Solutions to Chapter 8 Exercise Problems

Problem 8.1

A cam that is designed for cycloidal motion drives a flat-faced follower. During the rise, the follower displaces 1 in for 180° of cam rotation. If the cam angular velocity is constant at 100 rpm, determine the displacement, velocity, and acceleration of the follower at a cam angle of 60°.

Solution:

The equation for cycloidal motion is:

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)$$

For L = 1, and $\beta = 180° = \pi$, then

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right) = 1\left(\frac{\theta}{\pi} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\pi}\right) = \left(\frac{\theta}{\pi} - \frac{1}{2\pi}\sin 2\theta\right)$$

$$\dot{y} = \frac{L\omega}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right) = \frac{\omega}{\pi}(1 - \cos 2\theta)$$

$$\ddot{y} = 2L\pi\left(\frac{\omega}{\beta}\right)^2 \sin\frac{2\pi\theta}{\beta} = 2\pi\left(\frac{\omega}{\pi}\right)^2 \sin 2\theta$$

The angular velocity is $\dot{\theta} = 100\ rpm = 100\frac{2\pi}{60} = 10.472\ rad/s$

When $\theta = 60° = \frac{\pi}{3}$,

$$y = \left(\frac{\pi/3}{\pi} - \frac{1}{2\pi}\sin 2(\pi/3)\right) = \left(\frac{1}{3} - \frac{1}{2\pi}\sin(2\pi/3)\right) = 0.195 \text{ in}$$

$$\dot{y} = \frac{\omega}{\pi}(1 - \cos 2\theta) = \frac{10.472}{\pi}(1 - \cos(2\pi/3)) = 5.000\frac{in.}{\text{sec.}}$$

$$\ddot{y} = 2\pi\left(\frac{\omega}{\pi}\right)^2 \sin 2\theta = 2\pi\left(\frac{10.472}{\pi}\right)^2 \sin(2\pi/3) = 60.46\frac{\text{in.}}{\text{sec}^2}$$

Problem 8.2

A constant-velocity cam is designed for simple harmonic motion. If the flat-faced follower displaces 2 in for 180° of cam rotation and the cam angular velocity is 100 rpm, determine the displacement, velocity, and acceleration when the cam angle is 45°.

Solution:

The equation for simple harmonic motion is:

$$y = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right)$$

For L = 2, and $\beta = 180^\circ = \pi$, then

$$y = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right) = \frac{2}{2}\left(1 - \cos\frac{\pi\theta}{\pi}\right) = (1 - \cos\theta)$$

$$\dot{y} = \frac{dy}{dt} = \frac{d}{dt}(1 - \cos\theta) = \dot{\theta}\sin\theta$$

$$\ddot{y} = \frac{d^2y}{dt^2} = \frac{d}{dt}\left(\dot{\theta}\sin\theta\right) = \dot{\theta}^2\cos\theta$$

The angular velocity is $\dot{\theta} = 100\ rpm = 100\frac{2\pi}{60} = 10.472\ \text{rad / s}$

When $\theta = 45^\circ$,

$$y = (1 - \cos\theta) = (1 - \cos 45^\circ) = 1 - 0.707 = 0.292\ \text{in},$$

$$\dot{y} = \dot{\theta}\sin\theta = 10.472\sin 45^\circ = 10.472\ (0.707) = 7.405\frac{\text{in}}{\text{s}}$$

$$\ddot{y} = \dot{\theta}^2\cos\theta = 10.472^2\ (0.707) = 77.531\frac{\text{in}}{\text{s}^2}$$

Problem 8.3

A cam drives a radial, knife-edged follower through a 1.5-in rise in 180° of cycloidal motion. Give the displacement at 60° and 100°. If this cam is rotating at 200 rpm, what are the velocity (ds/dt) and the acceleration (d^2s/dt^2) at $\theta = 60°$?

<u>Solution:</u>

The equation for cycloidal motion is:

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)$$

For L = 1.5, and $\beta = 180^\circ = \pi$, then

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right) = 1.5\left(\frac{\theta}{\pi} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\pi}\right) = 1.5\left(\frac{\theta}{\pi} - \frac{1}{2\pi}\sin 2\theta\right)$$

$$\dot{y} = \frac{L\omega}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right) = \frac{1.5\omega}{\pi}(1 - \cos 2\theta)$$

$$\ddot{y} = 2L\pi\left(\frac{\omega}{\beta}\right)^2 \sin\frac{2\pi\theta}{\beta} = 2(1.5)\pi\left(\frac{\omega}{\pi}\right)^2 \sin 2\theta = 3\pi\left(\frac{\omega}{\pi}\right)^2 \sin 2\theta$$

The angular velocity is $\dot{\theta} = 200\ rpm = 200\frac{2\pi}{60} = 20.944\ \text{rad / s}$

When $\theta = 60° = \frac{\pi}{3}$,

$$y = 1.5\left(\frac{\theta}{\pi} - \frac{1}{2\pi}\sin 2\theta\right) = 1.5\left(\frac{\pi/3}{\pi} - \frac{1}{2\pi}\sin\frac{2\pi}{3}\right) = 0.293\ \text{in}$$

$$\dot{y} = \frac{1.5\omega}{\pi}(1 - \cos 2\theta) = \frac{1.5(20.944)}{\pi}\left(1 - \cos\frac{2\pi}{3}\right) = 15.00\frac{\text{in}}{\text{s}}$$

$$\ddot{y} = 3\pi\left(\frac{\omega}{\pi}\right)^2 \sin 2\theta = 3\pi\left(\frac{20.944}{\pi}\right)^2 \sin\frac{2\pi}{3} = 362.76\frac{\text{in}}{\text{s}^2}$$

When $\theta = 100° = \frac{100\pi}{180} = \frac{5\pi}{9}$,

$$y = 1.5\left(\frac{\theta}{\pi} - \frac{1}{2\pi}\sin 2\theta\right) = 1.5\left(\frac{5\pi/9}{\pi} - \frac{1}{2\pi}\sin\frac{10\pi}{9}\right) = 0.915\ \text{in}$$

Problem 8.4

Draw the displacement schedule for a follower that rises through a total displacement of 1.5 inches with constant acceleration for 1/4th revolution, constant velocity for 1/8th revolution, and constant deceleration for 1/4th revolution of the cam. The cam then dwells for 1/8th revolution, and returns with simple harmonic motion in 1/4th revolution of the cam.

Solution:

The displacement profile can be easily computed using the equations in Chapter 8 using Matlab. The curves are matched at the endpoints of each segment. The profile equations are:

For $0 \le \theta \le \frac{\pi}{2}$

$$y_1 = a_0 + a_1\theta + a_2\theta^2$$

The boundary conditions at $\theta = 0$ are $y_1 = 0$ and $y'_1 = 0$. Therefore,

$$a_0 = a_1 = 0$$

So,

$$y_1 = a_2\theta^2$$

and

$$y'_1 = 2a_2\theta$$

where a_2 is yet to be determined.

For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$

$$y_2 = b_0 + b_1\theta$$

The boundary conditions at $\theta = \frac{\pi}{2}$ are $y_1 = a_2\left(\frac{\pi}{2}\right)^2$ and $y'_1 = 2a_2\frac{\pi}{2} = a_2\pi$. Then

$$b_0 + b_1\frac{\pi}{2} = a_2\left(\frac{\pi}{2}\right)^2$$

and

$$0 = -a_2\left(\frac{\pi}{2}\right)^2 + b_0 + b_1\frac{\pi}{2}$$

Also

$$y'_2 = b_1 = a_2\pi$$

or

$$0 = a_2\pi - b_1$$

For $\frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

$$y_3 = c_0 + c_1\theta + c_2\theta^2$$
$$y'_3 = c_1 + 2c_2\theta$$
$$y''_3 = 2c_2$$

The boundary conditions at $\theta = \frac{3\pi}{4}$ are $y_2 = a_2\pi\left(-\frac{\pi}{4} + \theta\right) = a_2\pi\left(-\frac{\pi}{4} + \frac{3\pi}{4}\right) = a_2\frac{\pi^2}{2}$ and $y'_2 = a_2\pi$. Also, at $\theta = \frac{5\pi}{4}$, $y_3 = 1.5$ and $y'_3 = 0$. Then, matching the conditions,

$$y_3 = a_2\frac{\pi^2}{2} = c_0 + c_1\frac{3\pi}{4} + c_2\left(\frac{3\pi}{4}\right)^2$$
$$y'_3 = c_1 + 2c_2\frac{3\pi}{4} = a_2\pi$$
$$1.5 = c_0 + c_1\frac{5\pi}{4} + c_2\left(\frac{5\pi}{4}\right)^2$$
$$y'_3 = c_1 + 2c_2\frac{5\pi}{4} = 0$$

The boundary condition equations can be written as:

$$0 = -a_2\left(\frac{\pi}{2}\right)^2 + b_0 + b_1\frac{\pi}{2}$$

$$0 = a_2\pi - b_1$$

$$0 - a_2\frac{\pi^2}{2} + c_0 + c_1\frac{3\pi}{4} + c_2\left(\frac{3\pi}{4}\right)^2$$

$$0 = -a_2\pi + c_1 + 2c_2\frac{3\pi}{4}$$

$$1.5 = c_0 + c_1\frac{5\pi}{4} + c_2\left(\frac{5\pi}{4}\right)^2$$

$$0 = c_1 + 2c_2\frac{5\pi}{4}$$

In matrix form,

$$\begin{bmatrix}0\\0\\0\\0\\0\\1.5\end{bmatrix} = \begin{bmatrix} -\left(\frac{\pi}{2}\right)^2 & 1 & \frac{\pi}{2} & 0 & 0 & 0 \\ \pi & 0 & -1 & 0 & 0 & 0 \\ -\frac{\pi^2}{2} & 0 & 0 & 1 & \frac{3\pi}{4} & \left(\frac{3\pi}{4}\right)^2 \\ -\pi & 0 & 0 & 0 & 1 & \frac{3\pi}{2} \\ 0 & 0 & 0 & 0 & 1 & \frac{5\pi}{2} \\ 0 & 0 & 0 & 1 & \frac{5\pi}{4} & \left(\frac{5\pi}{4}\right)^2 \end{bmatrix} \begin{bmatrix}a_2\\b_0\\b_1\\c_0\\c_1\\c_2\end{bmatrix}$$

Solving for the constraints using Matlab,

$$\begin{bmatrix}a_2\\b_0\\b_1\\c_0\\c_1\\c_2\end{bmatrix} = \begin{bmatrix}0.2026\\-0.5000\\0.6366\\-1.6250\\1.5915\\-0.2026\end{bmatrix}$$

The equations are then given in the following:

For $0 \le \theta \le \frac{\pi}{2}$

$$y_1 = a_2\theta^2 = 0.2026\theta^2$$

For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$

$$y_2 = b_0 + b_1\theta = -0.5000 + 0.6366\theta$$

For $\frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

$$y_3 = c_0 + c_1\theta + c_2\theta^2 = -1.6250 + 1.5915\theta - 0.2026\theta^2$$

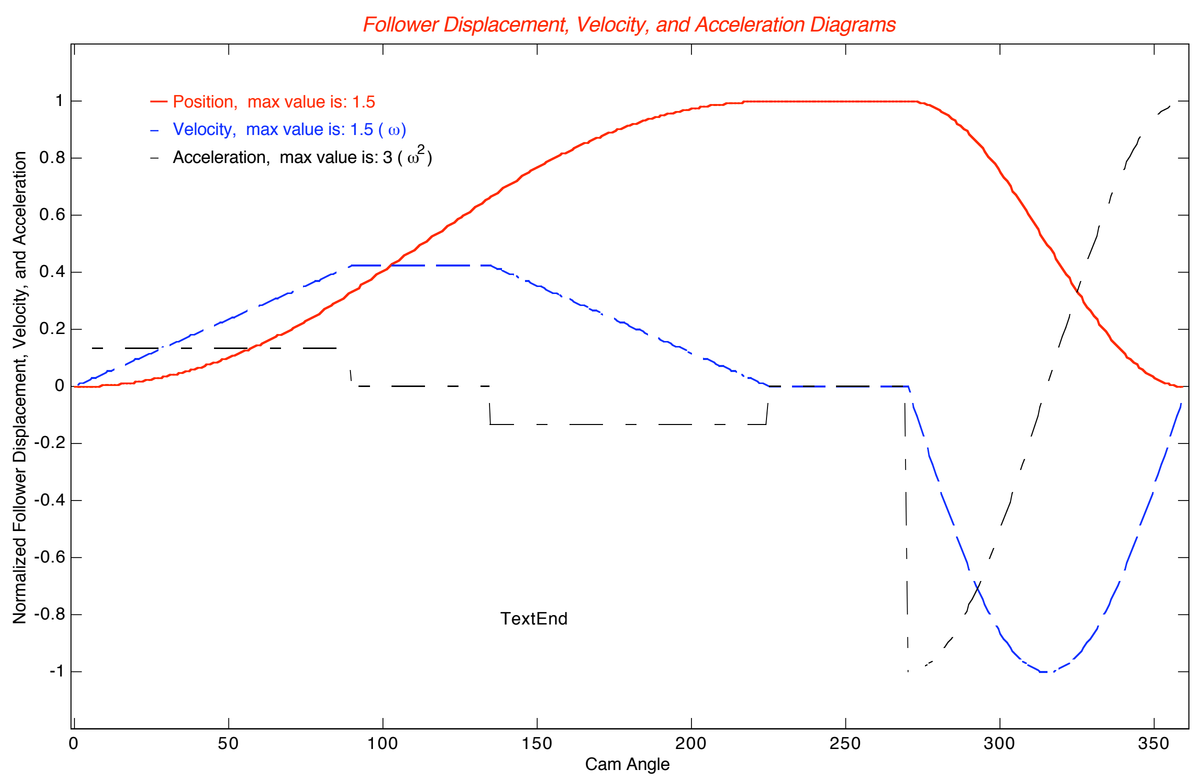
For $\frac{5\pi}{4} \le \theta \le \frac{3\pi}{2}$

$$y_4 = 1.5$$

For the return, $\frac{3\pi}{2} \le \theta \le 2\pi$, and

$$y_5 = \frac{L}{2}\left(1 + cos\frac{\pi\theta}{\beta}\right) = \frac{3}{4}(1 + cos\,2\theta)$$

The displacement diagram is plotted in the following:



Problem 8.5

Draw the displacement schedule for a follower that rises through a total displacement of 20 mm with constant acceleration for 1/8th revolution, constant velocity for 1/4th revolution, and constant deceleration for 1/8th revolution of the cam. The cam then dwells for 1/4th revolution, and returns with simple harmonic motion in 1/4th revolution of the cam.

Solution:

The displacement profile can be easily computed using the equations in Chapter 8 using Matlab. The curves are matched at the endpoints of each segment. The profile equations are:

For $0 \le \theta \le \frac{\pi}{4}$

$$y_1 = a_0 + a_1\theta + a_2\theta^2$$

The boundary conditions at $\theta = 0$ are $y_1 = 0$ and $y'_1 = 0$. Therefore,

$$a_0 = a_1 = 0$$

So,

$$y_1 = a_2\theta^2$$

and

$$y'_1 = 2a_2\theta$$

where a_2 is yet to be determined.

For $\frac{\pi}{4} \le \theta \le \frac{3\pi}{8}$

$$y_2 = b_0 + b_1\theta$$

The bounary conditions at $\theta = \frac{\pi}{4}$ are $y_1 = a_2\left(\frac{\pi}{4}\right)^2$ and $y'_1 = 2a_2\frac{\pi}{4} = a_2\frac{\pi}{2}$. Then,

$$b_0 + b_1\frac{\pi}{4} = a_2\left(\frac{\pi}{4}\right)^2$$

and

$$0 = -a_2\left(\frac{\pi}{4}\right)^2 + b_0 + b_1\frac{\pi}{4}$$

Also

$$y'_2 = b_1 = a_2\frac{\pi}{2}$$

or

$$0 = a_2\frac{\pi}{2} - b_1$$

For $\frac{3\pi}{8} \le \theta \le \pi$

$$y_3 = c_0 + c_1\theta + c_2\theta^2$$
$$y'_3 = c_1 + 2c_2\theta$$
$$y''_3 = 2c_2$$

The boundary conditions at $\theta = \frac{3\pi}{8}$ are $y_2 = b_0 + b_1\frac{3\pi}{8}$ and $y'_2 = b_1$. Also, at $\theta = \pi$, $y_3 = 20$, and $y'_3 = 0$. Then matching the conditions,

$$y_3 = b_0 + b_1\frac{3\pi}{8} = c_0 + c_1\frac{3\pi}{8} + c_2\left(\frac{3\pi}{8}\right)^2$$
$$y'_3 = c_1 + 2c_2\frac{3\pi}{8} = b_1$$
$$20 = c_0 + c_1\pi + c_2\pi^2$$
$$y'_3 = c_1 + 2c_2\pi = 0$$

The boundary condition equations can be written as:

$$0 = -a_2\left(\frac{\pi}{4}\right)^2 + b_0 + b_1\frac{\pi}{4}$$
$$0 = a_2\frac{\pi}{2} - b_1$$
$$0 - b_0 - b_1\frac{3\pi}{8} + c_0 + c_1\frac{3\pi}{8} + c_2\left(\frac{3\pi}{8}\right)^2$$
$$0 = -b_1 + c_1 + c_2\frac{3\pi}{4}$$
$$20 = c_0 + c_1\pi + c_2\pi^2$$
$$0 = c_1 + 2c_2\pi$$

In matrix form,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} -\left(\frac{\pi}{4}\right)^2 & 1 & \frac{\pi}{4} & 0 & 0 & 0 \\ \frac{\pi}{2} & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -\frac{3\pi}{8} & 1 & \frac{3\pi}{8} & \left(\frac{3\pi}{8}\right)^2 \\ 0 & 0 & -1 & 0 & 1 & \frac{3\pi}{4} \\ 0 & 0 & 0 & 0 & 1 & 2\pi \\ 0 & 0 & 0 & 1 & \pi & \pi^2 \end{bmatrix} \begin{bmatrix} a_2 \\ b_0 \\ b_1 \\ c_0 \\ c_1 \\ c_2 \end{bmatrix}$$

Solving for the constraints using Matlab,

$$\begin{bmatrix} a_2 \\ b_0 \\ b_1 \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 7.2051 \\ -4.4444 \\ 11.3177 \\ -8.4444 \\ 18.1083 \\ -2.8820 \end{bmatrix}$$

The equations are then given in the following:

For $0 \le \theta \le \frac{\pi}{4}$

$$y_1 = a_2\theta^2 = 7.2051\theta^2$$

For $\frac{\pi}{4} \le \theta \le \frac{3\pi}{8}$

$$y_2 = b_0 + b_1\theta = \ -4.4444 + 11.3177\theta$$

For $\frac{3\pi}{8} \le \theta \le \pi$

$$y_3 = c_0 + c_1\theta + c_2\theta^2 = -8.4444 + 18.1083\theta - 2.8820\theta^2$$

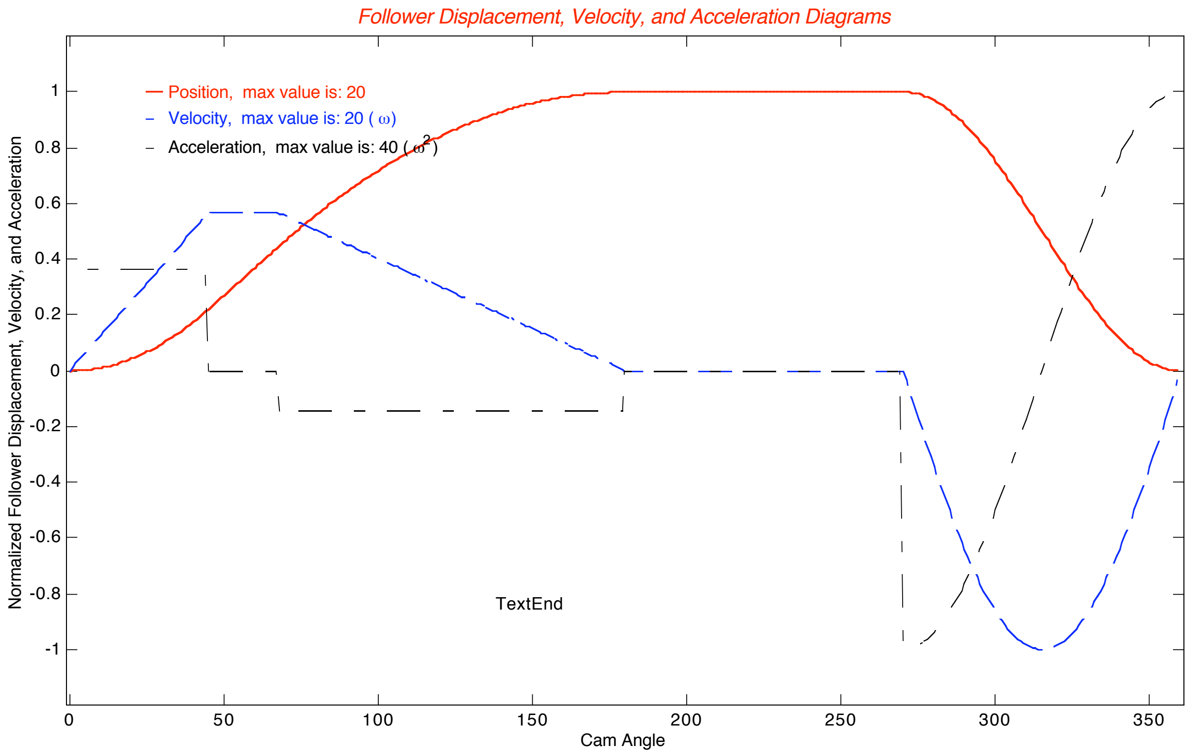
For $\pi \le \theta \le \frac{3\pi}{2}$

$$y_4 = 20\ mm$$

For the return, $\frac{3\pi}{2} \le \theta \le 2\pi$, and

$$y_5 = \frac{L}{2}\left(1 + cos\frac{\pi\theta}{\beta}\right) = 10(1 + cos\,2\theta)$$

The displacement diagram is plotted in the following:



Problem 8.6

Draw the displacement schedule for a follower that rises through a total displacement of 30 mm with constant acceleration for 90˚ of rotation and constant deceleration for 45˚ of cam rotation. The follower returns 15 mm with simple harmonic motion in 90˚ of cam rotation and dwells for 45˚ of cam rotation. It then returns the remaining 15 mm with simple harmonic motion during the remaining 90˚ of cam rotation.

Solution:

The displacement profile can be easily computed using the equations in Chapter 8 using Matlab. The curves are matched at the endpoints of each segment. The profile equations are:

For $0 \le \theta \le \dfrac{\pi}{2}$

$$y_1 = a_0 + a_1\theta + a_2\theta^2$$

The boundary conditions at $\theta = 0$ are $y_1 = 0$ and $y_1' = 0$. Therefore,

$$a_0 = a_1 = 0$$

So,

$$y_1 = a_2\theta^2$$

and

$$y_1' = 2a_2\theta$$

where a_2 is yet to be determined.

For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$

$$y_2 = b_0 + b_1\theta + b_2\theta^2$$

The boundary conditions at $\theta = \frac{\pi}{2}$ and $y_1 = a_2\left(\frac{\pi}{2}\right)^2$ and $y'_1 = 2a_2\frac{\pi}{2} = a_2\pi$. Then,

$$b_0 + b_1\frac{\pi}{2} + b_2\left(\frac{\pi}{2}\right)^2 = a_2\left(\frac{\pi}{2}\right)^2$$

and

$$0 = -a_2\left(\frac{\pi}{2}\right)^2 + b_0 + b_1\frac{\pi}{2} + b_2\left(\frac{\pi}{2}\right)^2$$

Also

$$y'_2 = b_1 + b_2\pi = a_2\pi$$

or

$$0 = -a_2\pi + b_1 + b_2\pi$$

The boundary conditions at $\theta = \frac{3\pi}{4}$ are $y_2 = 30$ and $y'_2 = 0$. Then,

$$b_0 + b_1\frac{3\pi}{4} + b_2\left(\frac{3\pi}{4}\right)^2 = 30$$

and

$$0 = b_1 + 2b_2\left(\frac{3\pi}{4}\right) = b_1 + b_2\left(\frac{3\pi}{2}\right)$$

The four boundary condition equations can be summarized as:

$$0 = -a_2\left(\frac{\pi}{2}\right)^2 + b_0 + b_1\frac{\pi}{2} + b_2\left(\frac{\pi}{2}\right)^2$$

$$0 = -a_2\pi + b_1 + b_2\pi$$

$$30 = b_0 + b_1\frac{3\pi}{4} + b_2\left(\frac{3\pi}{4}\right)^2$$

$$0 = b_1 + b_2\left(\frac{3\pi}{2}\right)$$

In matrix form,

$$\begin{bmatrix} 0 \\ 0 \\ 30 \\ 0 \end{bmatrix} = \begin{bmatrix} -\left(\frac{\pi}{2}\right)^2 & 1 & \frac{\pi}{2} & \left(\frac{\pi}{2}\right)^2 \\ -\pi & 0 & 1 & \pi \\ 0 & 1 & \frac{3\pi}{4} & \left(\frac{3\pi}{4}\right)^2 \\ 0 & 0 & 1 & \frac{3\pi}{2} \end{bmatrix} \begin{bmatrix} a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

7.5

Solving for the constraints using Matlab,

$$\begin{bmatrix} a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 8.1057 \\ -60.0000 \\ 76.3944 \\ -16.2114 \end{bmatrix}$$

The equations are then given in the following:

For $0 \le \theta \le \frac{\pi}{2}$

$$y_1 = a_2\theta^2 = 8.1057\theta^2$$

For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$

$$y_2 = b_0 + b_1\theta + b_2\theta^2 = -60.0000 + 76.3944\theta + -16.2114\theta^2$$

For $\frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

$$\beta = \frac{\pi}{2}$$

and

$$y_3 = \frac{L}{2}\left(1 + cos\frac{\pi\theta}{\beta}\right) = \frac{15}{2}(1 + cos\,2\theta)$$

This equation assumes that the curve is 15 mm high and that the curve falls to zero. However, the actual curve begins at a height of 30 mm and returns to only 15 mm. Because of this, we need to add 15 mm to the value for y_3. Then,

$$y_3 = 15 + 7.5(1 + cos\,2\theta)$$

For $\frac{5\pi}{4} \le \theta \le \frac{6\pi}{4}$

$$y_4 = 15$$

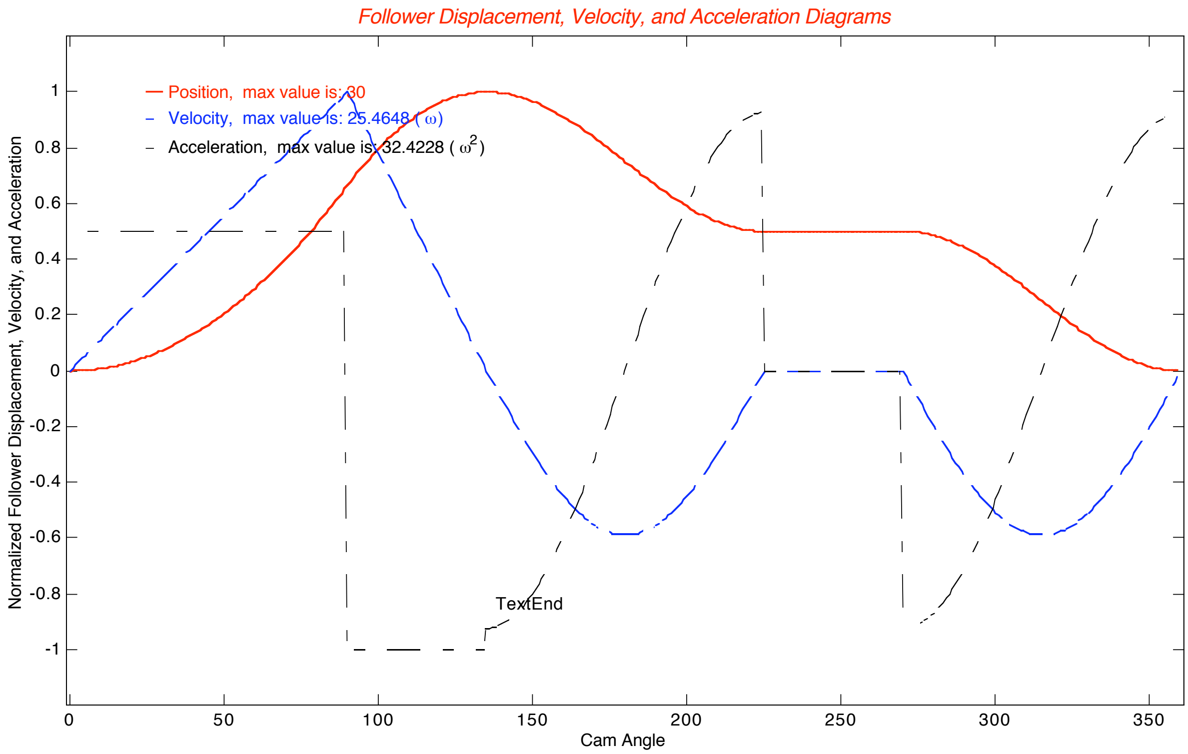
For $\frac{6\pi}{4} \le \theta \le 2\pi$

$$\beta = \frac{\pi}{2}$$

and

$$y_5 = \frac{L}{2}\left(1 + cos\frac{\pi\theta}{\beta}\right) = \frac{15}{2}(1 + cos\,2\theta)$$

The displacement diagram is plotted in the following:



Problem 8.7

Draw the displacement schedule for a follower that rises through a total displacement of 3 inches with cycloidal motion in 120 degrees of cam rotation. The follower then dwells for 90˚ and returns to zero with simple harmonic motion in 90˚ of cam rotation. The follower then dwells for 60˚ before repeating the cycle.

Solution:

The displacement profile can be easily computed using the equations in Chapter 8 using Matlab. The curves are matched at the endpoints of each segment. The profile equations are:

For $0 \le \theta \le \dfrac{2\pi}{3}$

$$\beta = \frac{2\pi}{3}$$

and

$$y_1 = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right) = 3\left(\frac{3\theta}{2\pi} - \frac{1}{2\pi}\sin 3\theta\right)$$

$$y'_1 = L\left(\frac{1}{\beta} - \frac{1}{\beta}cos\frac{2\pi\theta}{\beta}\right) = 3\left(\frac{3}{2\pi} - \frac{3}{2\pi}cos\,3\theta\right)$$
$$y''_1 = L\left(\frac{2\pi}{\beta^2}sin\frac{2\pi\theta}{\beta}\right) = 3\left(\frac{9}{2\pi}sin\,3\theta\right)$$

For $\frac{2\pi}{3} \le \theta \le \frac{7\pi}{6}$

$$y_2 = 3$$

For the return, $\frac{7\pi}{6} \le \theta \le \frac{5\pi}{3}$ –

$$\beta = \frac{\pi}{2}$$

and

$$y_3 = \frac{L}{2}\left(1 + cos\frac{\pi\theta}{\beta}\right) = 1.5(1 + cos\,2\theta)$$

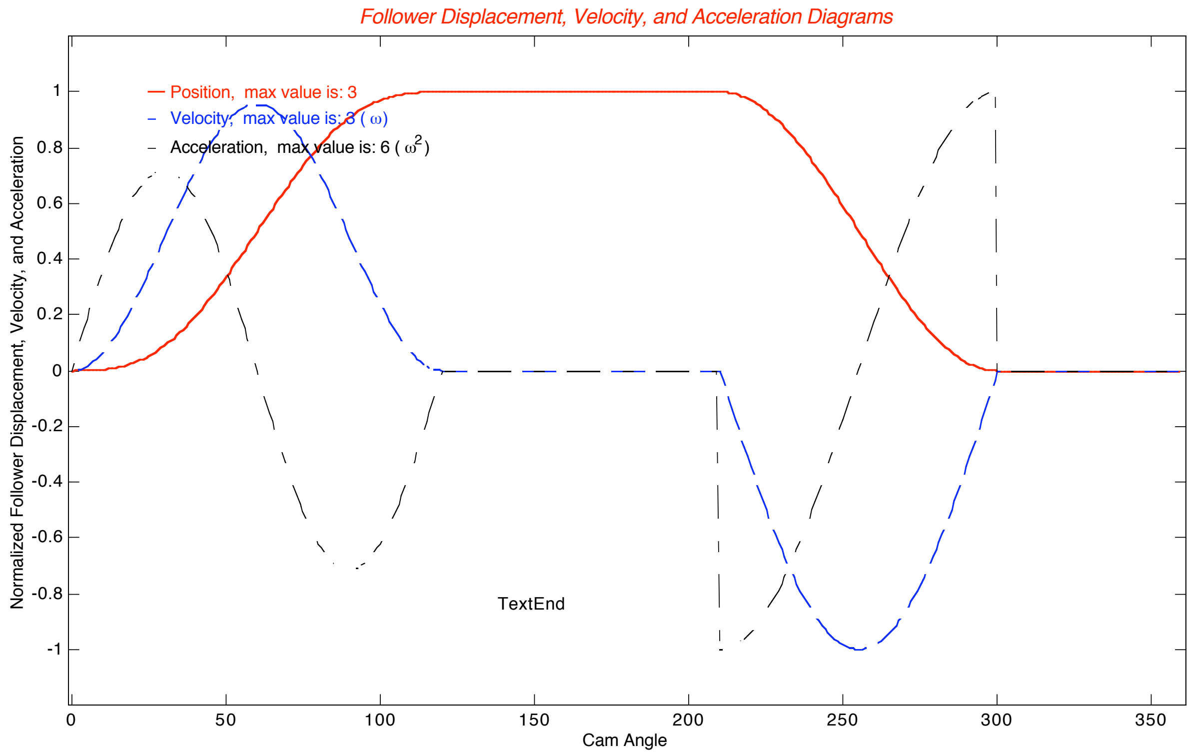
For the remainder of the cycle,

$$\frac{5\pi}{3} \le \theta \le 2\pi$$

and

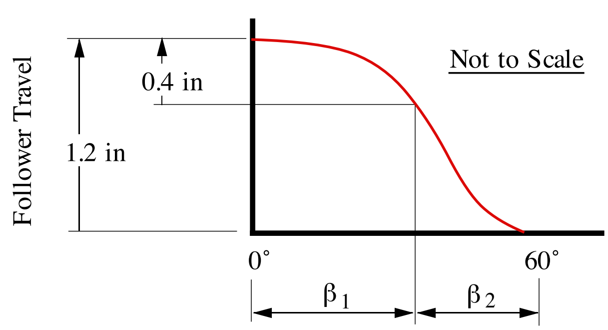
$$y_4 = 0$$

The displacement diagram is plotted in the following:



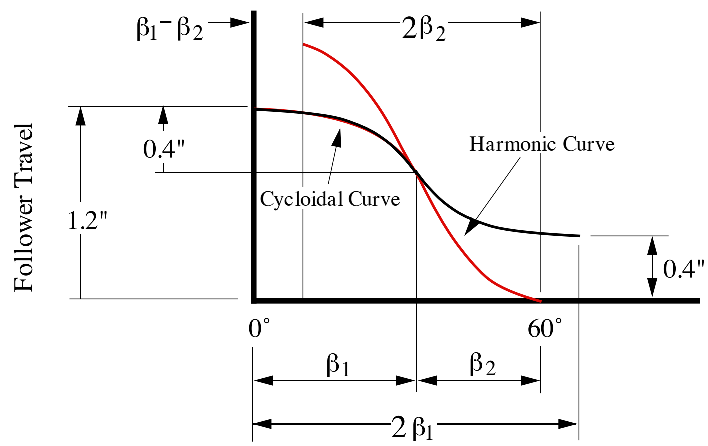
Problem 8.8

A cam returns from a full lift of 1.2 in during its initial 60° rotation. The first 0.4 in of the return is half-cycloidal. This is followed by a half-harmonic return. Determine β_1 and β_2 so that the motion has continuous first and second derivatives. Draw a freehand sketch of y', y'', and y''' indicating any possible mismatch in the third derivative.



Solution:

The first part of the return is made up of a cycloidal curve and the second part is made up of a harmonic curve. This is shown schematically in the figure below.



The range for the cycloidal curve is given by

$$0 \le \theta_1 \le 2\beta_1$$

and the range for the harmonic curve is given by

$$\beta_1 - \beta_2 \le \theta_2 \le \beta_1 + \beta_2$$

Also,

$$\theta_2 = \theta_1 - (\beta_1 - \beta_2)$$

The general form for the cycloidal equation for a return is given in Section 7.8 as

$$y = L\left[1 - \frac{\theta}{\beta} + \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right]$$

Half of the cycloidal return is 0.4 so return is 0.8. The β range for θ_1 is $2\beta_1$. As indicated in the figure above, the cycloidal curve is offset from the horizontal axis by 0.4". Therefore, this much must be added to y. The cycloidal equation for the return is

$$y_1 = L_1\left[1 - \frac{\theta_1}{2\beta_1} + \frac{1}{2\pi}\sin\frac{2\pi\theta_1}{2\beta_1}\right] + 0.4 = 0.8\left[1 - \frac{\theta_1}{2\beta_1} + \frac{1}{2\pi}\sin\frac{\pi\theta_1}{\beta_1}\right] + 0.4$$

$$= 0.8\left[1.5 - \frac{\theta_1}{2\beta_1} + \frac{1}{2\pi}\sin\frac{\pi\theta_1}{\beta_1}\right] \qquad (1)$$

The harmonic curve is given by Eq. (812). Half of the harmonic return is (1.2"-0.4") = 0.8 so that the whole return is 1.6". The β range for θ_2 is $2\beta_2$. Therefore, the equation for the harmonic part of the return is:

$$y_2 = \frac{L_2}{2}\left[1 + \cos\frac{\pi\theta_2}{2\beta_2}\right] = 0.8\left[1 + \cos\frac{\pi\theta_2}{2\beta_2}\right] \qquad (2)$$

We also know that $\theta_2 = \theta_1 - (\beta_1 - \beta_2)$ and $\beta_2 = \frac{\pi}{3} - \beta_1$

Eqs. (1) and (2) can be reduced so that the only unknown is β_1. To solve for the unknown, we can equate the slopes at $\theta_1 = \beta_1$. For the cycloidal equation,

$$y'_1 = -\frac{0.8}{2\beta_1}\left[1-\cos\frac{\pi\theta_1}{\beta_1}\right]$$

and at $\theta_1 = \beta_1$

$$y'_1 = -\frac{0.8}{2\beta_1}\left[1-\cos\frac{\pi\beta_1}{\beta_1}\right] = -\frac{0.8}{\beta_1} \tag{3}$$

For the harmonic equation

$$y'_2 = -\frac{0.4\pi}{\beta_2}\left[\sin\frac{\pi\theta_2}{2\beta_2}\right] \tag{4}$$

At $\theta_1 = \beta_1, \theta_2 = \beta_2$. Therefore,

$$y'_2 = -\frac{0.4\pi}{\beta_2}\left[\sin\frac{\pi\theta_2}{2\beta_2}\right] = -\frac{0.4\pi}{\beta_2}\left[\sin\frac{\pi\beta_2}{2\beta_2}\right] = -\frac{0.4\pi}{\beta_2} \tag{4}$$

Equation Eqs. (3) and (4) give the following equation

$$-\frac{0.8}{\beta_1} = -\frac{0.4\pi}{\beta_2}$$

or

$$\frac{2}{\beta_1} = \frac{\pi}{\beta_2} = \frac{\pi}{\pi/3-\beta_1}$$

This equation can be easily solved for β_1. The result is: $\beta_1 = \dfrac{2\pi}{3(\pi+2)} = 0.4073436$ radians.

We can now write y, y', and y" for each part of the curve. For $\theta_1 \le 0.4073436$

$$y = 0.8\left[1.5-\frac{\theta_1}{2\beta_1}+\frac{1}{2\pi}\sin\frac{\pi\theta_1}{\beta_1}\right]$$

$$y' = -\frac{0.8}{2\beta_1}\left[1-\cos\frac{\pi\theta_1}{\beta_1}\right]$$

$$y'' = -\frac{0.8\pi}{2\beta_1^2}\left[\sin\frac{\pi\theta_1}{\beta_1}\right]$$

and for $0.4073436 \le \theta \le \pi/3$

$$y = 0.8\left[1+\cos\frac{\pi\theta_2}{2\beta_2}\right]$$

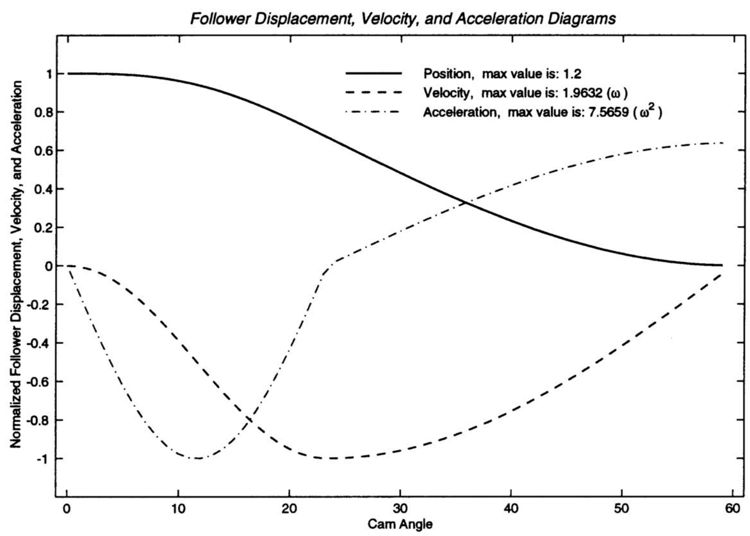
$$y' = -\frac{0.4\pi}{\beta_2}\left[\sin\frac{\pi\theta_2}{2\beta_2}\right]$$

$$y'' = -\frac{0.2\pi^2}{\beta_2^2}\left[\cos\frac{\pi\theta_2}{2\beta_2}\right]$$

where

$$\theta_2 = \theta_1 - (\beta_1 - \beta_2) \text{ and } \beta_2 = \frac{\pi}{3} - \beta_1$$

The results are plotted in the following

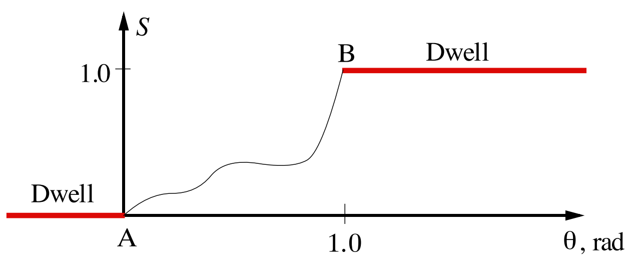


Problem 8.9

Assume that s is the cam-follower displacement and θ is the cam rotation. The rise is 1.0 cm after 1.0 radian of rotation, and the rise begins and ends at a dwell. The displacement equation for the follower during the rise period is

$$s = h\sum_{i=0}^{n} C_i\left(\frac{\theta}{\beta}\right)^i$$

If the position, velocity, and acceleration are continuous at $\theta = 0$, and the position and velocity are continuous at $\theta = 1.0$ rad, determine the value of n required in the equation, and find the coefficients C_i if $\dot{\theta} = 2$ rad/s. Note: Use the minimum possible number of terms.



Solution:

First determine the number of terms required. There are a total of five conditions to match; therefore, the number of terms is 5 making n = 4.

The conditions to match are:

At $\theta = 0$, $\quad s = \dfrac{ds}{d\theta} = \dfrac{d^2s}{d\theta^2} = 0$

At $\theta = \beta$, $\quad$ s = h = 1.0

$$\frac{ds}{d\theta} = 0$$

Now,

$$s = \sum_{i=0}^{n} C_i\left(\frac{\theta}{\beta}\right)^i = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4$$

$$\frac{ds}{d\theta} = \frac{C_1}{\beta} + \frac{2C_2}{\beta}\left(\frac{\theta}{\beta}\right) + \frac{3C_3}{\beta}\left(\frac{\theta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\theta}{\beta}\right)^3$$

and

$$\frac{d^2s}{d\theta^2} = \frac{2C_2}{\beta^2} + \frac{6C_3}{\beta^2}\left(\frac{\theta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\theta}{\beta}\right)^2$$

Applying the conditions at $\theta = 0$,

$$s = C_0 = 0$$

$$\frac{ds}{d\theta} = \frac{C_1}{\beta} \Rightarrow C_1 = 0$$

$$\frac{d^2s}{d\theta^2} = \frac{2C_2}{\beta^2} = 0 \Rightarrow C_2 = 0$$

Applying the conditions at $\theta = \beta$,

$$s = C_3\left(\frac{\beta}{\beta}\right)^3 + C_4\left(\frac{\beta}{\beta}\right)^4 = C_3 + C_4 = 1$$

$$\frac{ds}{d\theta} = \frac{3C_3}{\beta}\left(\frac{\beta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\beta}{\beta}\right)^3 = \frac{3C_3}{\beta} + \frac{4C_4}{\beta} = 0$$

or

$$3C_3 + 4C_4 = 0$$

Solving for the constants,

$$C_3 = 4$$

and

$$C_4 = -3$$

Therefore,

$$s = 4\left(\frac{\theta}{\beta}\right)^3 - 3\left(\frac{\theta}{\beta}\right)^4$$

Problem 8.10

Resolve Problem 8.9 if $\theta = 0.8$ rad and $\dot{\theta} = 200$ rad/s.

Solution:

First determine the number of terms required. There are a total of five conditions to match; therefore, the number of terms is 5 making n = 4.

The conditions to match are:

At $\theta = 0$, $s = \frac{ds}{d\theta} = \frac{d^2s}{d\theta^2} = 0$

At $\theta = \beta$, s = h = 1.0

$$\frac{ds}{d\theta} = 0$$

Now,

$$s = \sum_{i=0}^{n} C_i \left(\frac{\theta}{\beta}\right)^i = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4$$

$$\frac{ds}{d\theta} = \frac{C_1}{\beta} + \frac{2C_2}{\beta}\left(\frac{\theta}{\beta}\right) + \frac{3C_3}{\beta}\left(\frac{\theta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\theta}{\beta}\right)^3$$

and

$$\frac{d^2s}{d\theta^2} = \frac{2C_2}{\beta^2} + \frac{6C_3}{\beta^2}\left(\frac{\theta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\theta}{\beta}\right)^2$$

Applying the conditions at $\theta = 0$,

$$s = C_0 = 0$$

$$\frac{ds}{d\theta} = \frac{C_1}{\beta} \Rightarrow C_1 = 0$$

$$\frac{d^2s}{d\theta^2} = \frac{2C_2}{\beta^2} = 0 \Rightarrow C_2 = 0$$

Applying the conditions at $\theta = \beta$,

$$s = C_3\left(\frac{\beta}{\beta}\right)^3 + C_4\left(\frac{\beta}{\beta}\right)^4 = C_3 + C_4 = 1$$

$$\frac{ds}{d\theta} = \frac{3C_3}{\beta}\left(\frac{\beta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\beta}{\beta}\right)^3 = \frac{3C_3}{\beta} + \frac{4C_4}{\beta} = 0$$

or

$$3C_3 + 4C_4 = 0$$

Solving for the constants,

$$C_3 = 4$$

and

$$C_4 = -3$$

Therefore,

$$s = 4\left(\frac{\theta}{\beta}\right)^3 - 3\left(\frac{\theta}{\beta}\right)^4$$

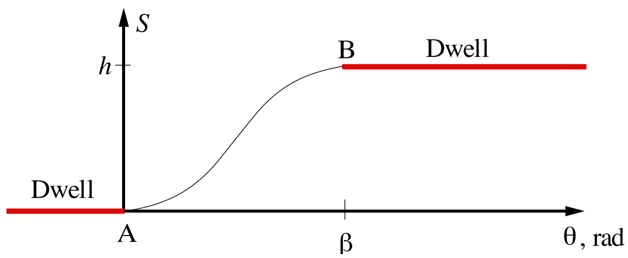
Notice that this solution is EXACTLY the same as that for 8.9. The results are independent of both θ and $\dot{\theta}$. This is one of the reasons for normalizing the problem with respect to β.

Problem 8.11

For the cam displacement schedule given, h is the rise, β is the angle through which the rise takes place, and s is the displacement at any given angle θ. The displacement equation for the follower during the rise period is

$$s = h\sum_{i=0}^{5} a_i\left(\frac{\theta}{\beta}\right)^i$$

Determine the required values for a_0 ... a_5 such that the displacement, velocity, and acceleration functions are continuous at the end points of the rise portion.



<u>Solution:</u>

There are a total of six conditions to match; therefore, the number of terms is 6 making n = 5.

The conditions to match are:

At $\theta = 0$, $\quad s = \dfrac{ds}{d\theta} = \dfrac{d^2s}{d\theta^2} = 0$

At $\theta = \beta$, $\quad$ s = h

and

$$\frac{ds}{d\theta} = \frac{d^2s}{d\theta^2} = 0$$

Now,

$$s = h\sum_{i=0}^{n} C_i\left(\frac{\theta}{\beta}\right)^i = h\left[C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5\right]$$

$$\frac{ds}{d\theta} = h\left[\frac{C_1}{\beta} + \frac{2C_2}{\beta}\left(\frac{\theta}{\beta}\right) + \frac{3C_3}{\beta}\left(\frac{\theta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\theta}{\beta}\right)^3 + \frac{5C_5}{\beta}\left(\frac{\theta}{\beta}\right)^4\right]$$

and

$$\frac{d^2s}{d\theta^2} = h\left[\frac{2C_2}{\beta^2} + \frac{6C_3}{\beta^2}\left(\frac{\theta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\theta}{\beta}\right)^2 + \frac{20C_5}{\beta^2}\left(\frac{\theta}{\beta}\right)^3\right]$$

Applying the conditions at $\theta = 0$,

$$s = C_0 = 0$$

$$\frac{ds}{d\theta} = h\frac{C_1}{\beta} \Rightarrow C_1 = 0$$

$$\frac{d^2s}{d\theta^2} = h\frac{2C_2}{\beta^2} = 0 \Rightarrow C_2 = 0$$

Applying the conditions at $\theta = \beta$,

$$s = h\left[C_3\left(\frac{\beta}{\beta}\right)^3 + C_4\left(\frac{\beta}{\beta}\right)^4 + C_5\left(\frac{\beta}{\beta}\right)^5\right] = h[C_3 + C_4 + C_5] = h$$

or

$$C_3 + C_4 + C_5 = 1$$

$$\frac{ds}{d\theta} = h\left[\frac{3C_3}{\beta}\left(\frac{\beta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\beta}{\beta}\right)^3\right] = h\left[\frac{3C_3}{\beta} + \frac{4C_4}{\beta} + \frac{5C_5}{\beta}\right] = 0$$

or

$$3C_3 + 4C_4 + 5C_5 = 0$$

$$\frac{d^2s}{d\theta^2} = h\left[\frac{6C_3}{\beta^2}\left(\frac{\beta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\beta}{\beta}\right)^2 + \frac{20C_5}{\beta^2}\left(\frac{\beta}{\beta}\right)^3\right] = h\left[\frac{6C_3}{\beta^2} + \frac{12C_4}{\beta^2} + \frac{20C_5}{\beta^2}\right] = 0$$

or

$$6C_3 + 12C_4 + 20C_5 = 0$$

Solving for the constants,

$$C_3 = 10;\ C_4 = -15;\ C_5 = 6$$

Therefore,

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

Problem 8.12

Resolve Problem 8.11 if $h = 20$ mm and $\beta = 120°$.

Solution:

There are a total of six conditions to match; therefore, the number of terms is 6 making n = 5.

The conditions to match are:

At $\theta = 0$, $\quad s = \frac{ds}{d\theta} = \frac{d^2s}{d\theta^2} = 0$

At $\theta = \beta$, $s = h$

and

$$\frac{ds}{d\theta} = \frac{d^2s}{d\theta^2} = 0$$

Now,

$$s = h\sum_{i=0}^{n} C_i \left(\frac{\theta}{\beta}\right)^i = h\left[C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5\right]$$

$$\frac{ds}{d\theta} = h\left[\frac{C_1}{\beta} + \frac{2C_2}{\beta}\left(\frac{\theta}{\beta}\right) + \frac{3C_3}{\beta}\left(\frac{\theta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\theta}{\beta}\right)^3 + \frac{5C_5}{\beta}\left(\frac{\theta}{\beta}\right)^4\right]$$

and

$$\frac{d^2s}{d\theta^2} = h\left[\frac{2C_2}{\beta^2} + \frac{6C_3}{\beta^2}\left(\frac{\theta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\theta}{\beta}\right)^2 + \frac{20C_5}{\beta^2}\left(\frac{\theta}{\beta}\right)^3\right]$$

Applying the conditions at $\theta = 0$,

$$s = C_0 = 0$$

$$\frac{ds}{d\theta} = h\frac{C_1}{\beta} \Rightarrow C_1 = 0$$

$$\frac{d^2s}{d\theta^2} = h\frac{2C_2}{\beta^2} = 0 \Rightarrow C_2 = 0$$

Applying the conditions at $\theta = \beta$_

$$s = h\left[C_3\left(\frac{\beta}{\beta}\right)^3 + C_4\left(\frac{\beta}{\beta}\right)^4 + C_5\left(\frac{\beta}{\beta}\right)^5\right] = h[C_3 + C_4 + C_5] = h$$

or

$$C_3 + C_4 + C_5 = 1$$

$$\frac{ds}{d\theta} = h\left[\frac{3C_3}{\beta}\left(\frac{\beta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\beta}{\beta}\right)^3\right] = h\left[\frac{3C_3}{\beta} + \frac{4C_4}{\beta} + \frac{5C_5}{\beta}\right] = 0$$

or

$$3C_3 + 4C_4 + 5C_5 = 0$$

$$\frac{d^2s}{d\theta^2} = h\left[\frac{6C_3}{\beta^2}\left(\frac{\beta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\beta}{\beta}\right)^2 + \frac{20C_5}{\beta^2}\left(\frac{\beta}{\beta}\right)^3\right] = h\left[\frac{6C_3}{\beta^2} + \frac{12C_4}{\beta^2} + \frac{20C_5}{\beta^2}\right] = 0$$

or

$$6C_3 + 12C_4 + 20C_5 = 0$$

Solving for the constants,

$C_3 = 10;\ C_4 = -15;\ C_5 = 6$

Therefore,

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

If h=20 mm and $\beta = 120^\circ$, then,

$$s = 20\left[10\left(\frac{\theta}{120}\right)^3 - 15\left(\frac{\theta}{120}\right)^4 + 6\left(\frac{\theta}{120}\right)^5\right]$$

Here, θ is assumed to be given in degrees. Note that the values for h and β do not enter the problem until the last step.

Problem 8.13

Resolve Problem 8.11 if $h = 2$ in and $\beta = 90^\circ$.

Solution:

There are a total of six conditions to match; therefore, the number of terms is 6 making n = 5.

The conditions to match are:

At $\theta = 0$, $s = \dfrac{ds}{d\theta} = \dfrac{d^2s}{d\theta^2} = 0$

At $\theta = \beta$, $s = h$

and

$$\frac{ds}{d\theta} = \frac{d^2s}{d\theta^2} = 0$$

Now,

$$s = h\sum_{i=0}^{n} C_i\left(\frac{\theta}{\beta}\right)^i = h\left[C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5\right]$$

$$\frac{ds}{d\theta} = h\left[\frac{C_1}{\beta} + \frac{2C_2}{\beta}\left(\frac{\theta}{\beta}\right) + \frac{3C_3}{\beta}\left(\frac{\theta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\theta}{\beta}\right)^3 + \frac{5C_5}{\beta}\left(\frac{\theta}{\beta}\right)^4\right]$$

and

$$\frac{d^2s}{d\theta^2} = h\left[\frac{2C_2}{\beta^2} + \frac{6C_3}{\beta^2}\left(\frac{\theta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\theta}{\beta}\right)^2 + \frac{20C_5}{\beta^2}\left(\frac{\theta}{\beta}\right)^3\right]$$

Applying the conditions at $\theta = 0$,

$$s = C_0 = 0$$

$$\frac{ds}{d\theta} = h\frac{C_1}{\beta} \Rightarrow C_1 = 0$$

$$\frac{d^2 s}{d\theta^2} = h\frac{2C_2}{\beta^2} = 0 \Rightarrow C_2 = 0$$

Applying the conditions at $\theta = \beta$_

$$s = h\left[C_3\left(\frac{\beta}{\beta}\right)^3 + C_4\left(\frac{\beta}{\beta}\right)^4 + C_5\left(\frac{\beta}{\beta}\right)^5\right] = h[C_3 + C_4 + C_5] = h$$

or

$$C_3 + C_4 + C_5 = 1$$

$$\frac{ds}{d\theta} = h\left[\frac{3C_3}{\beta}\left(\frac{\beta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\beta}{\beta}\right)^3\right] = h\left[\frac{3C_3}{\beta} + \frac{4C_4}{\beta} + \frac{5C_5}{\beta}\right] = 0$$

or

$$3C_3 + 4C_4 + 5C_5 = 0$$

$$\frac{d^2 s}{d\theta^2} = h\left[\frac{6C_3}{\beta^2}\left(\frac{\beta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\beta}{\beta}\right)^2 + \frac{20C_5}{\beta^2}\left(\frac{\beta}{\beta}\right)^3\right] = h\left[\frac{6C_3}{\beta^2} + \frac{12C_4}{\beta^2} + \frac{20C_5}{\beta^2}\right] = 0$$

or

$$6C_3 + 12C_4 + 20C_5 = 0$$

Solving for the constants,

$$C_3 = 10;\ C_4 = -15;\ C_5 = 6$$

Therefore,

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

If h=2 in and $\beta = 90^\circ$, then,

$$s = 2\left[10\left(\frac{\theta}{90}\right)^3 - 15\left(\frac{\theta}{90}\right)^4 + 6\left(\frac{\theta}{90}\right)^5\right]$$

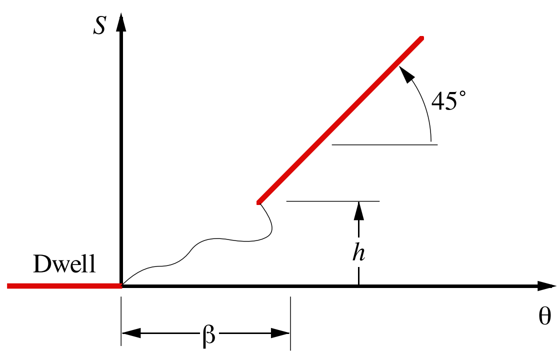
Here, θ is assumed to be given in degrees. Note that the values for h and β do not enter the problem until the last step.

Problem 8.14

Assume that s is the cam-follower displacement and θ is the cam rotation. The rise is h after β degrees of rotation, and the rise begins at a dwell and ends with a constant velocity segment. The displacement equation for the follower during the rise period is

$$s = h\sum_{i=0}^{n} C_i\left(\frac{\theta}{\beta}\right)^i$$

If the position, velocity, and acceleration are continuous at $\theta = 0$ and the position and velocity are continuous at $\theta = \beta$, determine the n required in the equation, and find the coefficients C_i that will satisfy the requirements if $s = h = 1.0$.



Solution:

First determine the number of terms required. There are a total of five conditions to match; therefore, the number of terms is 5 making n = 4.

The conditions to match are:

At $\theta = 0$, $s = \frac{ds}{d\theta} = \frac{d^2s}{d\theta^2} = 0$

At $\theta = \beta$, s = h = 1.0

$$\frac{ds}{d\theta} = \tan 45° = 1.0$$

Now,

$$s = \sum_{i=0}^{n} C_i\left(\frac{\theta}{\beta}\right)^i = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4$$

$$\frac{ds}{d\theta} = \frac{C_1}{\beta} + \frac{2C_2}{\beta}\left(\frac{\theta}{\beta}\right) + \frac{3C_3}{\beta}\left(\frac{\theta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\theta}{\beta}\right)^3$$

and

$$\frac{d^2s}{d\theta^2} = \frac{2C_2}{\beta^2} + \frac{6C_3}{\beta^2}\left(\frac{\theta}{\beta}\right) + \frac{12C_4}{\beta^2}\left(\frac{\theta}{\beta}\right)^2$$

Applying the conditions at $\theta = 0$,

$$s = C_0 = 0$$

$$\frac{ds}{d\theta} = \frac{C_1}{\beta} \Rightarrow C_1 = 0$$

$$\frac{d^2s}{d\theta^2} = \frac{2C_2}{\beta^2} = 0 \Rightarrow C_2 = 0$$

Applying the conditions at $\theta = \beta$,

$$s = C_3\left(\frac{\beta}{\beta}\right)^3 + C_4\left(\frac{\beta}{\beta}\right)^4 = C_3 + C_4 = 1$$

$$\frac{ds}{d\theta} = \frac{3C_3}{\beta}\left(\frac{\beta}{\beta}\right)^2 + \frac{4C_4}{\beta}\left(\frac{\beta}{\beta}\right)^3 = \frac{3C_3}{\beta} + \frac{4C_4}{\beta} = 1$$

or

$$3C_3 + 4C_4 = \beta$$

Solving for the constants,

$$C_3 = 4 - \beta$$

and

$$C_4 = \beta - 3$$

Therefore,

$$s = (4-\beta)\left(\frac{\theta}{\beta}\right)^3 + (\beta - 3)\left(\frac{\theta}{\beta}\right)^4$$

Problem 8.15

A follower moves with simple harmonic motion a distance of 20 mm in 45˚ of cam rotation. The follower then moves 20 mm more with cycloidal motion to complete its rise. The follower then dwells and returns 25 mm with cycloidal motion and then moves the remaining 15 mm with harmonic motion in 45˚. Find the intervals of cam rotation for the cycloidal motions and dwell by matching velocities and accelerations, then determine the equations for the displacement (S) as a function of θ for the entire motion cycle.

Solution:

This is a curve matching problem. To begin the problem, consider the equations for harmonic and cycloidal motions:

Harmonic:

$$y = \frac{L}{2}\left(1 - cos\frac{\pi\theta}{\beta}\right)$$

$$y' = \frac{\pi L}{2\beta}\left(sin\frac{\pi\theta}{\beta}\right)$$

$$y'' = \frac{\pi^2 L}{2\beta^2}\left(cos\frac{\pi\theta}{\beta}\right)$$

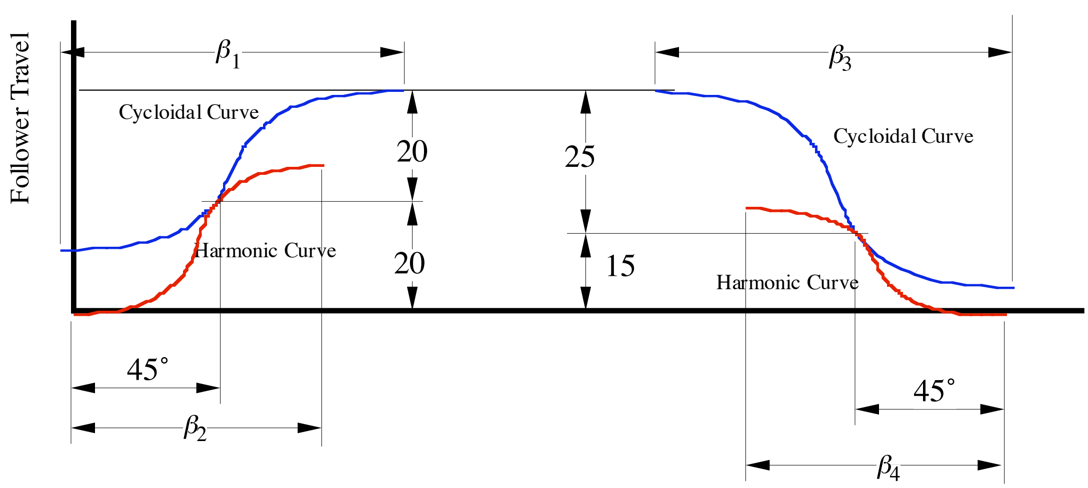
Cycloidal:

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}sin\frac{2\pi\theta}{\beta}\right)$$

$$y' = L\left(\frac{1}{\beta} - \frac{1}{\beta}cos\frac{2\pi\theta}{\beta}\right)$$

$$y'' = L\left(\frac{2\pi}{\beta^2}sin\frac{2\pi\theta}{\beta}\right)$$

For both the harmonic and cycloidal motions, we must determine L and β. There are a total of four curves, so we need to determine four L's and four β's The geometry is shown in the following figure.



We can treat the rise and return separately, and then determine the dwell to ensure that there is a full cycle of motion.

For the rise section, assume that the harmonic curve is half-harmonic, and the cycloidal curve is half cycloidal. This will allow us to match the curves at their inflections points and will ensure curvature continuity. For the harmonic curve,

$$\beta_2 = \frac{\pi}{2}$$

and

$$L_2 = 40$$

Therefore, the harmonic curve is

$$y = \frac{L}{2}\left(1 - cos\frac{\pi\theta}{\beta}\right) = \frac{40}{2}(1 - cos 2\theta) = 20(1 - cos 2\theta)$$

The slope equation is

$$y' = \frac{\pi L}{2\beta}\left(sin\frac{\pi\theta}{\beta}\right) = L(sin 2\theta)$$

Or the cycloidal curve,

$$L_1 = 40$$

and

$$y = L_1\left(\frac{\theta_1}{\beta_1} - \frac{1}{2\pi}sin\frac{2\pi\theta_1}{\beta_1}\right) = 40\left(\frac{\theta_1}{\beta_1} - \frac{1}{2\pi}sin\frac{2\pi\theta_1}{\beta_1}\right)$$

The slope equation is

$$y' = L_1\left(\frac{1}{\beta_1} - \frac{1}{\beta_1}cos\frac{2\pi\theta_1}{\beta_1}\right)$$

To find β_1, equate the slopes at the midpoint, then,

$$y' = L_2\left(sin 2\frac{\beta_2}{2}\right) = L_1\left(\frac{1}{\beta_1} - \frac{1}{\beta_1}cos\frac{2\pi}{\beta_1}\frac{\beta_1}{2}\right)$$

or

$$40(sin\beta_2) = 40\left(\frac{1}{\beta_1} - \frac{1}{\beta_1}cos\pi\right)$$

or

$$\left(sin\frac{\pi}{2}\right) = \left(\frac{1}{\beta_1} - \frac{1}{\beta_1}cos\pi\right)$$

Then,

$$\frac{2}{\beta_1} = 1$$

or

$$\beta_1 = 2$$

The rise part of the curve is given by,

For $\theta < \frac{\pi}{4}$

$$y = 20(1 - cos 2\theta)$$

The cycloidal curve starts at

$$\theta = (\beta_2 - \beta_1)/2 = \left(\frac{\pi}{2} - 2\right)/2 = -0.2146$$

Therefore,

$$\theta_1 = \theta + 0.2146$$

Then for $\frac{\pi}{4} < \theta < \left(\frac{\pi}{4} + \frac{\beta_1}{2}\right)$

$$y = 40\left(\frac{\theta_1}{\beta_1} - \frac{1}{2\pi} sin \frac{2\pi\theta_1}{\beta_1}\right) = 40\left(\frac{[\theta + 0.2146]}{2} - \frac{1}{2\pi} sin \pi[\theta + 0.2146]\right)$$

The angular distance to the dwell is

$$\theta = \left(\frac{\pi}{4} + \frac{\beta_1}{2}\right) = \left(\frac{\pi}{4} + \frac{2}{2}\right) = 1 + \frac{\pi}{4}$$

For the return, use the same procedure. The harmonic part of the return is given by

$$y_4 = \frac{L_4}{2}\left(1 + cos \frac{\pi\theta_4}{\beta_4}\right)$$

Where

$$L_4 = 2(15) = 30$$

and

$$\beta_4 = \frac{\pi}{2}$$

Therefore,

$$y_4 = 15(1 + cos 2\theta_4)$$

For the cycloidal curve,

$$y_3 = L_3 - L_3\left(\frac{\theta_3}{\beta_3} - \frac{1}{2\pi} sin \frac{2\pi\theta_3}{\beta_3}\right)$$

Where

$$L_3 = 2(25) = 50$$

To determine β_3, equate the slopes for the two curves at their midpoints. Then,

$$y'_4 = -\frac{\pi L_4}{2\beta_4}\left(sin \frac{\pi}{\beta_4} \frac{\beta_4}{2}\right) = y'_3 = -L_3\left(\frac{1}{\beta_3} - \frac{1}{\beta_3} cos \frac{2\pi}{\beta_3} \frac{\beta_3}{2}\right)$$

or

$$30\left(sin \frac{\pi}{2}\right) = 50\left(\frac{1}{\beta_3} - \frac{1}{\beta_3} cos \pi\right)$$

Solving for β_3,

$$\frac{2}{\beta_3} = \frac{3}{5}$$

Or

$$\beta_3 = \frac{10}{3}$$

The cycloidal part of the curve will start at

$$\theta = 2\pi - (\beta_3 + \beta_4)/2$$

Therefore, the dwell period will be

$$\left(\frac{\pi}{4} + \frac{\beta_1}{2}\right) < \theta < 2\pi - (\beta_3 + \beta_4)/2$$

And the dwell distance will b,

$$y = 40$$

The cycloidal part of the curve occurs when

$$2\pi - (\beta_3 + \beta_4)/2 < \theta < 2\pi - \beta_3/2$$

And

$$y_3 = L_3 - L_3\left(\frac{\theta_3}{\beta_3} - \frac{1}{2\pi} sin \frac{2\pi\theta_3}{\beta_3}\right)$$

The curve will be shifted because L_3 is 50. Therefore,

$$y = y_3 - 10 = L_3 - L_3\left(\frac{\theta_3}{\beta_3} - \frac{1}{2\pi} sin \frac{2\pi\theta_3}{\beta_3}\right) - 10$$

The second harmonic part of the curve occurs when

$$2\pi - \beta_3/2 < \theta < 2\pi$$

And

$$y_4 = \frac{L_4}{2}\left(1 + cos \frac{\pi\theta}{\beta_4}\right)$$

The displacement diagram is shown in the following: