut-and-tie model for silo wall.

Strut and tie

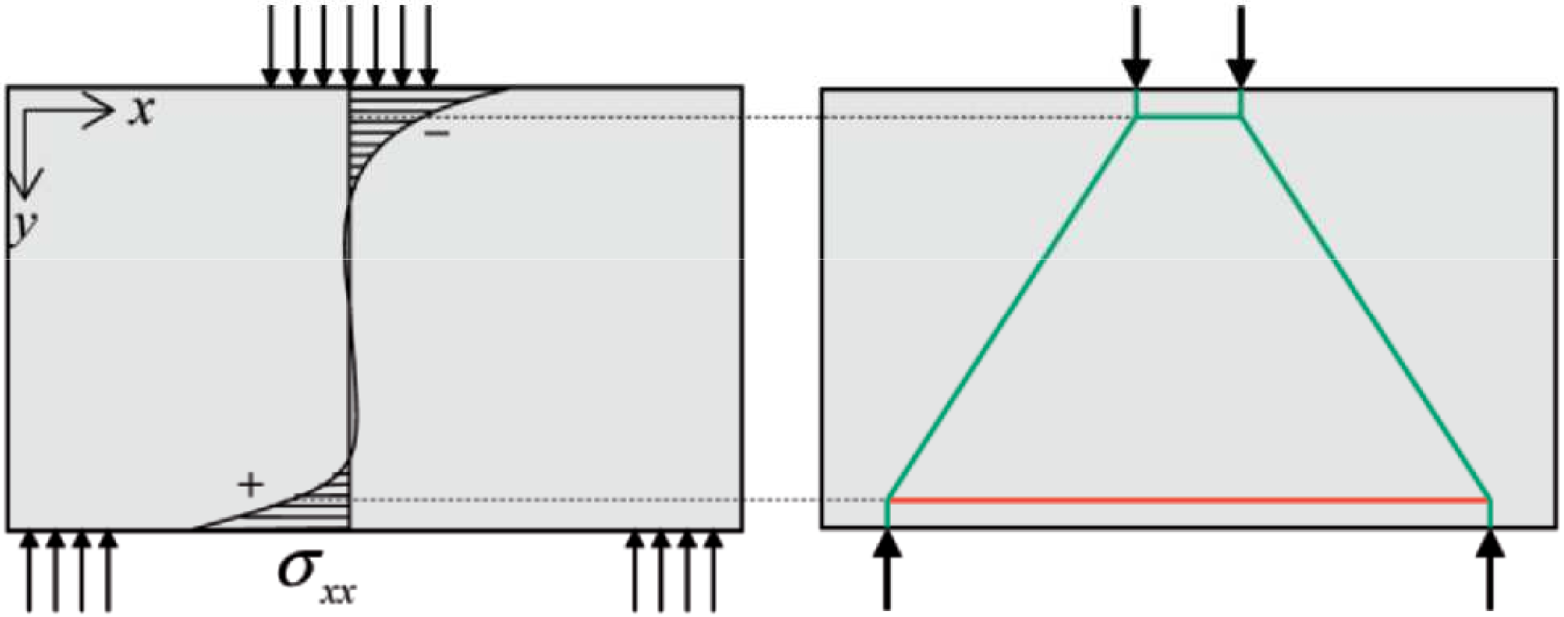


Figure 2.20 Foundation block. Stresses and strut-and-tie scheme.

Strut and tie

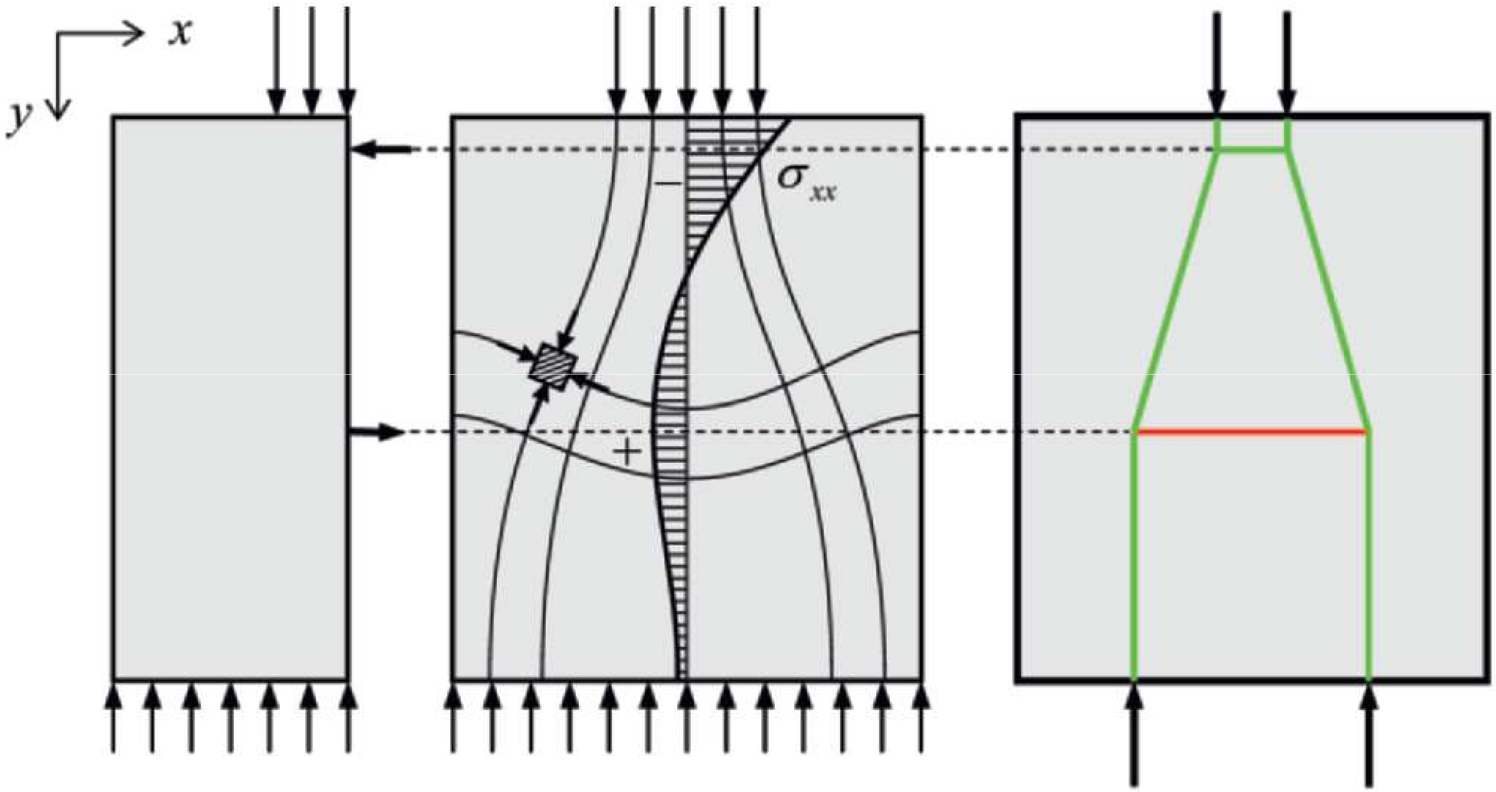


Figure 2.21 Load spreading (for example the anchorage of a pre-stressed cable in a beam).

Strut and tie

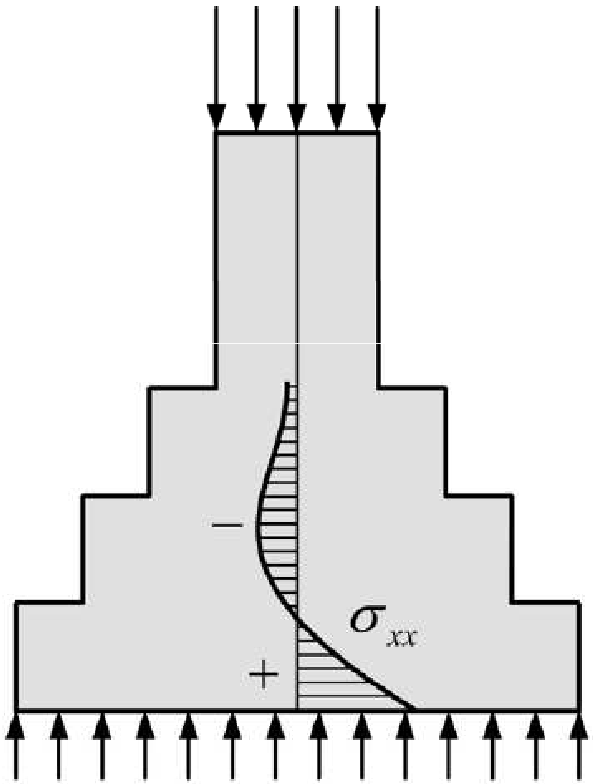


Figure 2.22 Foundation foot. Stresses and strut-and-tie scheme.

Strut and tie

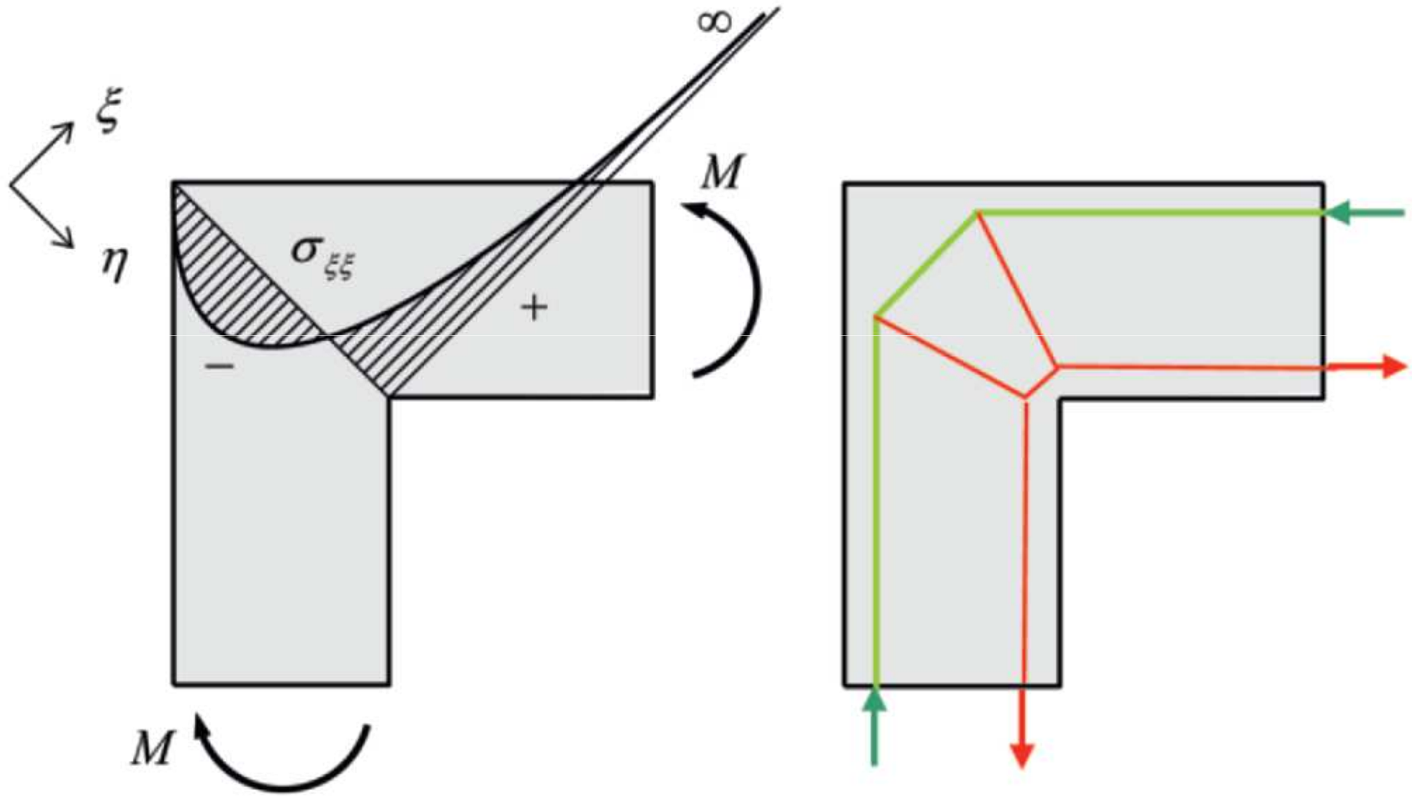


Figure 2.24 Beam-column connection. Stresses and strut-and-tie scheme.

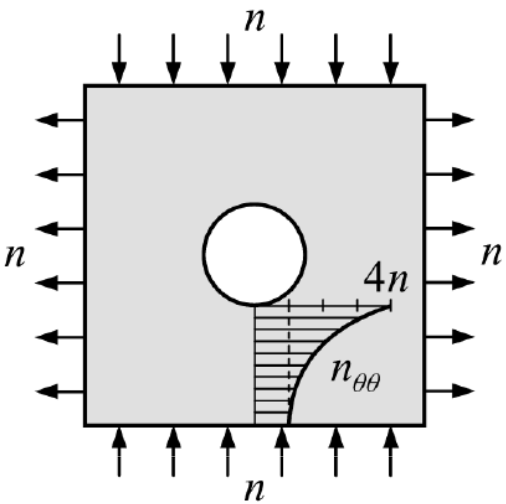
Strut and tie

The membrane solution for a constant shear force, in combination with a linearly varying bending moment, deviates from classical beam theory. Plane sections are no longer plane after loading. A linear distribution of bending stresses over the depth of the beam is accompanied by a distorted cross-section. The simple formulas for deflection and rotation in classical beam theory must be amended for shear deformation. This amendment is negligible if the cantilever length is over five times the beam depth.

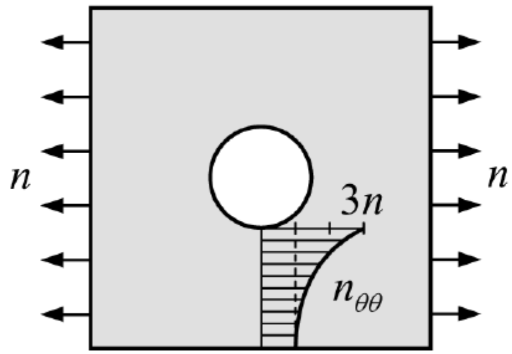
Strut and tie

The distribution of bending stresses in a shear wall is dependent on the ratio of the wall depth and span. Three aspect ratios are considered. For a high ratio (tall wall) the bending stress distribution is highly nonlinear, and the top part of the wall does not contribute to the load transfer. For a ratio in the order of unity (square wall) the distribution is still nonlinear, but the full cross-section participates in the transfer. For a low ratio (slender beam) the stress distribution approaches to the linear distribution of bending stress in classical beam theory.

Plates with holes



Constant shear



Uniaxial stress

$$n_{rr} = n\left(1 - 4\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)\cos 2\theta$$

$$n_{\theta\theta} = n\left(-1 - 3\frac{a^4}{r^4}\right)\cos 2\theta$$

$$n_{r\theta} = n\left(-1 - 2\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)\sin 2\theta$$

$$n_{rr} = \frac{n}{2}\left\{\left(1 - \frac{a^2}{r^2}\right) + \left(1 - 4\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)\cos 2\theta\right\}$$

$$n_{\theta\theta} = \frac{n}{2}\left\{\left(1 + \frac{a^2}{r^2}\right) - \left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta\right\}$$

$$n_{r\theta} = \frac{n}{2}\left\{-1 - 2\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right\}\sin 2\theta$$

Plates with holes

- **At a round hole in a homogeneous (hydrostatic) stress field the stress concentration factor is 2**

- **At a round hole in a uniaxial stress field the the stress concentration factor is 3**

- **For a constant shear stress field the stress concentration factor even gets the value 4**

FEM modelling of plates

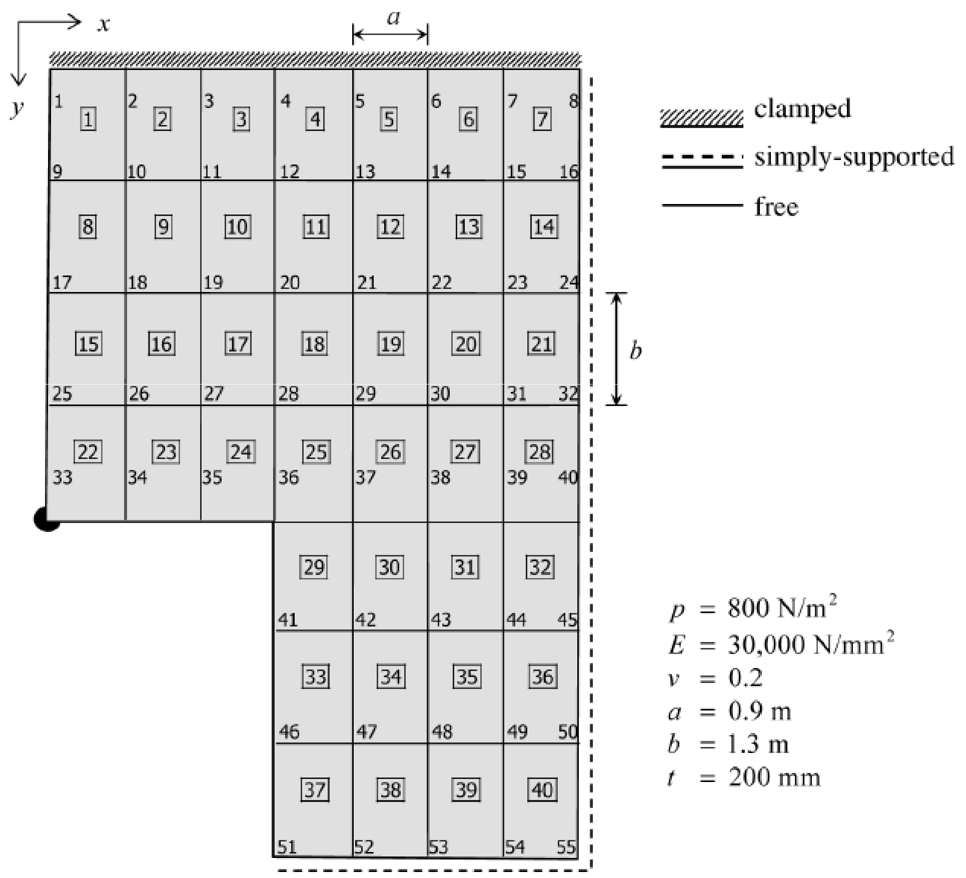
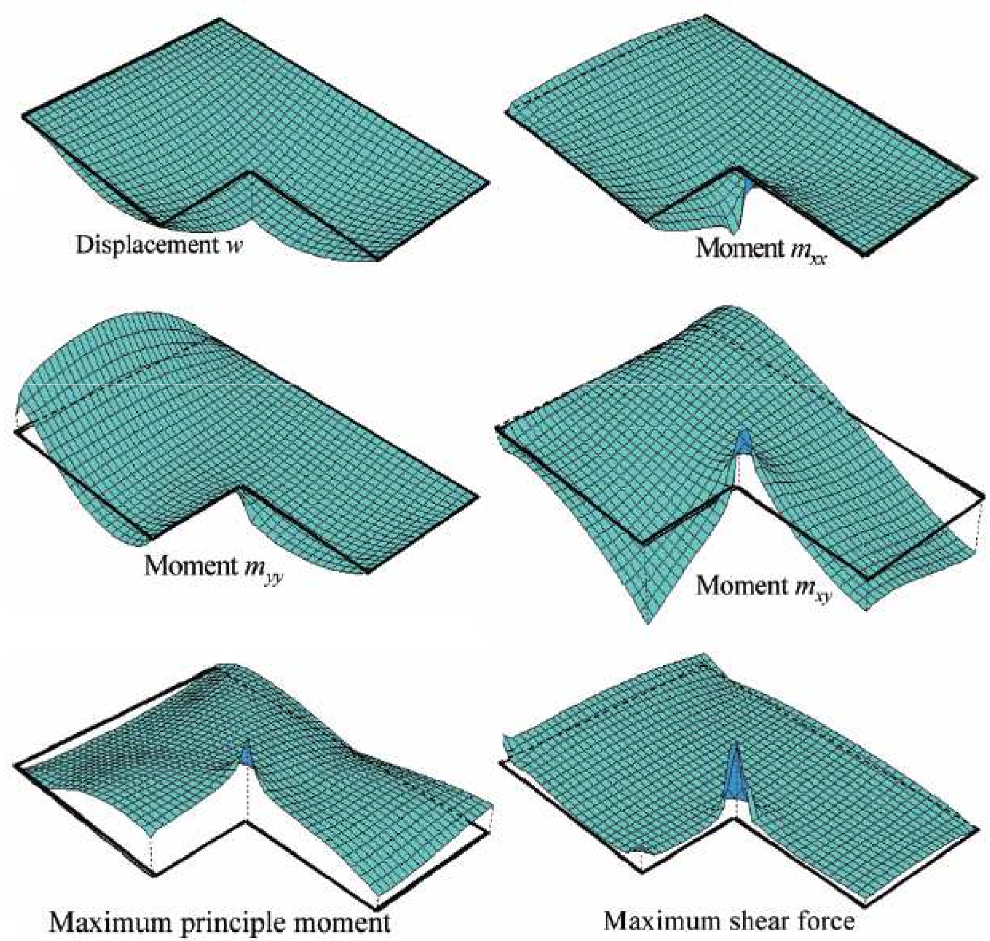
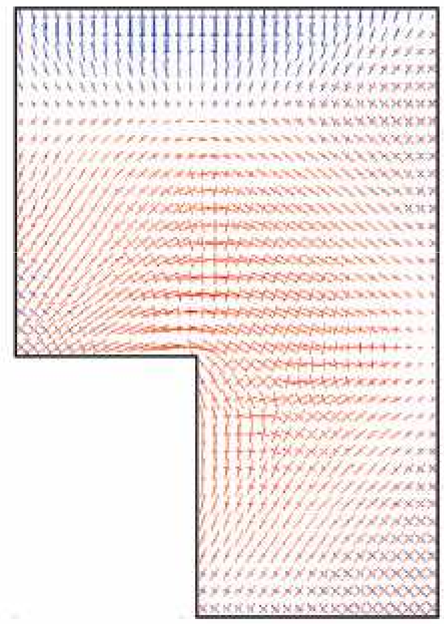


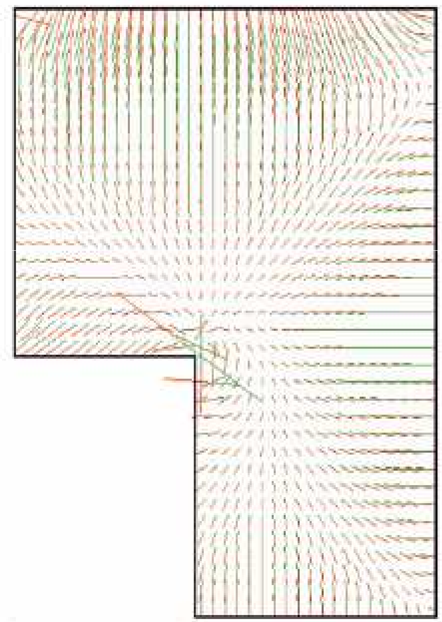
Figure 12.1 Mesh of a plate with node and element numbering.

FEM modelling of plates



FEM modelling of plates


Moment trajectories


Shear force trajectories

Lesson on singularities

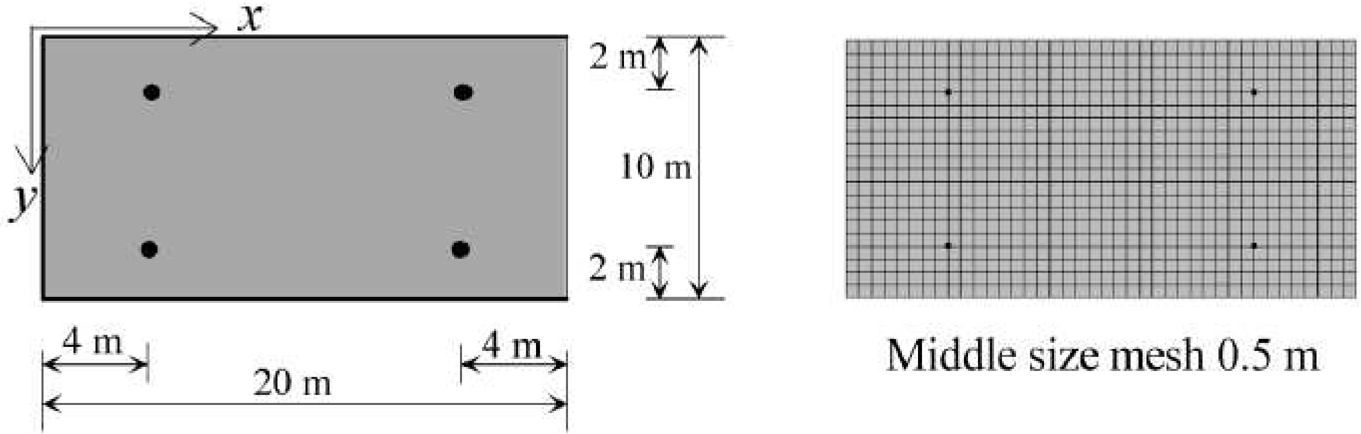
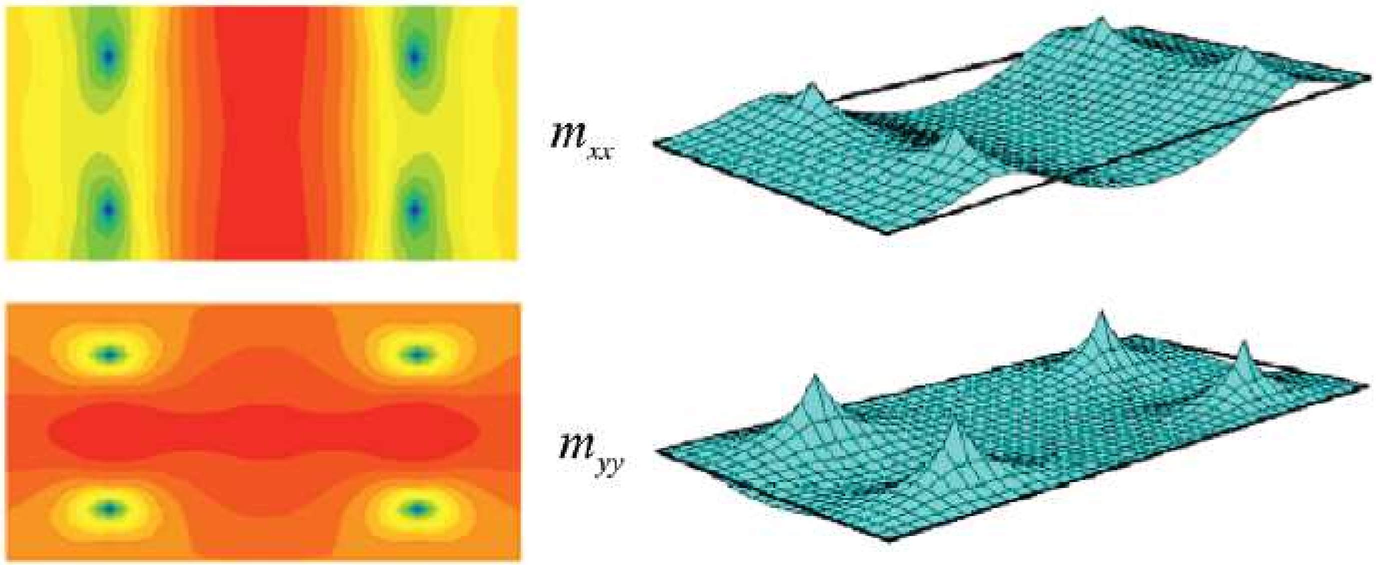


Figure 14.1 Rectangular slab on inward columns.



Lesson on singularities

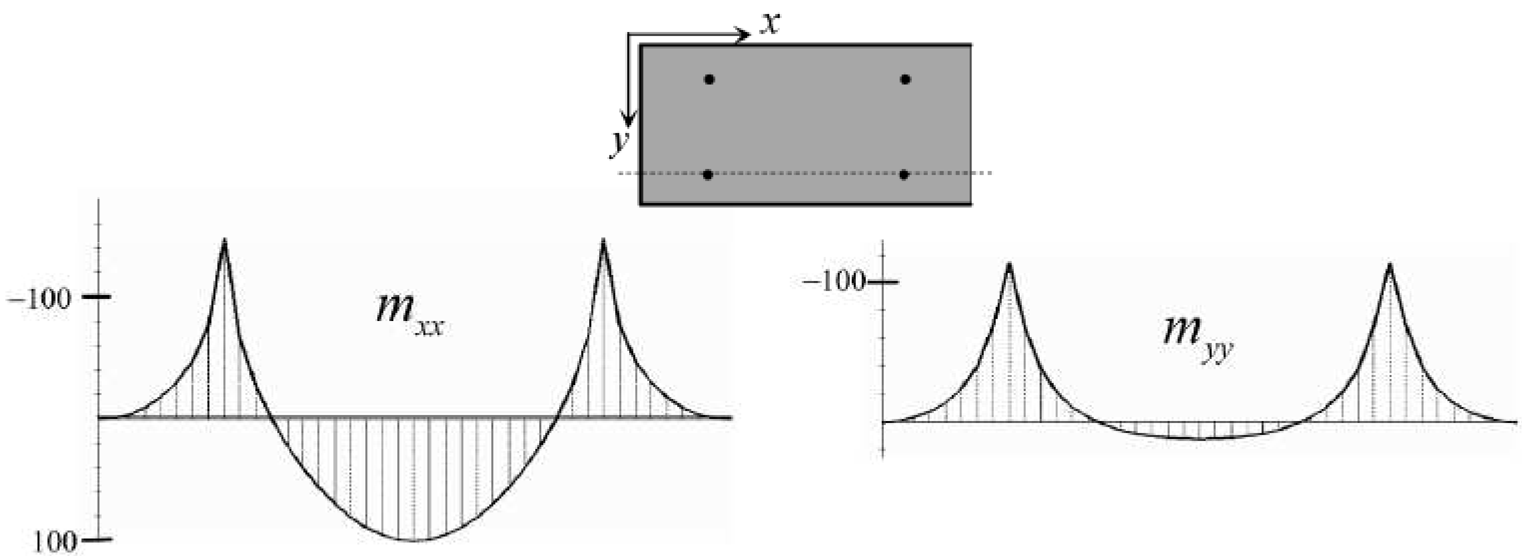


Figure 14.3 Moment diagrams in section over columns.

Lesson on singularities

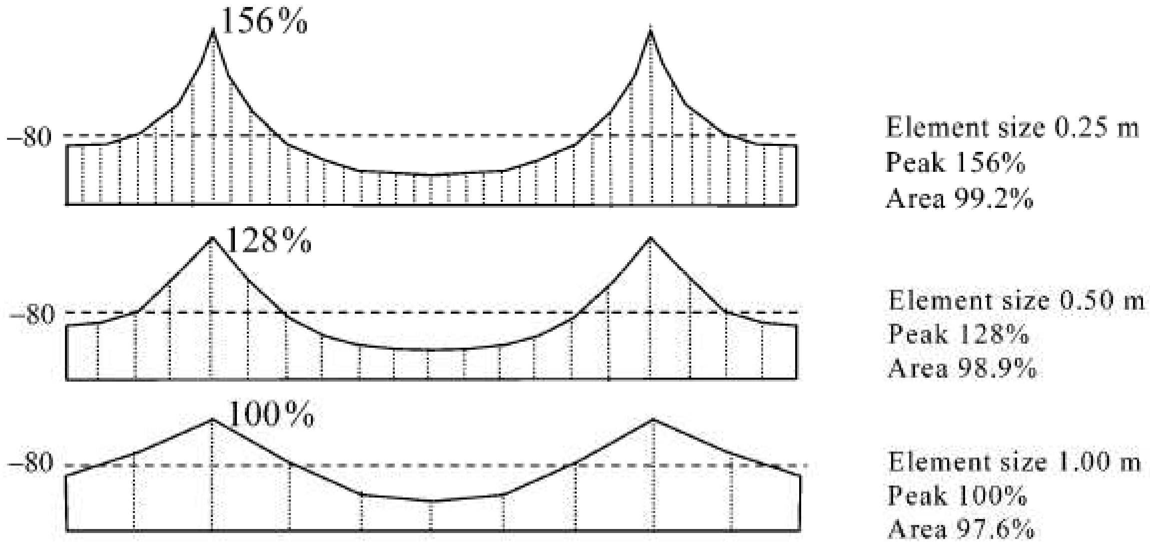


Figure 14.4 Moment distribution for different mesh fineness. The peak values differ much, the areas do not.

Lesson on singularities

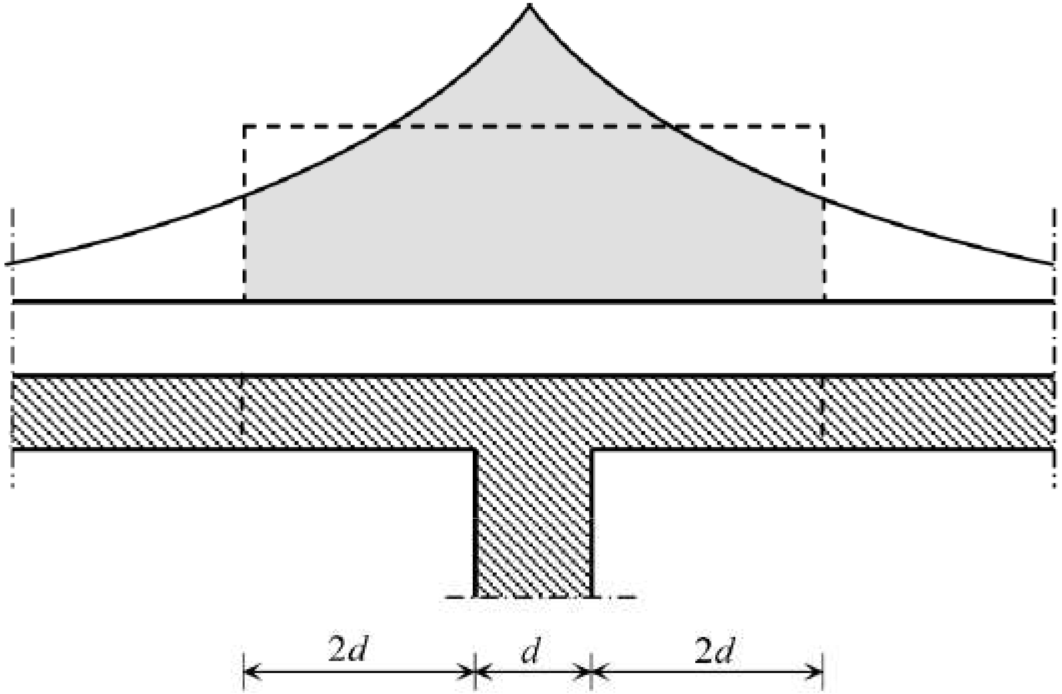


Figure 14.5 Smearing out of moment peak.

The integral over this section part determines the reinforcement which is needed in this section part. The structural engineer may spread this total amount equally over the width of the section part or choose to spread part of it and concentrate the remaining part above the column

Lesson on singularities

- **Convergence is obtained for stresses of finite value, but not at locations where the membrane plate theory predicts a singularity. Then no convergence will occur**

- **mesh refinement does not make sense.**

Discontinuities in thickness

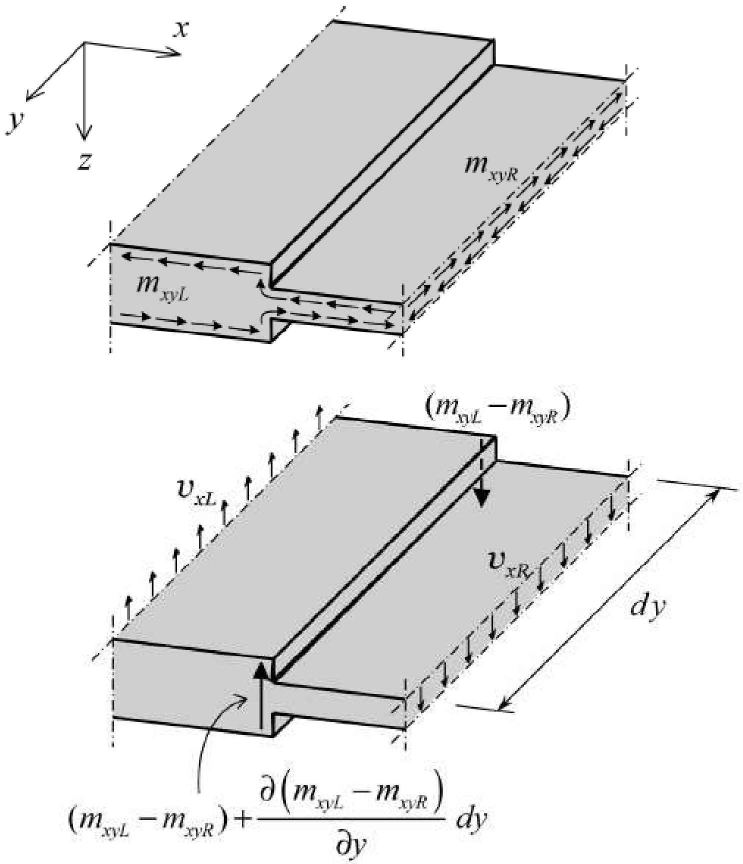


Figure 4.14 Twisting moment at discontinuity in thickness.

Discontinuities in thickness

At a boundary between plates of different thickness the bending moment normal to the boundary is continuous, and the bending moment parallel to the boundary and the twisting moment are discontinuous

The Kirchhoff shear force must be continuous. This implies that the shear forces normal to the boundary and parallel to the boundary will be discontinuous.

Kirchhoff vs Mindlin theories

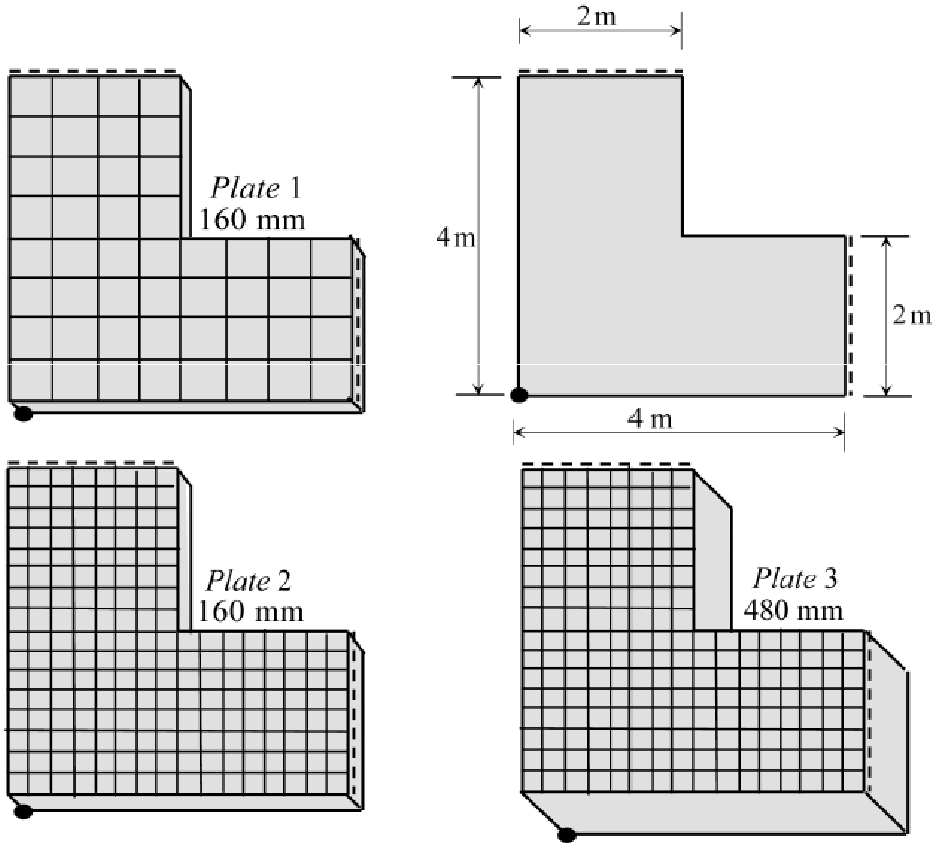


Figure 15.1 Three plates. Mesh fineness and thickness varied.

Kirchhoff vs Mindlin theories

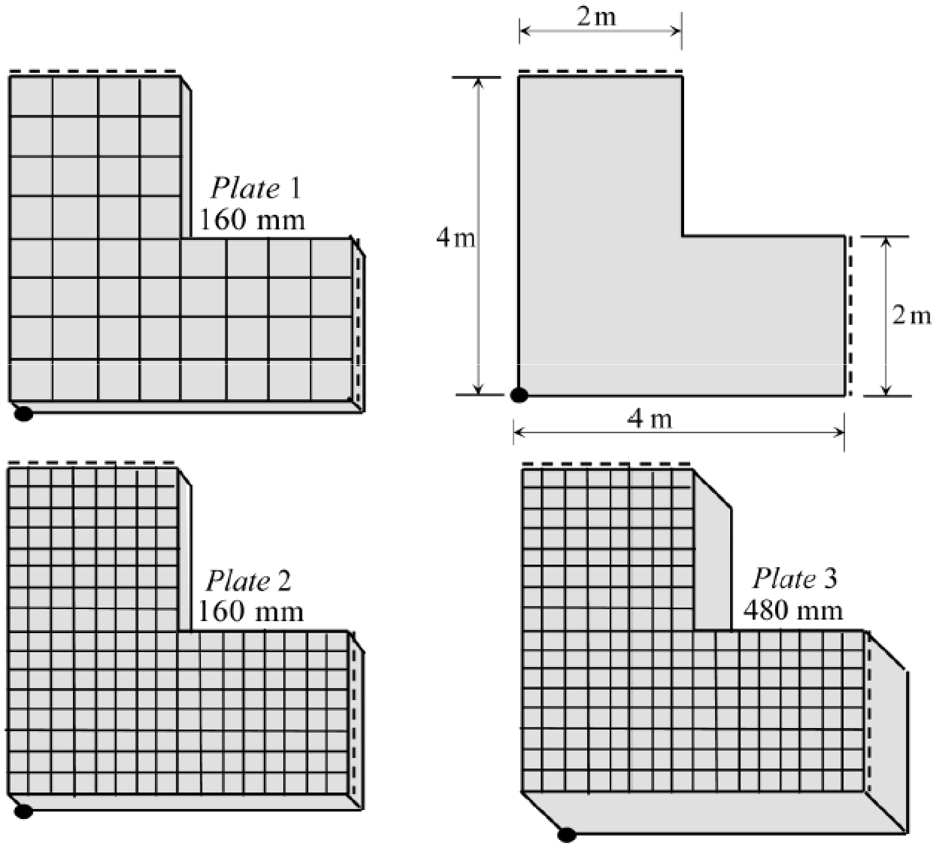


Figure 15.1 Three plates. Mesh fineness and thickness varied.

Kirchhoff vs Mindlin theories

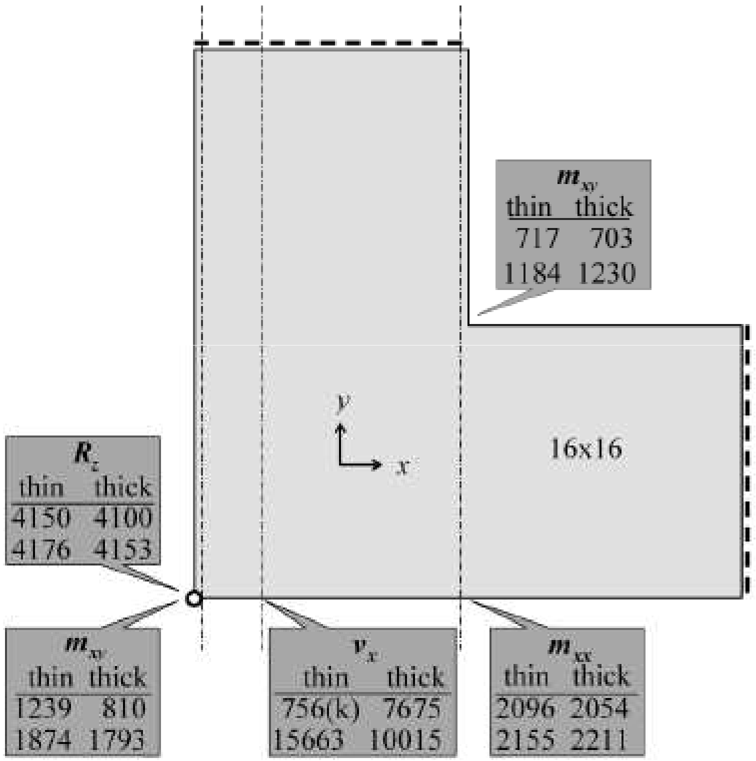


Figure 15.2 Big scatter in submitted results for twisting moment and shear force. Units in N and m.

Kirchhoff vs Mindlin theories

- **In a thin plate analysis we must use Kirchhoff**

- **The Mindlin analysis requires a senseless fine mesh to produce practically the same results**

- **Choosing Kirchhoff, we need never use element sizes smaller than the plate thickness**

Kirchhoff vs Mindlin theories

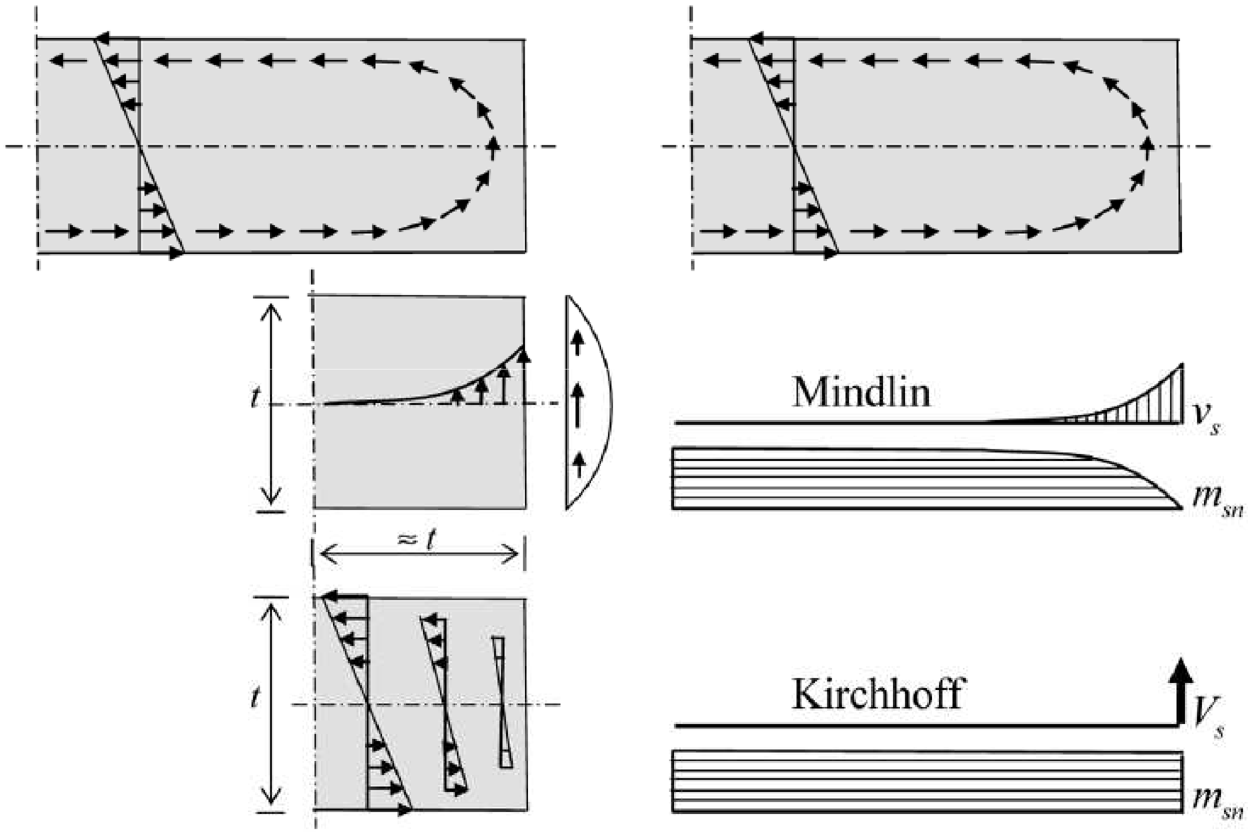


Figure 15.7 Close look at stress state near free edge.

Kirchhoff vs Mindlin theories

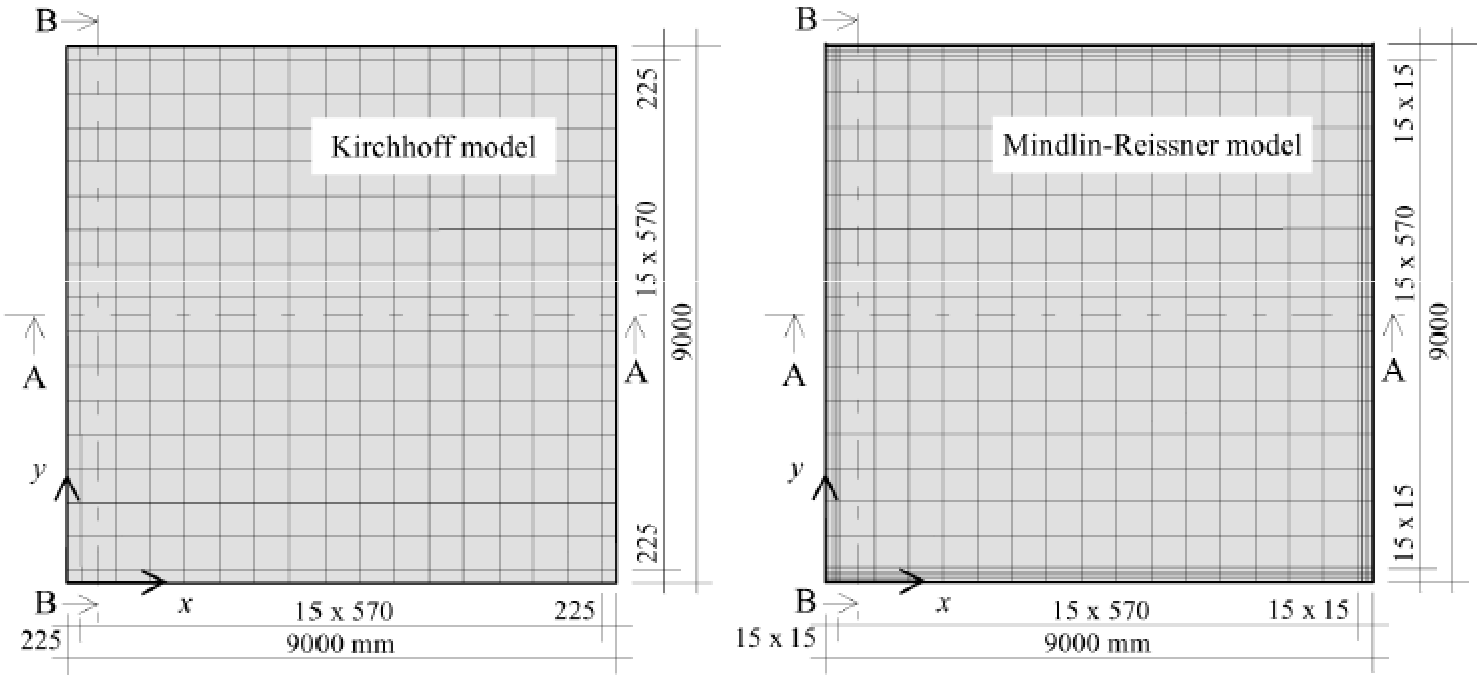


Figure 15.8 Mesh for thin plate analyses.

The thickness *t is 200 mm for the thin plate, and 2250 mm for the* thick plate.

Thin plates

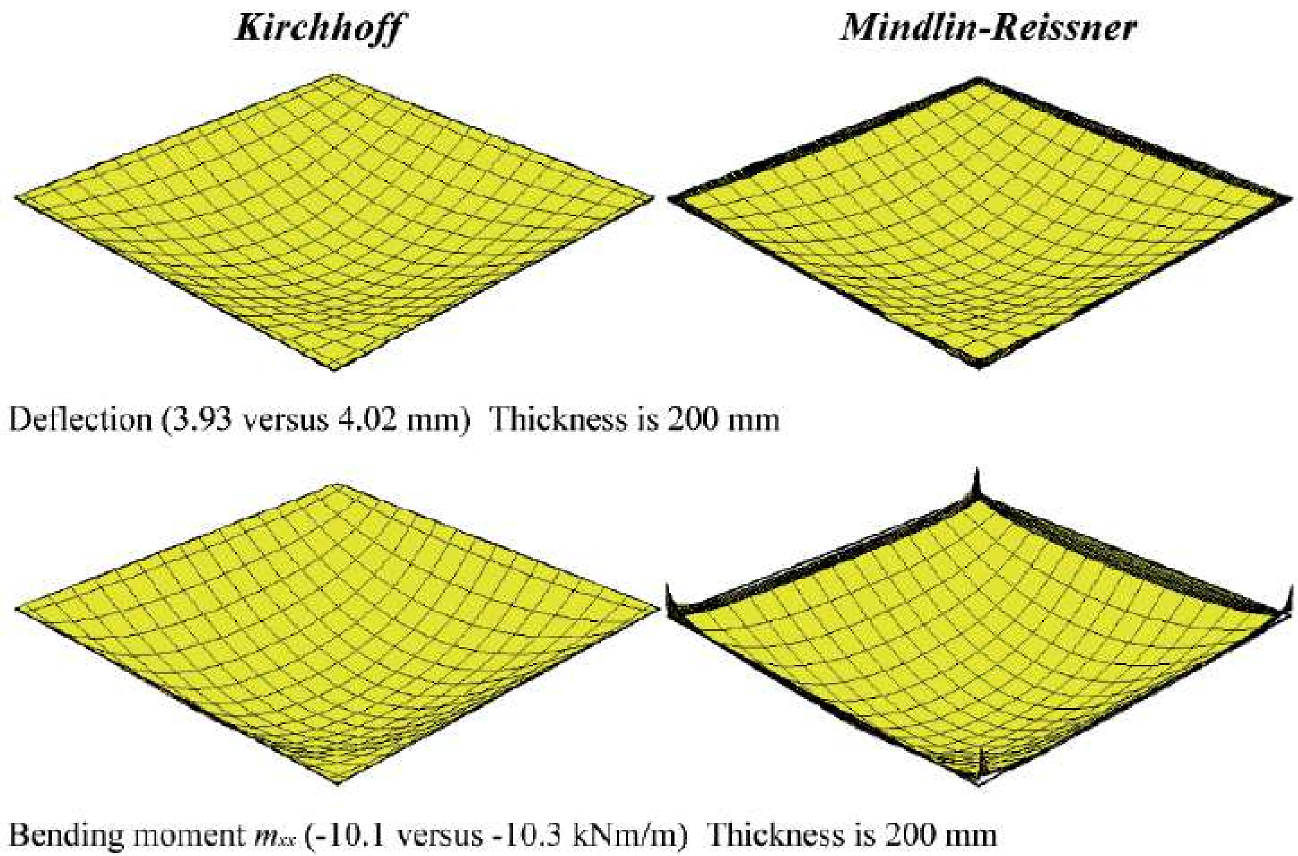
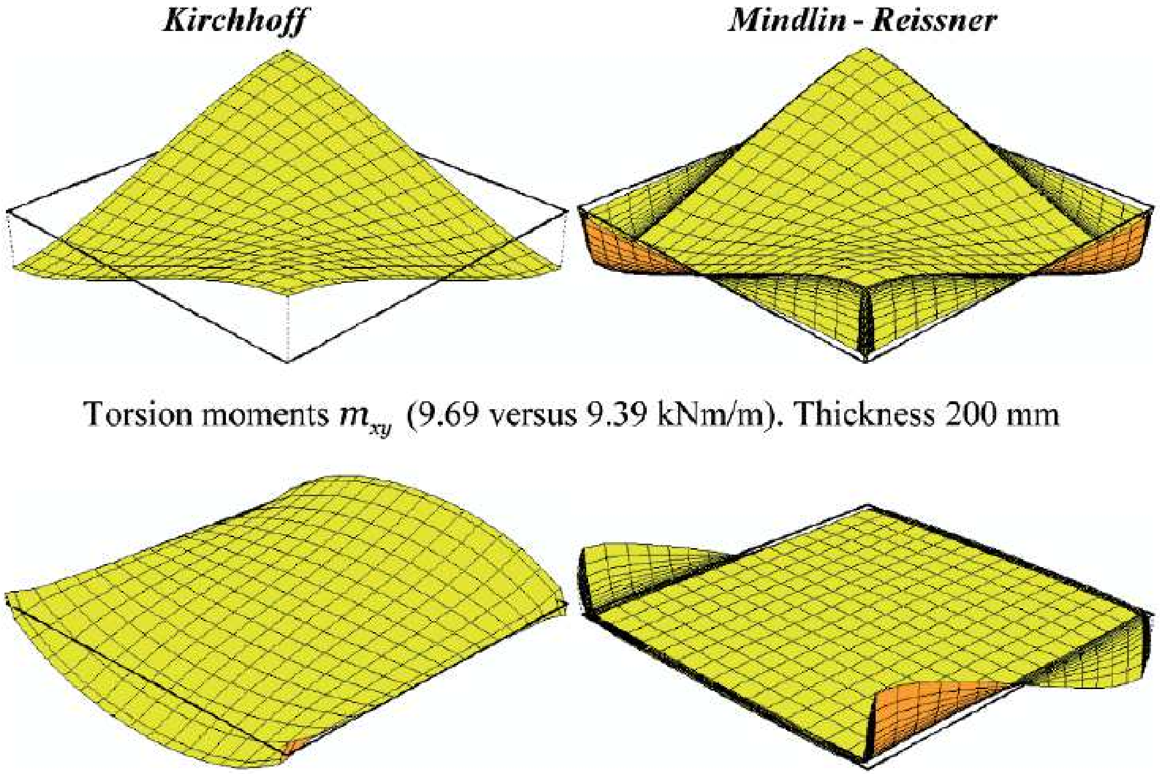


Figure 15.9 Deflection and bending moment in thin plate.

Thin plates



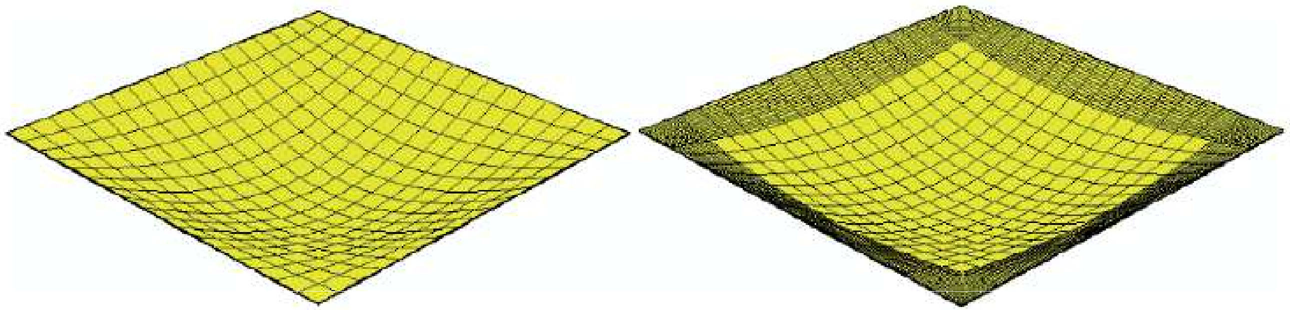
Torsion moments m_{xy} (9.69 versus 9.39 kNm/m). Thickness 200 mm

Shear force v_x (8.77 versus 11.53 kN/m). Thickness 200 mm

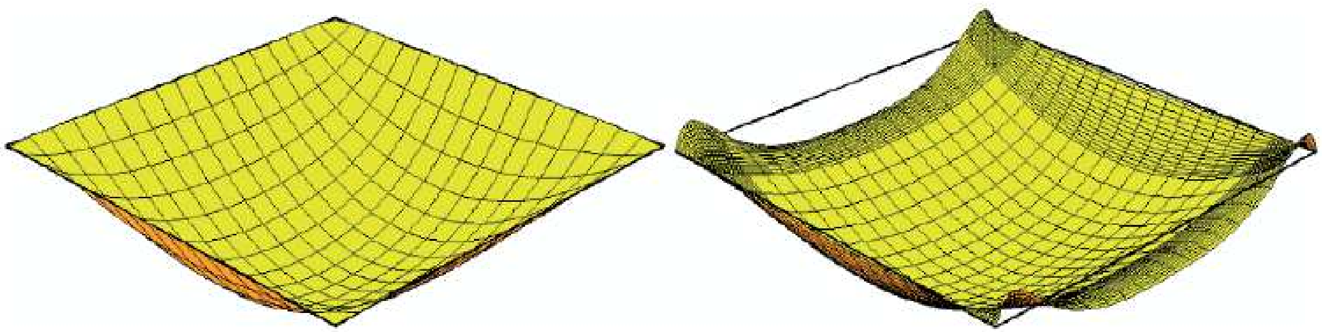
Figure 15.10 Twisting moment and shear force in thin plate.

Thick plates

Kirchhoff ***Mindlin-Reissner***



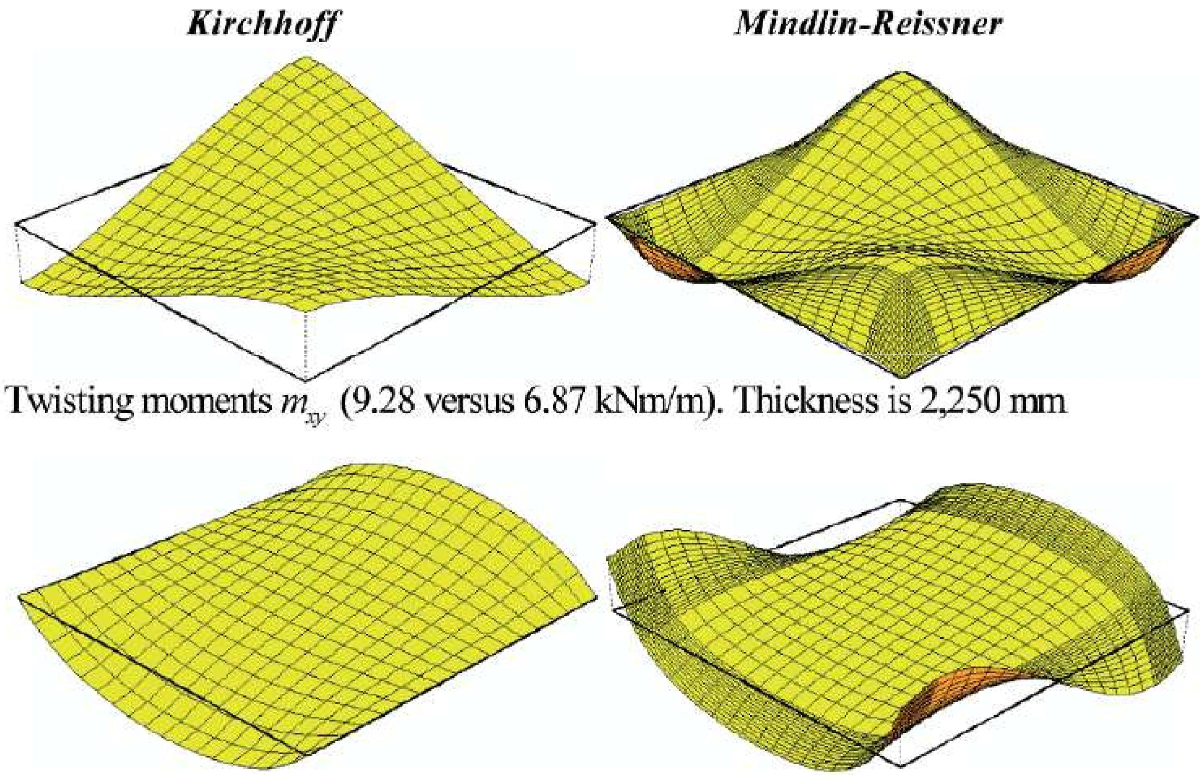
Deflection (0.00276 versus 0.00401 mm. Thickness is 2,250 mm



Bending moment m_{xx} (10.2 versus 11.9 kNm/m). Thickness is 2,250 mm

Figure 15.12 Mesh, deflection and bending moment for thick plate.

Thick plates



Twisting moments m_{xy} (9.28 versus 6.87 kNm/m). Thickness is 2,250 mm

Shear force v_y (7.73 versus 12.1 kN/m). Thickness is 2,250 mm

Figure 15.13 Twisting moment and shear force in thick plate.

Thin plates

- Thin plates should preferably be calculated with Kirchhoff theory.
- Mindlin theory is used for thin plates, this must be done at the cost of a very fine mesh, with results hardly different from Kirchhoff

Thin plates

- **Application of Kirchhoff theory requires an element size not smaller than about plate thickness**

- **If Kirchhoff theory is chosen and the FE-program offers the option of a graph for the shear force diagram across a section, also the concentrated edge shear force should be shown.**

- **If Kirchhoff theory is chosen and the FE-program is able to determine the resultant of shear forces and twisting moments (total force, total torque) over a section, also concentrated edge shear forces must be accounted for. Otherwise equilibrium is violated. This also holds at plate boundaries with edge beams**

- **If Kirchhoff theory is chosen and edge beams are applied, the bending moment in the beam is correct, but the shear force must be obtained as the sum of the concentrated edge shear force V_{edge} *and the* beam shear force V_{beam}**

Thick plates

- **A thick homogeneous isotropic plate must be analyzed by Mindlin theory**

- **An edge zone must be chosen of a width equal to about plate thickness, in which a sufficiently fine mesh is applied**

- **Sufficiently fine is five or more elements over the edge zone.**

Main differences between Mindlin theory and Kirchhoff theory

- **Mindlin is able to describe the discussed distribution of the shear force and twisting moment and Kirchhoff is not**
- **In Mindlin theory we can handle the boundary condition $M_{ns} = 0$, whereas we cannot in Kirchhoff theory**
- **Kirchhoff determines the integral of all the local vertical stress components and concentrates them into one shear force V_s *located at the very edge***
- **Kirchhoff is not able to have the twisting moment diminish to zero, and instead keeps it constant up to the edge**

Strengthened Strip Floor

A strengthened strip floor is a special case of a wide-slab floor. The floor is supported by a grid of columns. In one direction prefabricated strengthened strips are placed from one column to the other. In Figure 17.9 they are placed in the *y-direction. The width of these strips is bo. The strips are supports for* wide-slab floor units in the other direction (*x-direction). After the concrete* has been poured, the thickness of the slab is *h_1 at the strengthened strip, and h_o at the wide-slab*

If the strengthened strip and wide-slab floor units are pre-tensioned, we are justified in assuming that the wide-slab floor carries load in one direction. In that case no FE analysis is needed. If just mild steel reinforcement is applied, a FE-based analysis makes sense. Then more than one way is open for the analysis

Strengthened Strip Floor

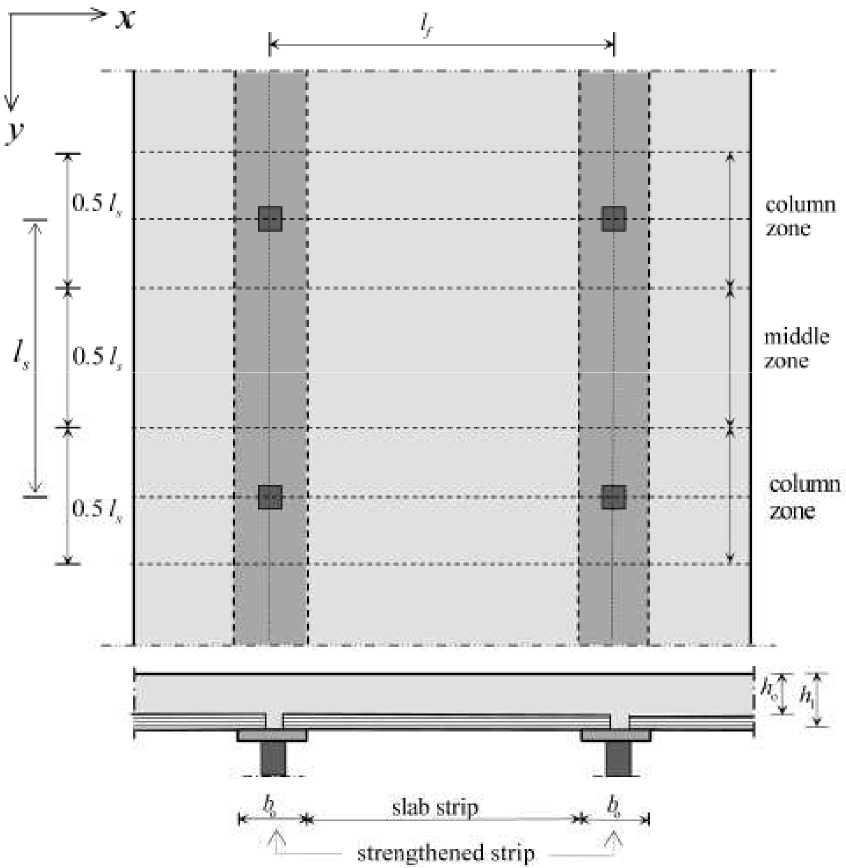


Figure 17.9 Strengthened strips over columns.

Strengthened Strip Floor

1 We may model the slab with just plate bending elements neglecting that the middle planes of the slab part with thickness ho and the slab part with thickness h1 do not coincide. The output consists of moments and transverse shear forces

2. We may model the slab with membrane-bending elements (in FE codes named shell elements). Now we automatically account for the different positions of middle planes. The output will consist of moments, transverse shear forces and membrane forces

3. We may apply three-dimensional volume elements. Now we are able to describe the geometry most truthfully, however receive the output in terms of stresses at nodes or Gaussian points→ not feasible

Strengthened Strip Floor

- To some extent we can account for the different positions of the middle planes in the first way of analysis, where we just use flexural plate elements

- This may be done by assigning the strengthened strip a larger thickness than h_1 or an adapted modulus of elasticity, here called h_e and E_e respectively

- Because of the different positions of the middle planes, part of the slab with thickness ho will act as a flange for the strengthened strip with thickness h_1

- Let us call the width of the flange b_f
- This extension occurs at both sides of the strengthened strip

Strengthened Strip Floor

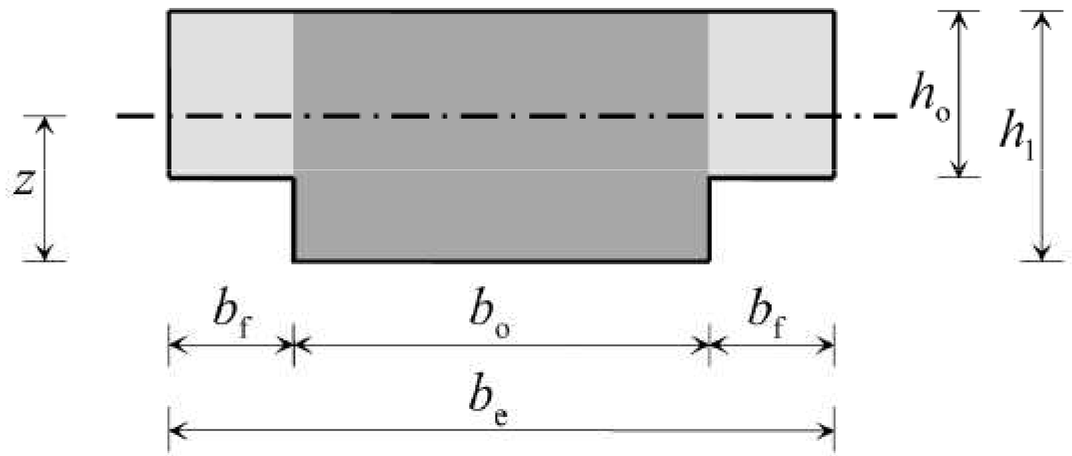


Figure 17.10 Effective width of strengthened strip.

Strengthened Strip Floor

A cross-section is obtained of width $b_e = b_o + 2b_f$. National codes of practice provide rules for calculation of the effective width b_e

We call the second moment of area of the extended cross- section I_e, and if the flanges are not included, I_o

The ratio I_e/I_o determines the multiplication factor for either the thickness of the strengthened strip or its elasticity modulus

Strengthened Strip Floor

We must take care not to duplicate the contribution of the slab parts of width b_f

If calculating I_e for the strengthened strip we must leave out the part $2b_f\,(h_o)^3/12$

Otherwise it occurs twice, both in the flexural rigidity of the elements in the strengthened strips and in the elements in the slab adjacent to the strengthened strip

Strengthened Strip Floor

We consider a floor and strengthened strip for which the following data hold:

$$l_s = 10\text{ m}, \quad l_f = 10\text{ m}, \quad E = 30{,}000\text{ N/mm}^2,$$

$$b_o = 2{,}400\text{ mm}, \quad h_o = 300\text{ mm}, \quad h_1 = 450\text{ mm}.$$

The assumed value b_f from the code of practice is 1,000 mm. Therefore

$$b_e = b_o + 2b_f = 2{,}400 + 2 \times 1{,}000 = 4{,}400\text{ mm}.$$

The position z of the neutral line is calculated from the formula

$$z = \frac{2b_f h_o(h_1 - \frac{1}{2}h_o) + b_o h_1(\frac{1}{2}h_1)}{2b_f h_o + b_o h_1} \qquad (17.11)$$

$$z = \frac{(6.0 \times 10^5)(300) + (10.8 \times 10^5)(225)}{6.0 \times 10^5 + 10.8 \times 10^5} = 251.8\text{ mm}$$

Strengthened Strip Floor

Calculation of I_e

$$I_e = 2b_f h_o\left(\left(h_1 - \frac{1}{2}h_o\right) - z\right)^2 + b_o h_1\left(\frac{1}{2}h_1 - z\right)^2 + \frac{1}{12}b_o h_1^3 \quad (17.12)$$

$$I_e = (6.00 \times 10^5)\,(300 - 251.8)^2$$

$$+ (10.8 \times 10^5)\,(225 - 251.8)^2 + \frac{1}{12} \times 2{,}400 \times 450^3$$

$$I_e = 1.39 \times 10^9 + 0.78 \times 10^9 + 18.23 \times 10^9 = 20.40 \times 10^9\ \text{mm}^4$$

The last of the three terms is the second moment of area for the strengthened strip without the flanges

$$I_o = 18.23 \times 10^9\ \text{mm}^4$$

Strengthened Strip Floor

The ratio is of I_e and I_o is

$$\frac{I_e}{I_o} = \frac{20.40}{18.23} = 1.119$$

Therefore, we either work with

$$h_e = \sqrt[3]{1.119}\, h_1 = 1.038\, h_1 = 1.038 \times 450 = 467 \text{ mm}$$

or with equal depths and

$$E_e = 1.119\, E = 1.119 \times 30{,}000 = 33{,}570 \text{ N/mm}^2$$