La seguente tabella riporta i valori del prodotto $\beta_j l$ per le condizioni di vincolo più comuni.

$j \leftrightarrow n$

$W_n(x) = (\varphi(x))_n = \varphi_j(x)$ $\qquad \alpha_n = \gamma_j$

FORME MODALI

$\varphi_j(x) = W_n(x)$

$\beta_j l = \delta_j$

EQ. CARATTERISTICA

End conditions of beam	Frequency equation	Mode shape (normal function)	Value of $\beta_n l$
* Pinned-pinned	$\sin \beta_n l = 0$	$\boxed{W_n(x) = C_n[\sin \beta_n x]}$	$\beta_1 l = \pi$ $\beta_2 l = 2\pi$ $\beta_3 l = 3\pi$ $\beta_4 l = 4\pi$
Free-free	$\cos \beta_n l \cdot \cosh \beta_n l = 1$	$W_n(x) = C_n[\sin \beta_n x + \sinh \beta_n x + \alpha_n (\cos \beta_n x + \cosh \beta_n x)]$ where $\alpha_n = \left(\dfrac{\sin \beta_n l - \sinh \beta_n l}{\cosh \beta_n l - \cos \beta_n l}\right)$	$\beta_1 l = 4.730041$ $\beta_2 l = 7.853205$ $\beta_3 l = 10.995608$ $\beta_4 l = 14.137165$ ($\beta l = 0$ for rigid body mode)
Fixed-fixed	$\cos \beta_n l \cdot \cosh \beta_n l = 1$	$W_n(x) = C_n[\sinh \beta_n x - \sin \beta_n x + \alpha_n (\cosh \beta_n x - \cos \beta_n x)]$ where $\alpha_n = \left(\dfrac{\sinh \beta_n l - \sin \beta_n l}{\cos \beta_n l - \cosh \beta_n l}\right)$	$\beta_1 l = 4.730041$ $\beta_2 l = 7.853205$ $\beta_3 l = 10.995608$ $\beta_4 l = 14.137165$
* Fixed-free	$\cos \beta_n l \cdot \cosh \beta_n l = -1$	$W_n(x) = C_n[\sin \beta_n x - \sinh \beta_n x - \alpha_n (\cos \beta_n x - \cosh \beta_n x)]$ where $\gamma_j \leftrightarrow \alpha_n = \left(\dfrac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l}\right)$	$\beta_1 l = 1.875104$ $\beta_2 l = 4.694091$ $\beta_3 l = 7.854757$ $\beta_4 l = 10.995541$
Fixed-pinned	$\tan \beta_n l - \tanh \beta_n l = 0$	$W_n(x) = C_n[\sin \beta_n x - \sinh \beta_n x + \alpha_n (\cosh \beta_n x - \cos \beta_n x)]$ where $\alpha_n = \left(\dfrac{\sin \beta_n l - \sinh \beta_n l}{\cos \beta_n l - \cosh \beta_n l}\right)$	$\beta_1 l = 3.926602$ $\beta_2 l = 7.068583$ $\beta_3 l = 10.210176$ $\beta_4 l = 13.351768$
Pinned-free	$\tan \beta_n l - \tanh \beta_n l = 0$	$W_n(x) = C_n[\sin \beta_n x + \alpha_n \sinh \beta_n x]$ where $\alpha_n = \left(\dfrac{\sin \beta_n l}{\sinh \beta_n l}\right)$	$\beta_1 l = 3.926602$ $\beta_2 l = 7.068583$ $\beta_3 l = 10.210176$ $\beta_4 l = 13.351768$ ($\beta l = 0$ for rigid body mode)

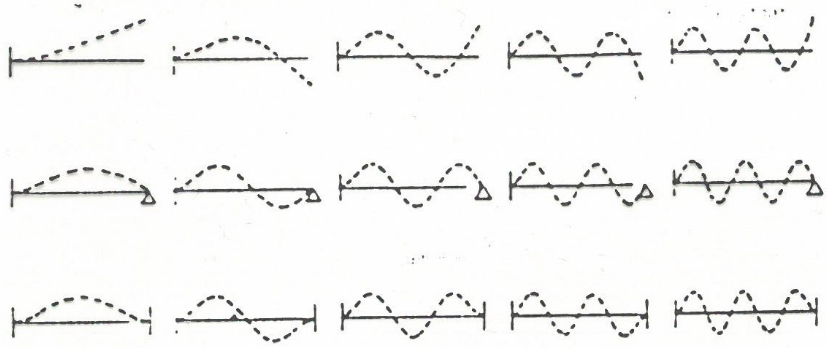
I primi cinque modi della trave a mensola sono rappresentati in figura.

$j = 1 \quad 2 \quad 3 \quad 4 \quad 5$

T. a mensola

Incastro - Appoggio

Incastro - Incastro

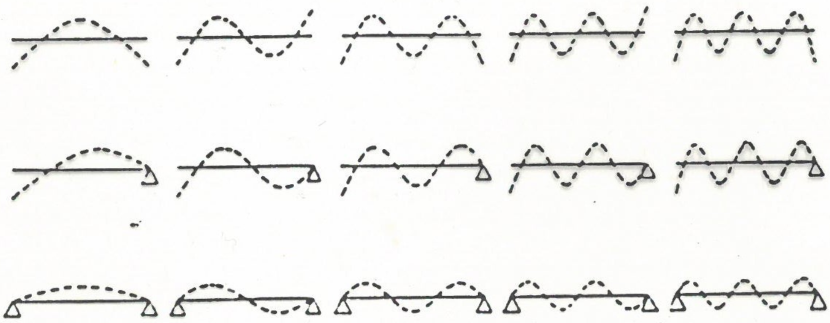


e per la trave diversamente appoggiata:

Trave libera

Un solo appoggio

Due appoggi



Le relazioni di ortogonalità dei modi propri sono:

$$\int_0^\ell \rho S \varphi_i \varphi_j \, dx = 0 \; ; \quad \int_0^\ell E I \frac{d^2\varphi_i}{dx^2} \frac{d^2\varphi_j}{dx^2} \, dx = 0 \qquad (i \neq j)$$

La massa modale e la rigidezza modale sono: $i = j$

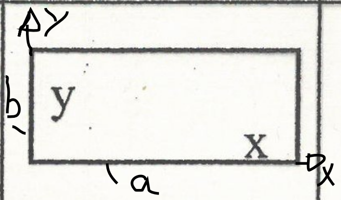
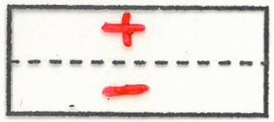
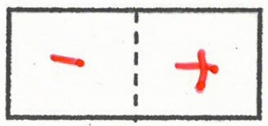
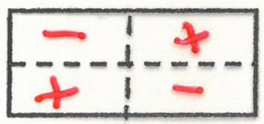
$$M_j = \int_0^\ell \rho S \varphi_j^2 \, dx \; ; \qquad k_j = \int_0^\ell E I \left(\frac{d^2\varphi_j}{dx^2} \right)^2 dx$$

e risulta:

$$\omega_j^2 = \frac{k_j}{M_j}$$

Trave a mensola - Forme modali (video):
https://www.youtube.com/watch?v=bLGW7cWQGEY

Tabella 6.6 - *Pulsazioni proprie, forme modali e linee nodali di una membrana rettangolare.* appoggiata ai bordi

Modo	1-1	1-2	2-1	2-2
Pulsazione	$\pi c\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$	$\pi c\sqrt{\frac{1}{a^2}+\frac{4}{b^2}}$	$\pi c\sqrt{\frac{4}{a^2}+\frac{1}{b^2}}$	$2\pi c\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$
Forme modali	$\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$	$\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi y}{b}\right)$	$\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$	$\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{2\pi y}{b}\right)$
Linee nodali				

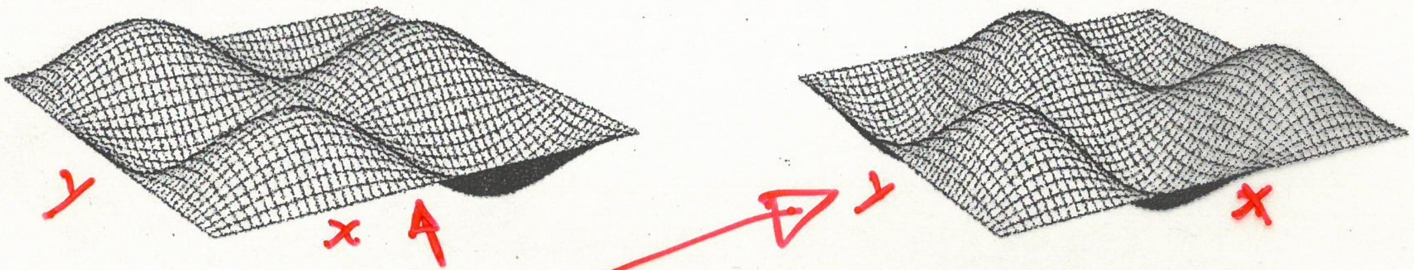


Fig. 6.14 - *Forme modali a) 2-3 e b) 3-2 di una membrana quadrata.*

La forma modale corrispondente alla generica ω_{rs} è:

$$\varphi_{rs}(x,y)=H_{rs}\sin\left(r\pi\frac{x}{a}\right)\sin\left(s\pi\frac{y}{b}\right),$$