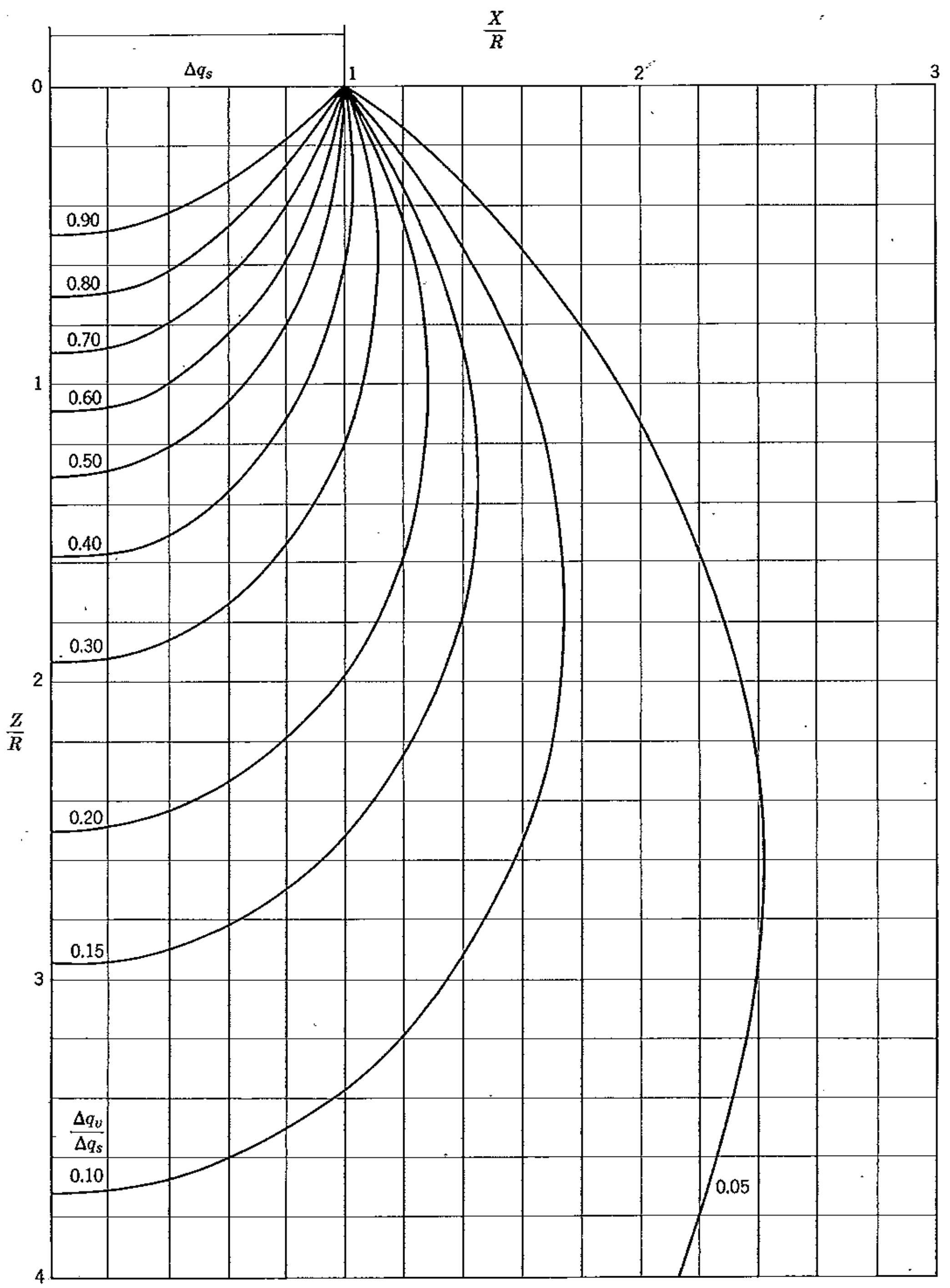

Fig. 8.4 Vertical stresses induced by uniform load on circular area.

It is a very tedious matter to obtain the elastic solution for a given loading and set of boundary conditions. In this book, we are concerned not with how to obtain solutions but rather with how to use these solutions. This section presents several solutions in graphical form.

Uniform load over a circular area. Figures 8.4 and 8.5 give the stresses caused by a uniformly distributed normal stress Δq_s acting over a circular area of radius R on the surface of an elastic half-space.[3] These stresses must be added to the initial geostatic stresses. Figure 8.4 gives

[3] In general, the stresses computed from the theory of elasticity are functions of Poisson's ratio μ. This quantity will be defined in Chapter 12. However, vertical stresses resulting from normal stresses applied to the surface are always independent of μ, and stresses caused by a strip load are also independent of μ. Thus of the charts presented in this chapter only those in Fig. 8.5 depend upon μ, and are for $\mu = 0.45$.

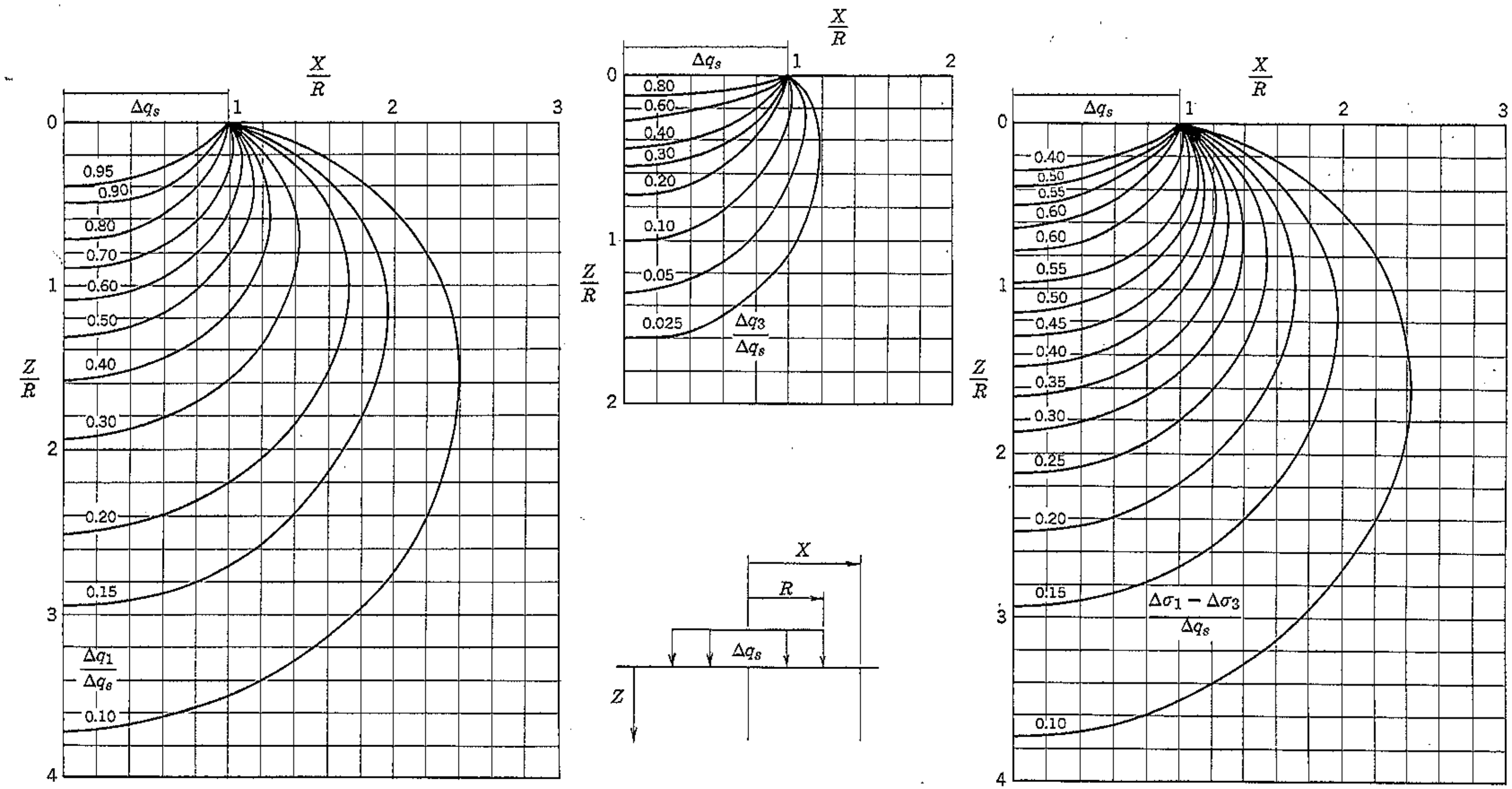

Fig. 8.5 Stresses under uniform load on circular area.

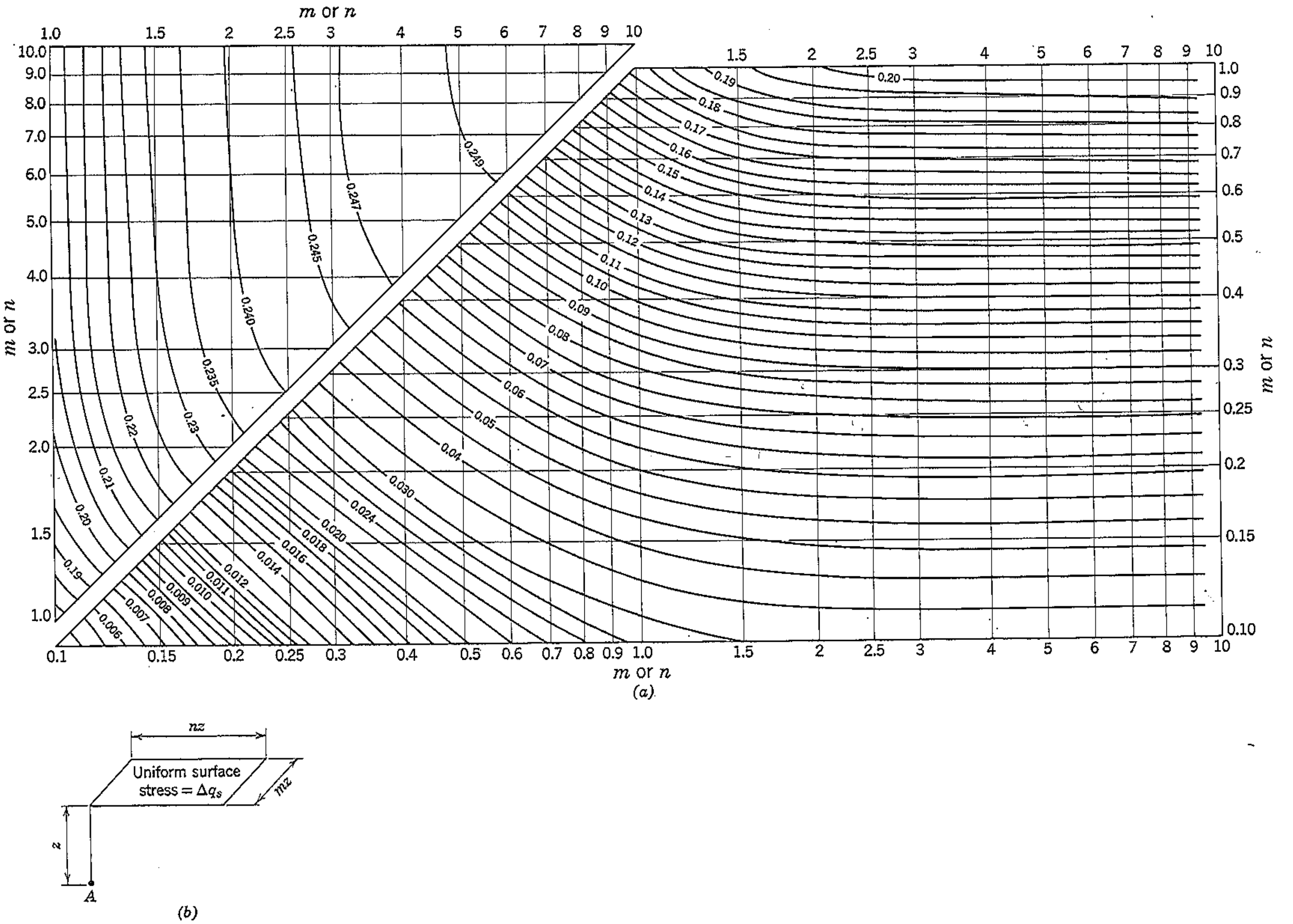

Fig. 8.6 (*a*) Chart for use in determining vertical stresses below corners of loaded rectangular surface areas on elastic, isotropic material. Chart gives $f(m, n)$. (*b*) At point A, $\Delta\sigma_v = \Delta q_s \times f(m, n)$. (From Newmark, 1942)

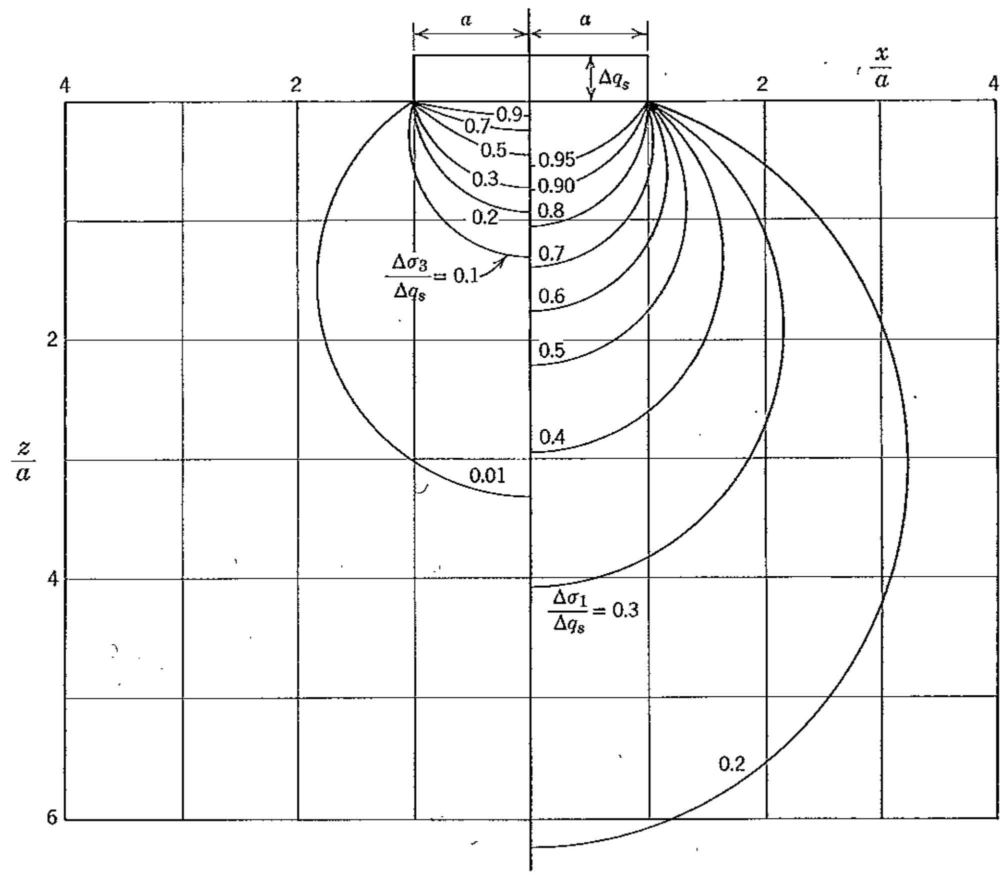

Fig. 8.7 Principal stresses under strip load.

obtain elastic stress distributions for almost any loading and boundary conditions. Charts such as those given here are useful for preliminary analysis of a problem or when the computer is not available.

Accuracy of calculated values of induced stresses. A critical question is: How accurate are the values of induced stresses as calculated from stress distribution theories? This question can be answered only by comparing calculated with actual stress increments for a number of field situations. Unfortunately, there are very few reliable sets of measured stress increments within soil masses (see Taylor, 1945 and Turnbull, Maxwell, and Ahlvin, 1961).

The relatively few good comparisons of calculated with measured stress increments indicate a surprisingly good agreement, especially in the case of vertical stresses. A great number of such comparisons are needed to establish the degree of reliability of calculated stress increments. At the present stage of knowledge, the soil engineer must continue to use stress distribution theories based on the theory of elasticity for lack of better techniques. He should realize, however, that his computed stress values may be in error by as much as ±25% or more.

8.4 PRINCIPAL STRESSES AND MOHR CIRCLE

As in any other material, the normal stress at a point within a soil mass is generally a function of the orientation of the plane chosen to define the stress. It is meaningless to talk of *the* normal stress or *the* shear stress at a point. Thus subscripts will usually be attached to the symbols σ and τ to qualify just how this stress is defined. More generally, of course, we should talk of the *stress tensor*, which provides a complete description for the state of stress at a point. This matter is discussed in textbooks on elementary mechanics, such as Crandall and Dahl (1959). The following paragraphs will state the essential concepts and definitions.

Principal Stresses

There exist at any stressed point three orthogonal (i.e., mutually perpendicular) planes on which there are zero shear stresses. These planes are called the *principal stress planes*. The normal stresses that act on these three planes are called the *principal stresses*. The largest of these three

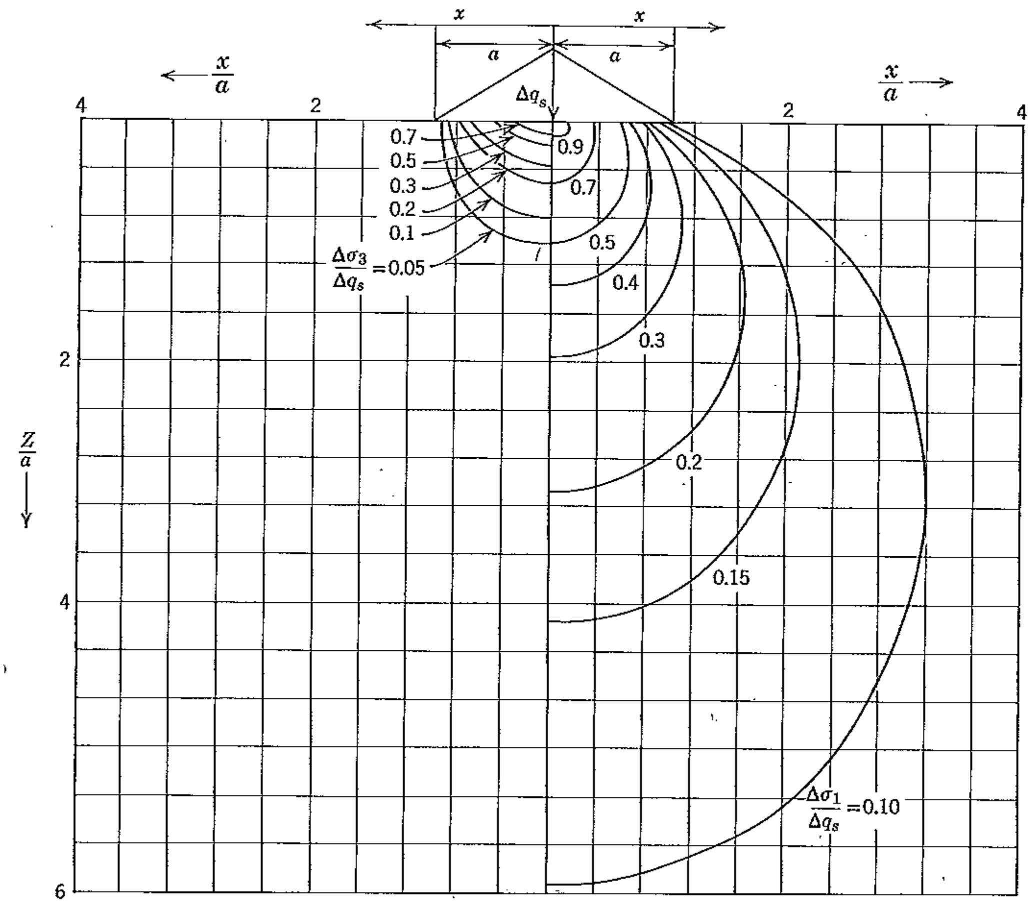

Fig. 8.8 Principal stresses under triangular strip load.

stresses is called the *major principal stress* σ_1, the smallest is called the *minor principal stress* σ_3, and the third is called the *intermediate principal stress* σ_2.

When the stresses in the ground are geostatic, the horizontal plane through a point is a principal plane and so too are all vertical planes through the point. When $K < 1$, $\sigma_v = \sigma_1$, $\sigma_h = \sigma_3$, and $\sigma_2 = \sigma_3 = \sigma_h$. When $K > 1$ the situation is reversed: $\sigma_h = \sigma_1$, $\sigma_v = \sigma_3$, and $\sigma_2 = \sigma_1 = \sigma_h$. When $K = 1$, $\sigma_v = \sigma_h = \sigma_1 = \sigma_2 = \sigma_3$ and the state of stress is *isotropic*.

We should also recall that the shear stresses on any two orthogonal planes (planes meeting at right angles) must be numerically equal. Returning to the definition of stresses given in Section 8.1, we must have $\tau_h = \tau_v$.

Mohr circle. Throughout most of this book, we shall be concerned only with the stresses existing in two dimensions rather than those in three dimensions.[4] More specifically, we shall be interested in the state of stress in the plane that contains the major and minor principal stresses, σ_1 and σ_3. Stresses will be considered positive when compressive. The remainder of the sign conventions are given in Fig. 8.9. The quantity $(\sigma_1 - \sigma_3)$ is called the *deviator stress* or *stress difference*.

Given the magnitude and direction of σ_1 and σ_3, it is possible to compute normal and shear stresses in any other direction using the equations developed from statics and shown in Fig. 8.9.[5] These equations, which provide a complete (in two dimensions) description for the state of stress, describe a circle. Any point on the circle, such as *A*, represents the stress on a plane whose normal is oriented at angle θ to the direction of the major principal stress. This graphical representation of the state of stress is known as the *Mohr circle* and is of the greatest importance in soil mechanics.

Given σ_1 and σ_3 and their directions, it is possible to find the stresses in any other direction by graphical

[4] The intermediate principal stress unquestionably has some influence upon the strength and stress-strain properties of soil. However, this influence is not well understood. Until this effect has been clarified, it seems best to work primarily in terms of σ_1 and σ_3.

[5] Equations 8.6 and 8.7 are derived in most mechanics texts; e.g., see Crandall and Dahl (1959), pp. 130–138.