Numerical methods for planar MBS

Coordinates and constraints

Kinematic analysis of multibody systems

	Pros	Cons
Graph...		
Analy...		
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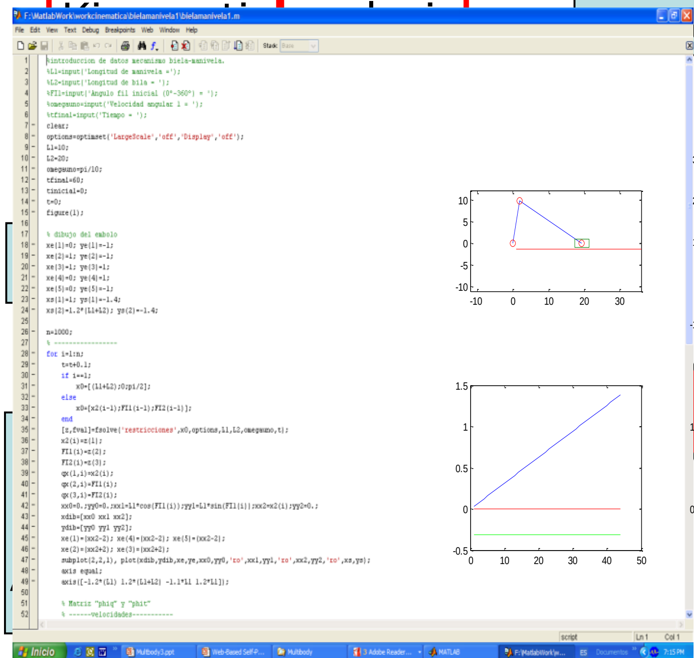
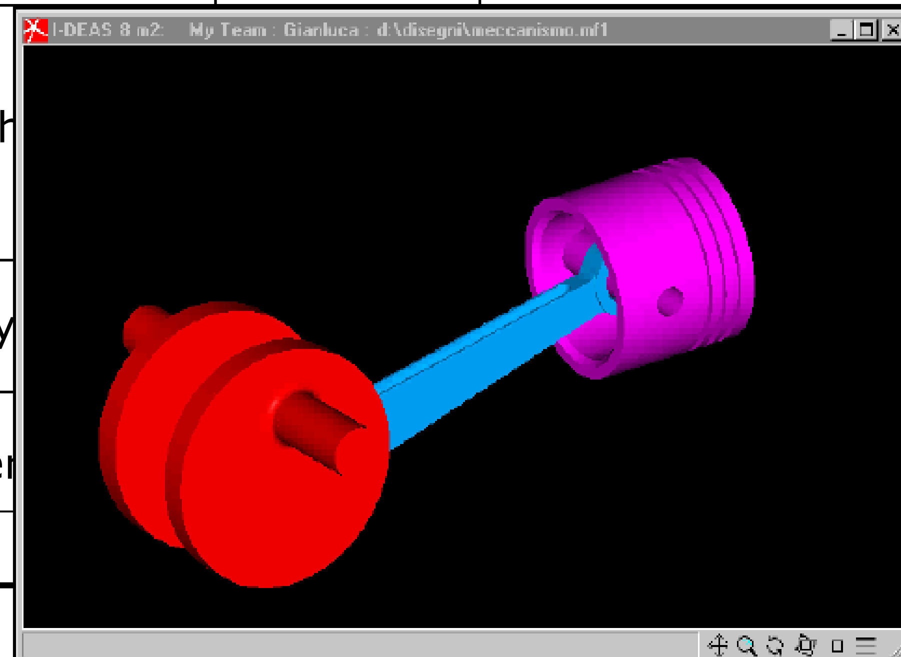
...ods





Dynamics

Loads

Synthesis

Approach

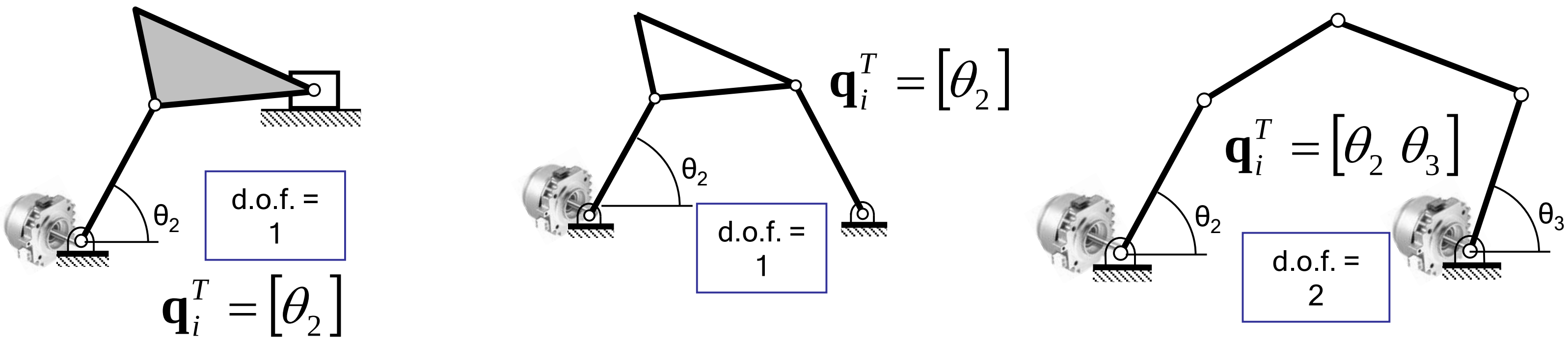
- User

- Position
- Velocity and acceleration

Independent coordinates

In order to describe a MBS a set of coordinates must be chosen.

These parameters must describe univocally the position, velocity and acceleration of the MBS at all times.

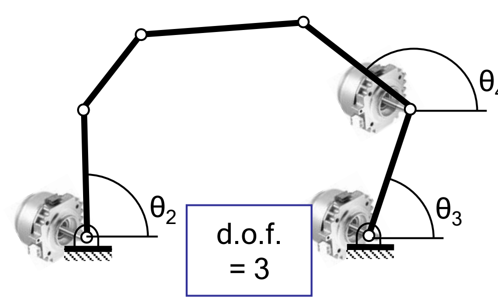


Minimum number of parameters is equal to no. d.o.f. → Independent coor. However, these independent coordinates do not always define univocally the MBS

They are dependent directly on time.

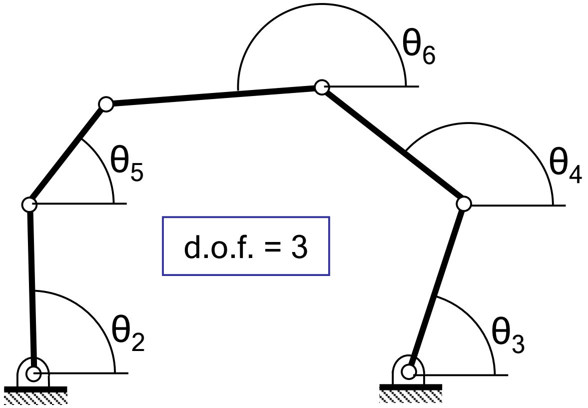
$$\left.\begin{aligned} \theta_1 &= \theta_1(\mathrm{t}) \\ \theta_2 &= \theta_2(\mathrm{t}) \\ \theta_3 &= \theta_3(\mathrm{t}) \end{aligned}\right\}$$

Time constraints



Dependent coordinates: Additional coordinates can be used in order to determine univocally the movement of the MBS.

$$\left.\begin{array}{l}\mathbf{q}_i^T = [\theta_2\ \theta_3\ \theta_4] \\ \mathbf{q}_d^T = [\theta_5\ \theta_6]\end{array}\right\} \quad \mathbf{q}^T = [\mathbf{q}_i^T\ \mathbf{q}_d^T]$$



These ones are related with independent coordinates by means of kinematic constraints. These constraints are usually non linear and their number is equal to the number of dependent coordinates

$$\left.\begin{array}{l}\mathbf{q}_i^T = [\theta_2\ \theta_3\ \theta_4] \\ \mathbf{q}_d^T = [\theta_5\ \theta_6]\end{array}\right\} \rightarrow \begin{Bmatrix}\Phi_1(q_i, q_d) \\ \Phi_2(q_i, q_d)\end{Bmatrix} = \mathbf{\Phi}(q) = 0$$

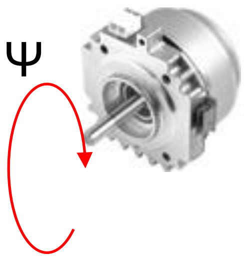
Kinematic constraints

Generalized coordinates

Independent + dependent coordinates = Generalized coordinates

The type of coordinates used for modeling MBS can be:

1. Relative coordinates.
2. Reference point coordinates.
3. Natural coordinates.
4. Combination.



The same MBS can be described with different types of coordinates, their definition will determine the efficiency and simplicity of the problem.

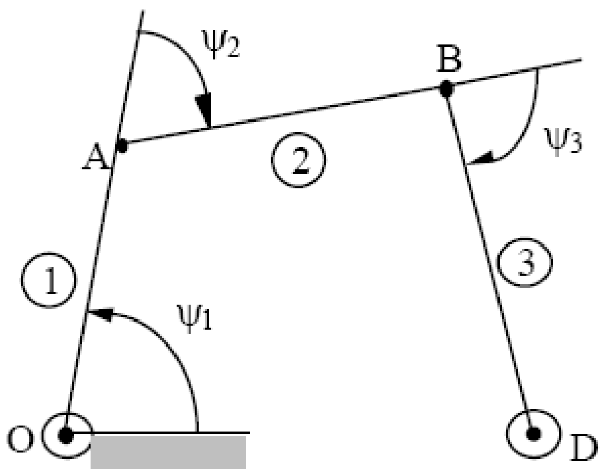
The inputs of a MBS are usually defined as coordinates

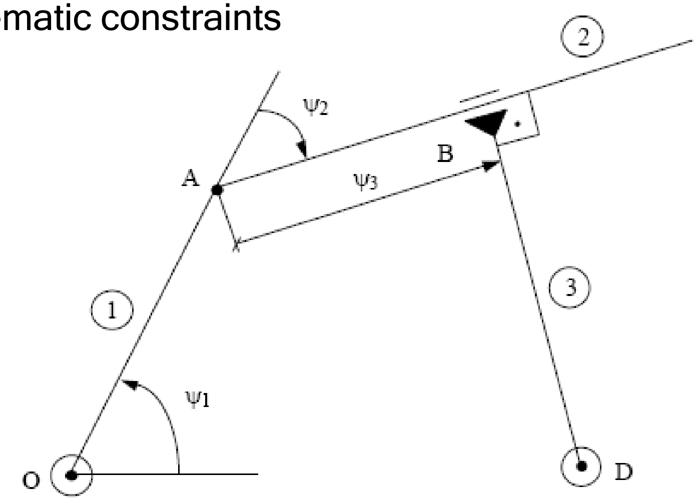
Exampl: Motor -> angle.
Actuator -> displacement.

Relative coordinates

Relative coordinates define the position of each element in relation to the previous element in the kinematic chain by using the parameters corresponding to the relative degrees of freedom allowed by the joint linking these elements

1 d.o.f.
2 dependent coord. = 2 kinematic constraints





$q_i^T=[\psi_1]$
$\mathbf{q}_d^T=[\psi_2\ \psi_3]$

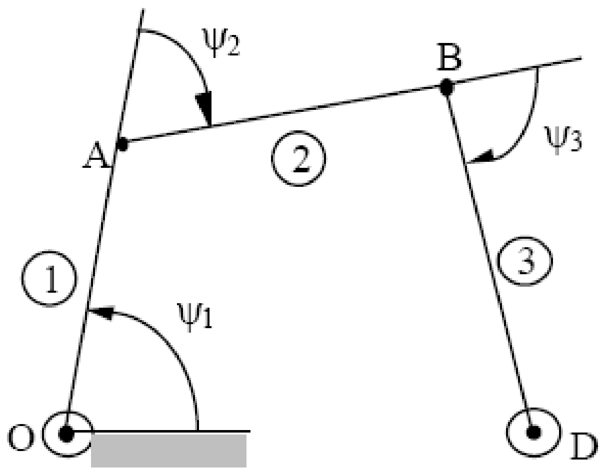
kinematic constraints arise from the condition of the vector closure of the kinematic loop

$q_i^T=[\psi_1]$
$\mathbf{q}_d^T=[\psi_2\ \psi_3]$

$$\mathbf{\Phi}(q)=\begin{Bmatrix} L_1 \cos \Psi_1 + L_2 \cos(\Psi_1+\Psi_2) + L_3 \cos(\Psi_1+\Psi_2+\Psi_3) - OD = 0 \\ L_1 \sin \Psi_1 + L_2 \sin(\Psi_1+\Psi_2) + L_3 \sin(\Psi_1+\Psi_2+\Psi_3) = 0 \end{Bmatrix}$$

$$\mathbf{\Phi}(q)=\begin{Bmatrix} L_1 \cos \Psi_1 + \Psi_3 \cos(\Psi_1+\Psi_2) + L_3 \cos(\Psi_1+\Psi_2-\pi/2) - OD = 0 \\ L_1 \sin \Psi_1 + \Psi_3 \sin(\Psi_1+\Psi_2) + L_3 \sin(\Psi_1+\Psi_2-\pi/2) = 0 \end{Bmatrix}$$

Independent + dependent coordinates = Generalized coordinates



1 d.o.f. = 1 Independent coor. = time constraint

$$\boldsymbol{\Phi}_1(\mathrm{t}) = \psi_1 - \omega\,\mathrm{t} = 0$$

2 dependent coor. = kinematic constraints

$$\boldsymbol{\Phi}_2(\mathrm{q}) = \begin{Bmatrix} L_1 \cos \Psi_1 + L_2 \cos(\Psi_1 + \Psi_2) + L_3 \cos(\Psi_1 + \Psi_2 + \Psi_3) - OD = 0 \\ L_1 \sin \Psi_1 + L_2 \sin(\Psi_1 + \Psi_2) + L_3 \sin(\Psi_1 + \Psi_2 + \Psi_3) = 0 \end{Bmatrix}$$

3 equations, 3 unknowns: Ψ_1, Ψ_2, Ψ_3.

$$\begin{Bmatrix} \boldsymbol{\Phi}_1(t) \\ \boldsymbol{\Phi}_2(q) \end{Bmatrix} = \boldsymbol{\Phi}(q, t) = 0$$

Numerical methods for planar MBS

Kinematics

Time = 0

The position of MBS is determined by solving a non-linear system of equations, namely, constraints, which can be expressed in a compact way as

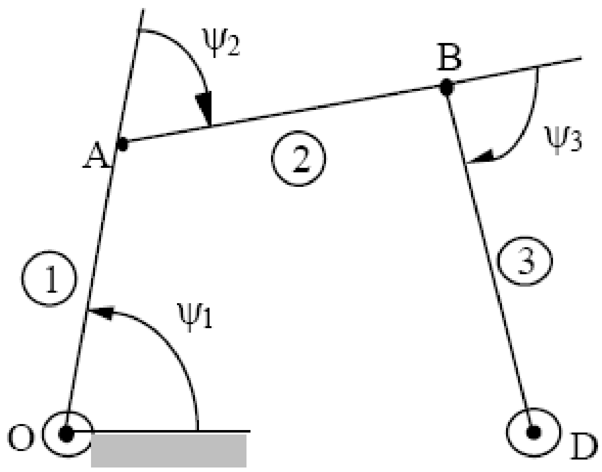
$$\mathbf{\Phi}\left(\mathbf{q}(t), t\right) = \mathbf{0}$$

Where *t* stands for time and **q** is the vector of generalized coordinates as seen before

$$\mathbf{q}^{\mathrm{T}} = \left[q_1 \ q_2 \ \ldots q_n\right]$$

Generalized coordinates are the unknowns of this problem. So, position of MBS is given by vector **q** when constraints equations are solved. However, solution of constraints can be tricky.

Time = 0



1 d.o.f. = 1 Independent coor. = time constraint

$$\mathbf{\Phi}_1(\mathrm{t}) = \psi_1 - \omega\,\mathrm{t} = 0$$

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$$\mathbf{\Phi}_2(\mathrm{q}) = \begin{Bmatrix} L_1 \cos\Psi_1 + L_2 \cos(\Psi_1 + \Psi_2) + L_3 \cos(\Psi_1 + \Psi_2 + \Psi_3) - OD = 0 \\ L_1 \sin\Psi_1 + L_2 \sin(\Psi_1 + \Psi_2) + L_3 \sin(\Psi_1 + \Psi_2 + \Psi_3) = 0 \end{Bmatrix}$$

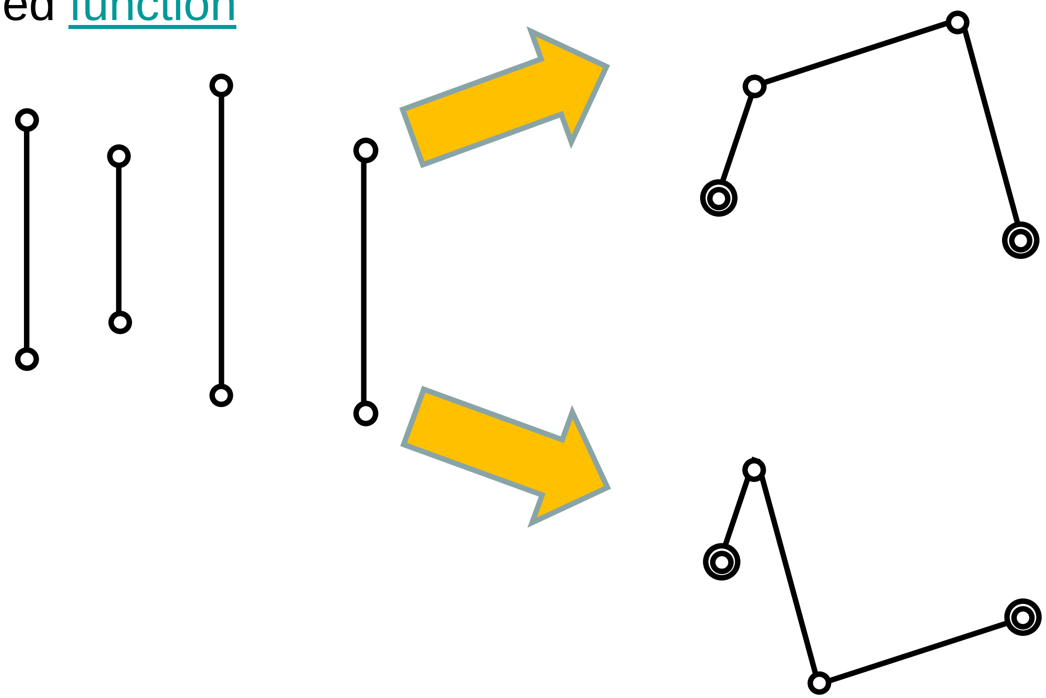
3 equations, 3 unknowns: Ψ_1, Ψ_2, Ψ_3.

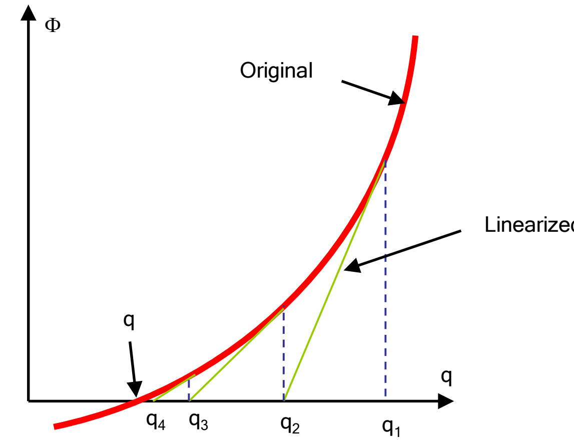
$$\begin{Bmatrix} \mathbf{\Phi}_1(t) \\ \mathbf{\Phi}_2(q) \end{Bmatrix} = \mathbf{\Phi}(q,t) = 0$$

Assembly problem

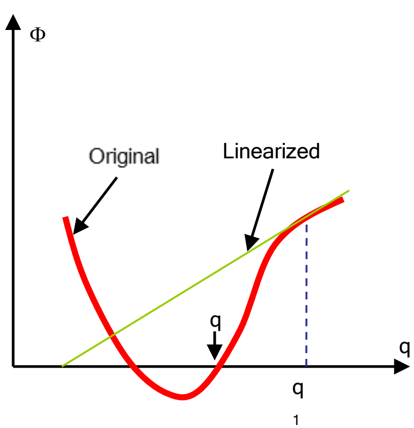
Newton-Raphson method is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function

Time = 0





The shape f the function defines the complexity of the solution process



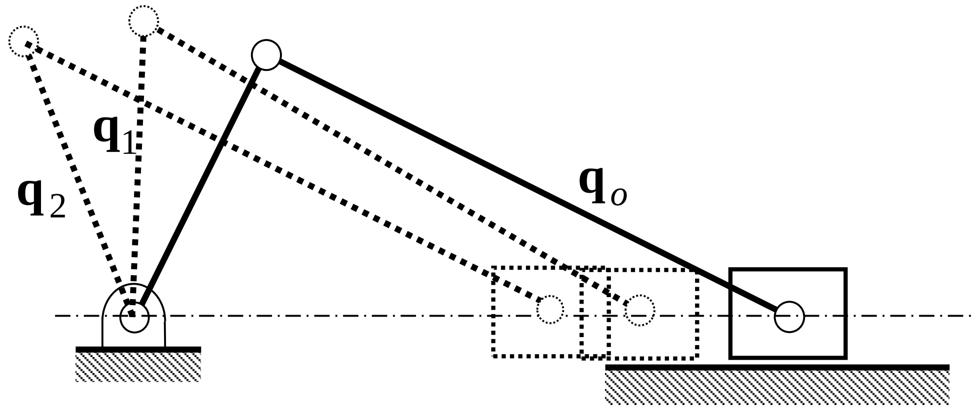
<u>Desired assembly is dependent on initial guess</u>

Next position problem

Time = $t_1, t_2, \ldots, t_n$

For the next position computation, the previous position can be used in order to use a valid initial guess.

As long as the increment of position between to consecutive instants is small enough, the previous position will be the best initial guess



Generalized velocities

Position problem $\mathbf{\Phi}(\mathbf{q}(t),t)=\mathbf{0}$ Solution is **q**

In order to obtain the equations for generalized velocities, the position equations must be differentiated following the chain rule

$$\dot{\mathbf{\Phi}}=\mathbf{\Phi}_{\mathrm{q}}\dot{\mathbf{q}}+\mathbf{\Phi}_{\mathrm{t}}=\mathbf{0}$$

Where,

$$\mathbf{\Phi}_q=\begin{bmatrix}\frac{\partial\phi_1}{\partial q_1} & \frac{\partial\phi_1}{\partial q_2} & \cdots & \frac{\partial\phi_1}{\partial q_n}\\ \frac{\partial\phi_2}{\partial q_1} & \frac{\partial\phi_2}{\partial q_2} & \cdots & \frac{\partial\phi_2}{\partial q_n}\\ \cdots & \cdots & \cdots & \cdots\\ \frac{\partial\phi_n}{\partial q_1} & \frac{\partial\phi_n}{\partial q_2} & \cdots & \frac{\partial\phi_n}{\partial q_n}\end{bmatrix}\qquad \mathbf{\Phi}_t=\begin{bmatrix}\frac{\partial\phi_1}{\partial t}\\ \frac{\partial\phi_2}{\partial t}\\ \cdots\\ \frac{\partial\phi_n}{\partial t}\end{bmatrix}$$

Jacobian matrix

Solution is

$$\dot{\mathbf{q}}=-\mathbf{\Phi}_{\mathrm{q}}^{-1}\mathbf{\Phi}_{\mathrm{t}}$$

Generalized accelerations

Position $\mathbf{\Phi}(\mathbf{q}(t),t)=\mathbf{0}$

Velocity (1st diff.) $\dot{\mathbf{\Phi}}=\mathbf{\Phi}_{\mathrm{q}}\dot{\mathbf{q}}+\mathbf{\Phi}_{\mathrm{t}}=\mathbf{0}$

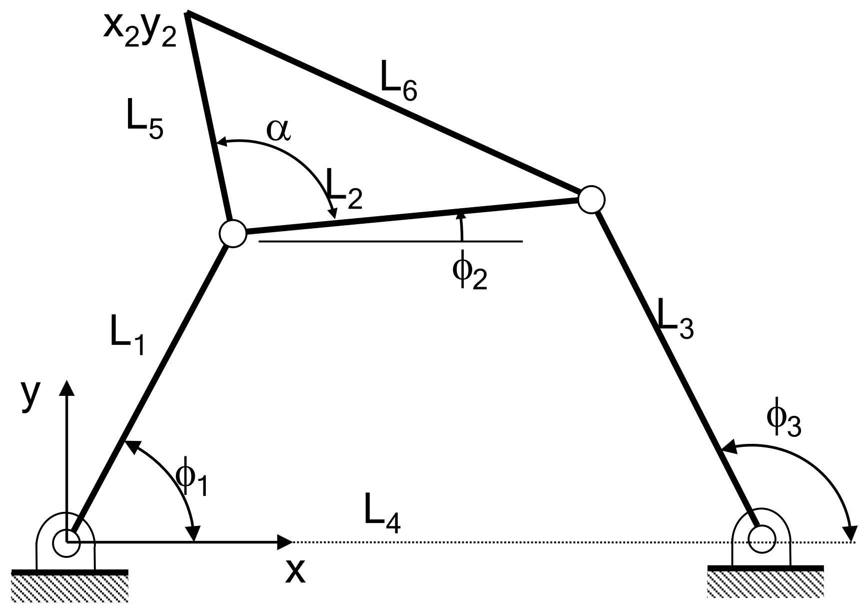
Acceleration (2nd diff.) $\frac{d}{dt}(\dot{\mathbf{\Phi}})=\frac{d}{dt}(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}+\mathbf{\Phi}_{t})=\mathbf{0}$

$$(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}+\mathbf{\Phi}_{t})_{\mathbf{q}}\dot{\mathbf{q}}+\frac{\partial}{\partial t}(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}+\mathbf{\Phi}_{t})=\mathbf{0}$$

$$(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}})_{q}\dot{\mathbf{q}}+\mathbf{\Phi}_{t\mathbf{q}}\dot{\mathbf{q}}+\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}}+\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}}+\mathbf{\Phi}_{tt}=\mathbf{0}$$

$$\mathbf{\Phi}_{\mathrm{q}}\ddot{\mathbf{q}}=-(\mathbf{\Phi}_{\mathrm{q}}\dot{\mathbf{q}})_{\mathrm{q}}\dot{\mathbf{q}}-2\mathbf{\Phi}_{\mathrm{qt}}\dot{\mathbf{q}}-\mathbf{\Phi}_{\mathrm{tt}}=\gamma$$

Example: Position



Generalized coordinates

$$\mathbf{q}^T = [x_2 \; y_2 \; \phi_1 \; \phi_2 \; \phi_3]$$

Independent coordinate is dependent of time

$$\phi_1 = \omega t$$

Constraint equations (time and kinematics) are:

$$\Phi(\mathbf{q}) = \begin{Bmatrix} x_2 - L_1 \cos\phi_1 - L_5 \cos(\alpha + \phi_2) \\ y_2 - L_1 \text{sen}\phi_1 - L_5 \text{sen}(\alpha + \phi_2) \\ -L_1 \cos\phi_1 - L_2 \cos\phi_2 + L_3 \cos\phi_3 + L_4 \\ L_1 \text{sen}\phi_1 + L_2 \text{sen}\phi_2 - L_3 \text{sen}\phi_3 \\ \phi_1 - \omega t \end{Bmatrix} = 0$$

Position x2 and y2

Closure equation

Time constraint

Example: Velocity

Jacobian matrix

$$\Phi_{\mathbf{q}} = \begin{bmatrix} 1 & 0 & L_1 \mathrm{sen}\phi_1 & L_5 \mathrm{sen}(\alpha+\phi_2) & 0 \\ 0 & 1 & -L_1\cos\phi_1 & -L_5\cos(\alpha+\phi_2) & 0 \\ 0 & 0 & L_1 \mathrm{sen}\phi_1 & L_2 \mathrm{sen}\phi_2 & -L_3 \mathrm{sen}\phi_3 \\ 0 & 0 & L_1\cos\phi_1 & L_2\cos\phi_2 & -L_3\cos\phi_3 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Constraints derived with respect to time

$$\Phi_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\omega \end{bmatrix}$$

Solving velocities equation $\dot{\mathrm{q}} = -\Phi_{\mathrm{q}}^{-1}\Phi_{\mathrm{t}}$

Leads to generalized velocities $\dot{q}^T = [\dot{x}_2,\ \dot{y}_2, \dot{\phi}_1, \dot{\phi}_2,\ \dot{\phi}_3]$

$$\boldsymbol{\Phi}_q\,\dot{q} = \begin{bmatrix} 1 & 0 & L_1 \mathrm{sen}\phi_1 & L_5 \mathrm{sen}(\alpha+\phi_2) & 0 \\ 0 & 1 & -L_1\cos\phi_1 & -L_5\cos(\alpha+\phi_2) & 0 \\ 0 & 0 & L_1 \mathrm{sen}\phi_1 & L_2 \mathrm{sen}\phi_2 & -L_3 \mathrm{sen}\phi_3 \\ 0 & 0 & L_1\cos\phi_1 & L_2\cos\phi_2 & -L_3\cos\phi_3 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{Bmatrix} = \begin{Bmatrix} \dot{x}_2 + \dot{\phi}_1 L_1 \mathrm{sen}\phi_1 + \dot{\phi}_2 L_5 \mathrm{sen}(\alpha+\phi_2) \\ \dot{y}_2 - \dot{\phi}_1 L_1\cos\phi_1 - \dot{\phi}_2 L_5\cos(\alpha+\phi_2) \\ \dot{\phi}_1 L_1 \mathrm{sen}\phi_1 + \dot{\phi}_2 L_2 \mathrm{sen}\phi_2 - \dot{\phi}_3 L_3 \mathrm{sen}\phi_3 \\ \dot{\phi}_1 L_1\cos\phi_1 + \dot{\phi}_2 L_2\cos\phi_2 - \dot{\phi}_3 L_3\cos\phi_3 \\ \dot{\phi}_1 \end{Bmatrix}$$

$$\gamma = -(\Phi_{\mathrm{q}}\dot{\mathrm{q}})_{\mathrm{q}}\dot{\mathrm{q}} - 2\Phi_{\mathrm{qt}}\dot{\mathrm{q}} - \Phi_{\mathrm{tt}}$$

$$\Phi_{\mathrm{tt}} = 0$$

$$\Phi_{\mathrm{qt}} = 0$$

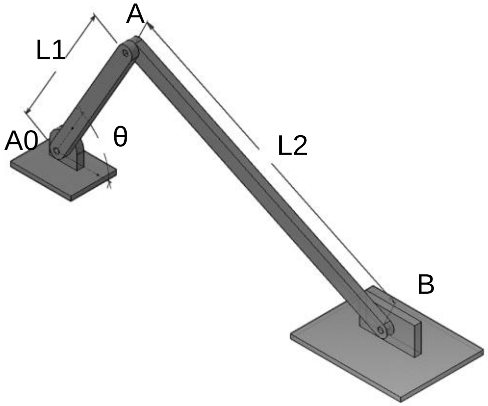
$$(\boldsymbol{\Phi}_{\mathbf{q}}\dot{\mathbf{q}})_{\mathbf{q}}\dot{\mathbf{q}} = \begin{bmatrix} 0 & 0 & \dot{\phi}_1 L_1\cos\phi_1 & \dot{\phi}_2 L_5\cos(\alpha+\phi_2) & 0 \\ 0 & 0 & \dot{\phi}_1 L_1 sen\phi_1 & \dot{\phi}_2 L_5 sen(\alpha+\phi_2) & 0 \\ 0 & 0 & \dot{\phi}_1 L_1\cos\phi_1 & \dot{\phi}_2 L_2\cos\phi_2 & -\dot{\phi}_3 L_3\cos\phi_3 \\ 0 & 0 & -\dot{\phi}_1 L_1 sen\phi_1 & -\dot{\phi}_2 L_2 sen\phi_2 & \dot{\phi}_3 L_3 sen\phi_3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{Bmatrix} = \begin{Bmatrix} \dot{\phi}_1^2 L_1\cos\phi_1 + \dot{\phi}_2^2 L_5\cos(\alpha+\phi_2) \\ \dot{\phi}_1^2 L_1 sen\phi_1 + \dot{\phi}_2^2 L_5 sen(\alpha+\phi_2) \\ \dot{\phi}_1^2 L_1\cos\phi_1 + \dot{\phi}_2^2 L_2\cos\phi_2 - \dot{\phi}_3^2 L_3\cos\phi_3 \\ -\dot{\phi}_1^2 L_1 sen\phi_1 - \dot{\phi}_2^2 L_2 sen\phi_2 + \dot{\phi}_3^2 L_3 sen\phi_3 \\ 0 \end{Bmatrix}$$

Numerical methods for planar MBS

Planar Dynamics

Types of Dynamic problems

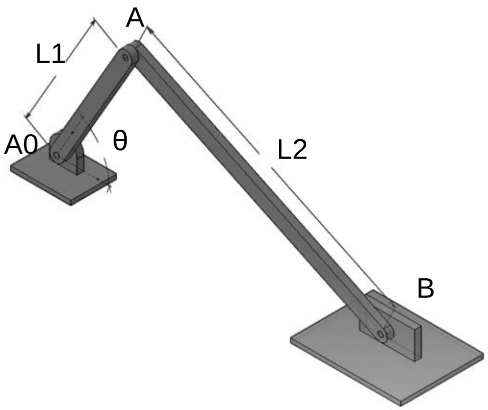
- Direct dynamic problem: Consists on determining the system's motion during a period of time by known external forces and/or kinematically driven d.o.f.



T, F

$\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$

- Inverse dynamic problem: Consist on determining the external forces that produce a specific movement.



T, F

$\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$

Dynamic Formalisms

Newton's equations

$$\begin{cases} \sum \mathbf{F} = m\mathbf{a} \\ \sum \mathbf{M}_G = \mathbf{I}_G \dfrac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{dt}} + \boldsymbol{\omega} \times \mathbf{I}_G \times \boldsymbol{\omega} \end{cases}$$

⇨ 6 x No. of bodies

Lagrange's equations (from Principle of Virtual Displacements) for both dependent and independent coordinates

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial T}{\partial \mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{t}} \boldsymbol{\lambda} = \mathbf{Q}$$

Where T is the kinetic energy $T = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{t}}\mathbf{M}\dot{\mathbf{q}}$

λ are the Lagrange's multipliers. They represent the effort needed in order to fulfill the kinematic constraints

Q are the generalized forces

This leads to $$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{t}\boldsymbol{\lambda} = \mathbf{Q}$$

This system has *n* equations and *n+m* unknowns (n DOFs and m is the dimension of $\boldsymbol{\lambda}$). For this reason, *m* more equations are needed (i.e the constraint equation)

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{t}\boldsymbol{\lambda} &= \mathbf{Q} \\ \mathbf{\Phi} &= \mathbf{0} \end{aligned}$$

DAE system

Differential Algebraic Equation involves an unknown function and its derivatives:

Solving the equations of motion (I): Stabilization

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{t}\boldsymbol{\lambda} = \mathbf{Q}$$
$$\mathbf{\Phi} = \mathbf{0}$$

Simpler solution than an DAE. However, this method is non stable because it is imposed on the second derivative

Baumgarte stabilization which leads to an equivalent system

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{t}\boldsymbol{\lambda} = \mathbf{Q}$$
$$\ddot{\mathbf{\Phi}} + 2\xi\omega\dot{\mathbf{\Phi}} + \omega^{2}\mathbf{\Phi} = \mathbf{0}$$

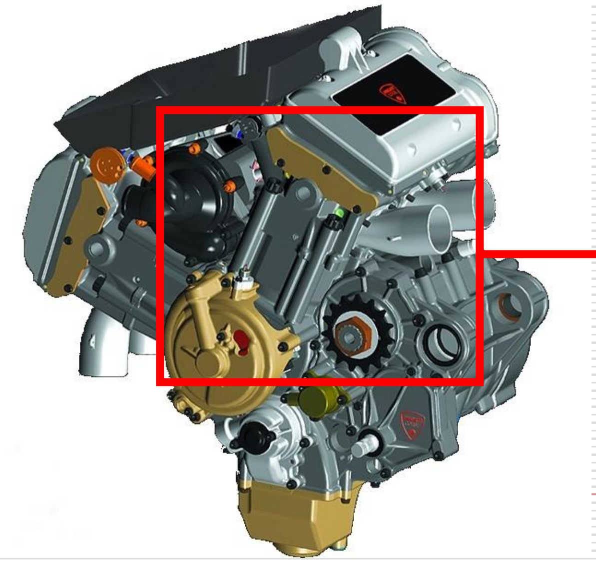
$$\xi = 1 \quad ; \quad \omega = 10$$

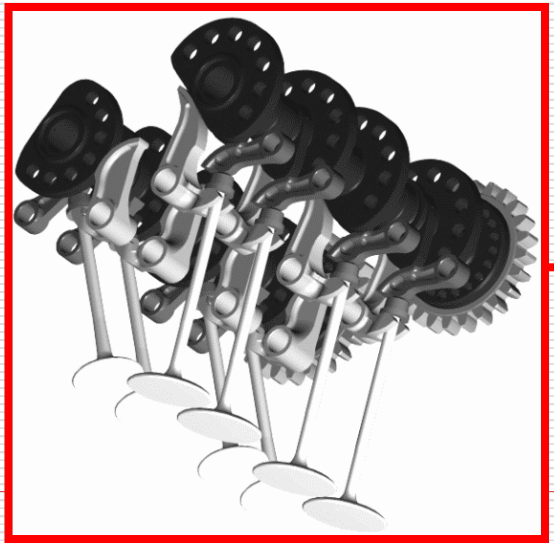
Values for mechanical systems

Other methods: Penalty method, Augmented Method

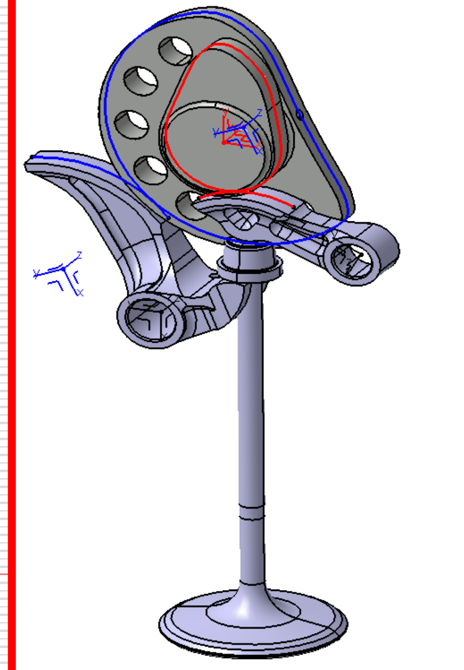
OGGETTO IN STUDIO:

- L'oggetto in studio è uno degli equipaggi della testata GP6
- Albero di aspirazione orizzontale DBGP6 V4
- Profilo camma CAGP005





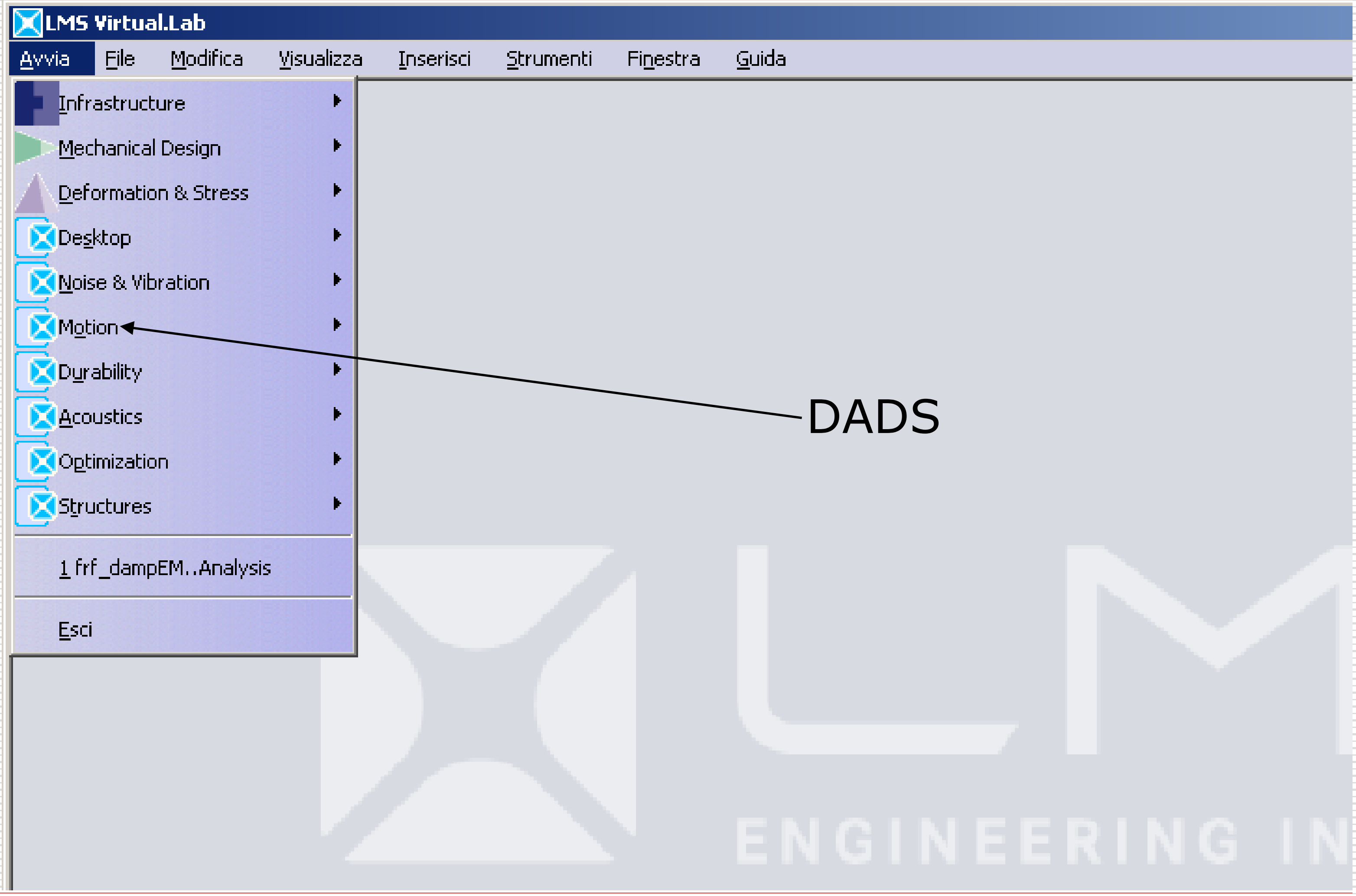
OGGETTO IN STUDIO



Overview

- Presentazione dei software impiegati
- Introduzione al MB
- Sintesi Cinematica e cinematica diretta
- Analisi dinamica a corpi rigidi
- Flessibilità nel MB
- Analisi dinamica a corpi flessibili

LMS Virtual.Lab

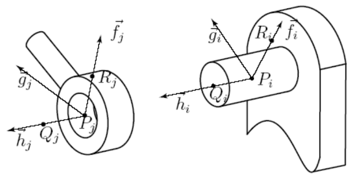


Steps di una analisi MB

- Disegnare i membri al CAD (sono solo oggetti solidi)
- Creare i Body(corpi rigidi) e deciso il membro-telaio
- Applicare i **Joints**
- Applicare un moto imposto- driver(se necessario)
- Condizioni iniziali
- Tipo di soluzione (cinematica, dinamica, ecc), la direzione della gravità, l’intervallo di integrazione
- Computo della soluzione
- Grafici, animazioni

Joints

1.2.2 Coppia Rotoidale



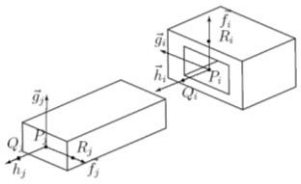


Figura 1.2: Coppia prismatica

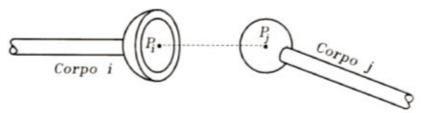


Figura 1.4: Coppia sferica

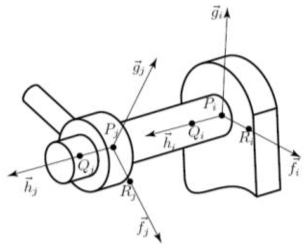


Figura 1.3: Coppia cilindrica

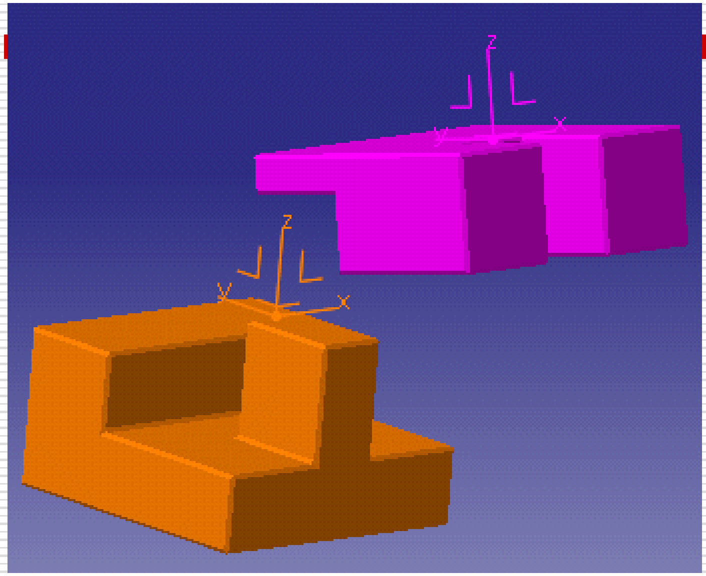
Tabella 1.1: Gradi di libertà delle coppie cinematiche

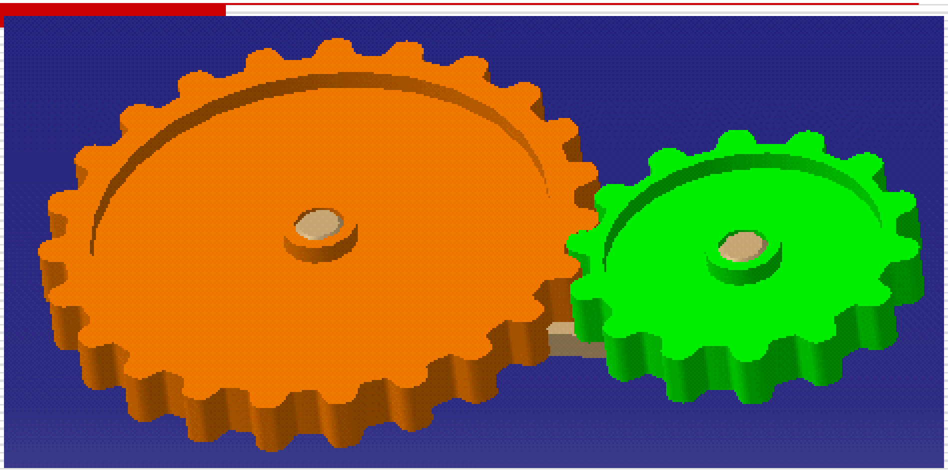
Coppia cinematica	f	Moti relativi inibiti
Rotoidale	1	3 trasl. + 2 rot.
Prismatica	1	2 tasl. + 3 rot.
Cilindrica	2	2 trasl. + 2 rot.
Sferica	3	3 traslazioni
Sfera nel cilindro	4	2 traslazioni
Giunto cardanico	2	3 trasl. + 1 rot.

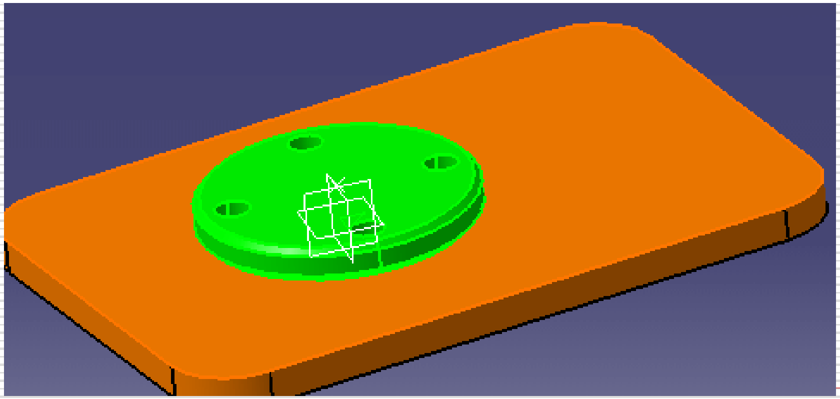
Per ogni coppia è possibile scrivere una eq. di vicolo

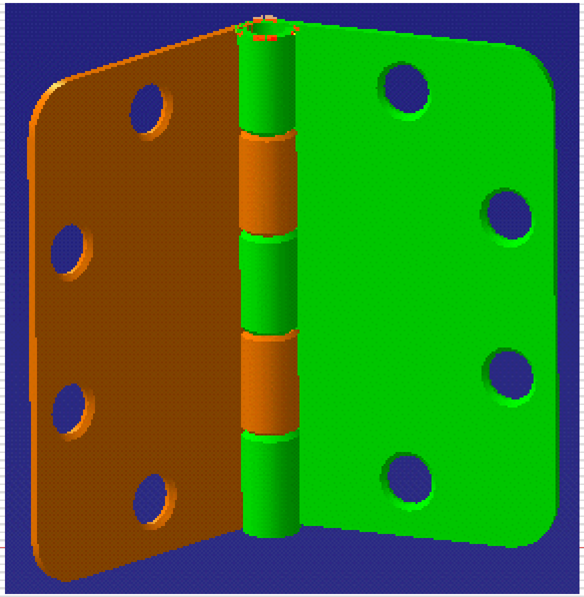
$$\Psi_k = \begin{vmatrix} X_{Ai} & Y_{Ai} & 1 \\ X_{Bi} & Y_{Bi} & 1 \\ X_{Cj} & Y_{Cj} & 1 \end{vmatrix} = 0$$

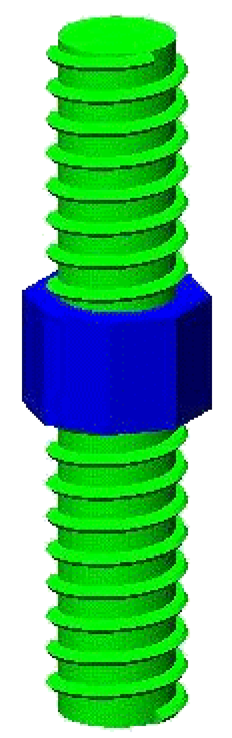
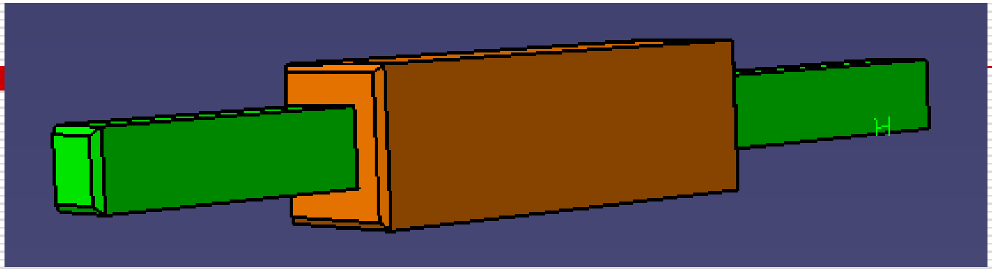
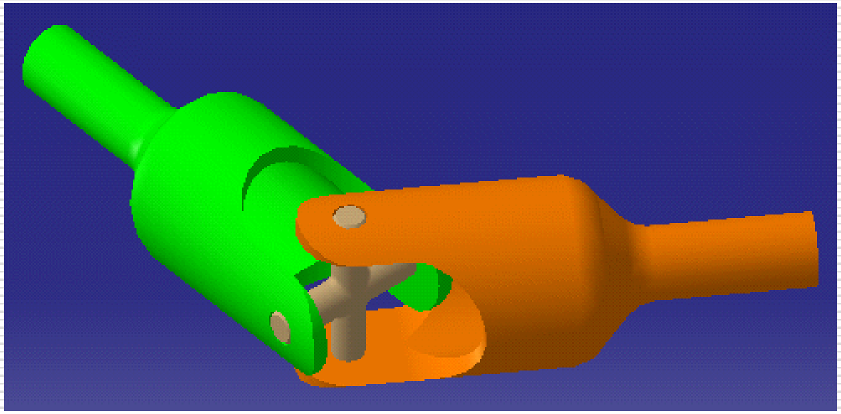
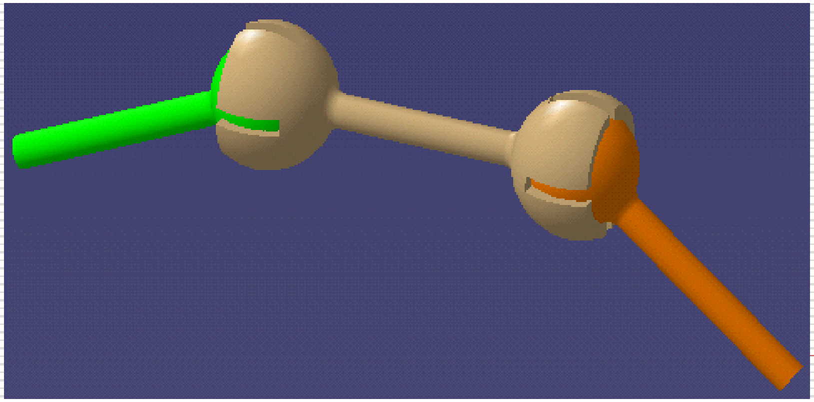
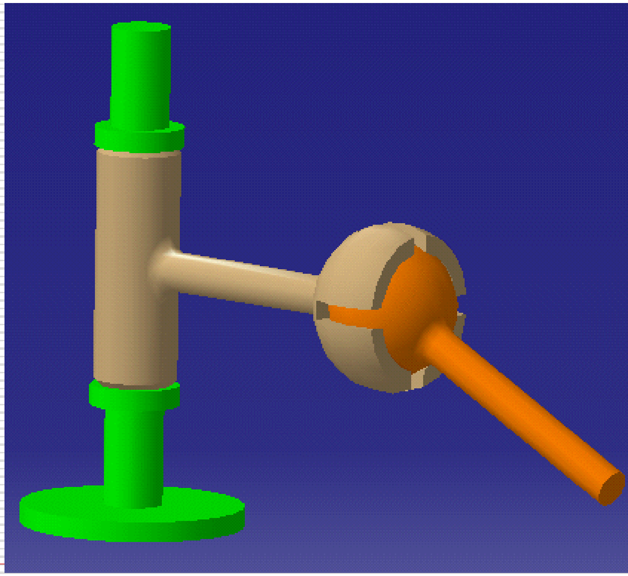
Joints

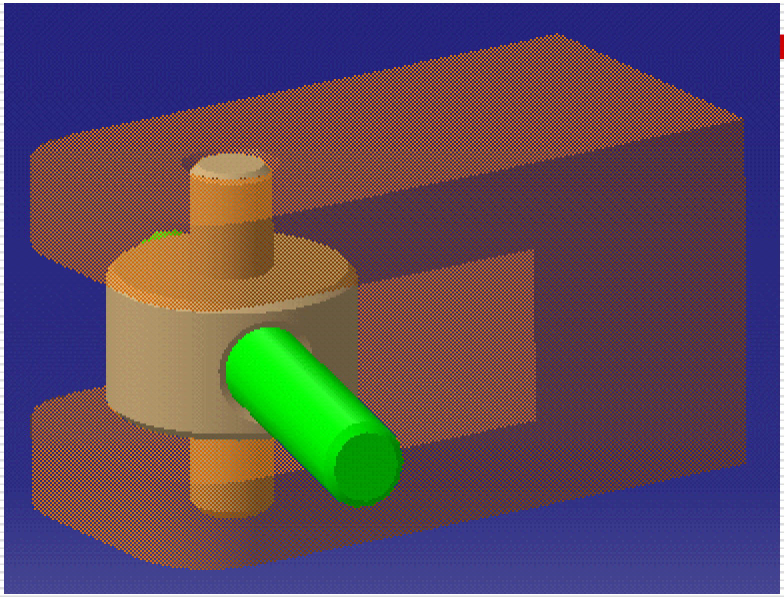
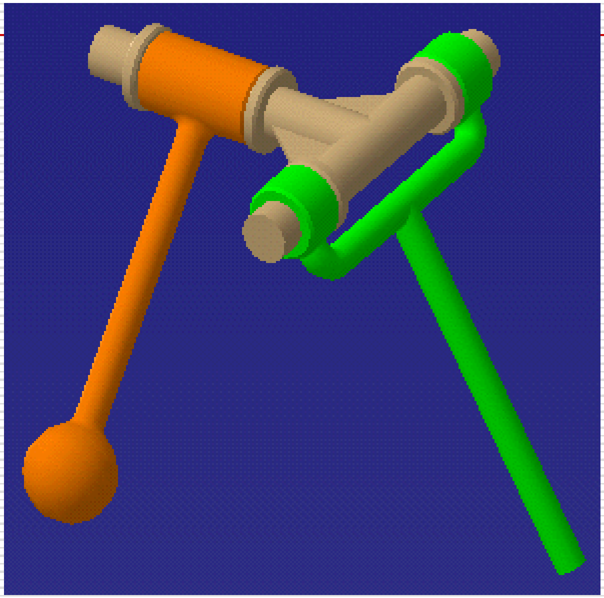
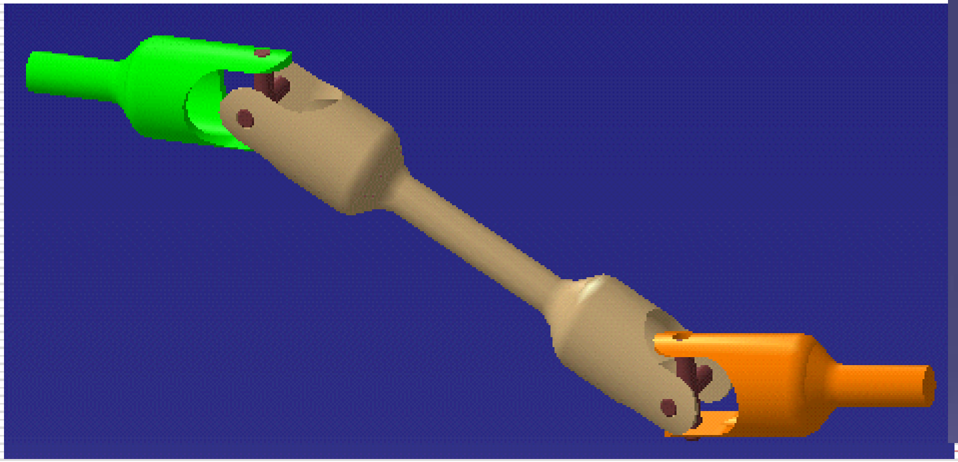
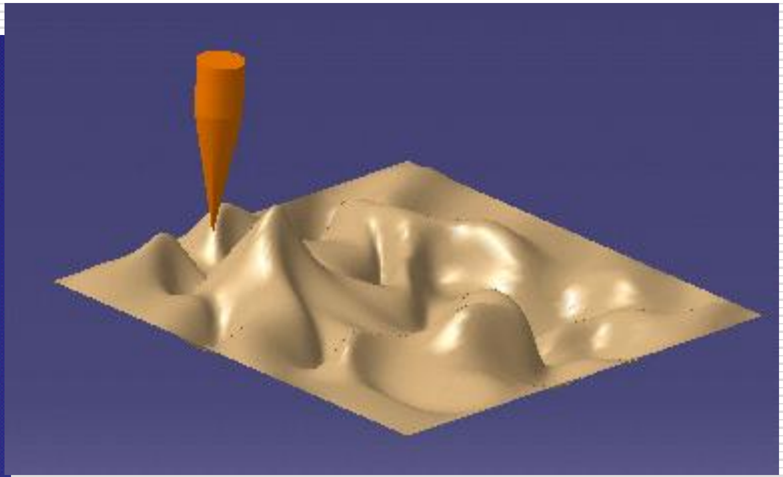


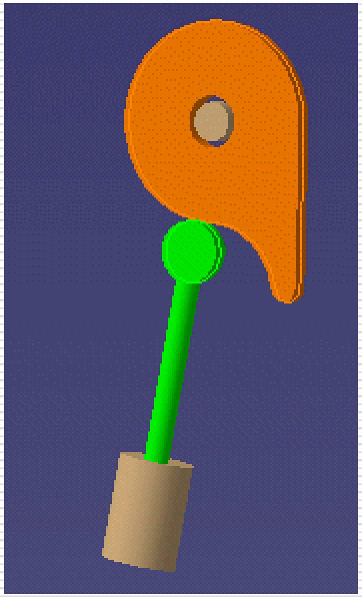


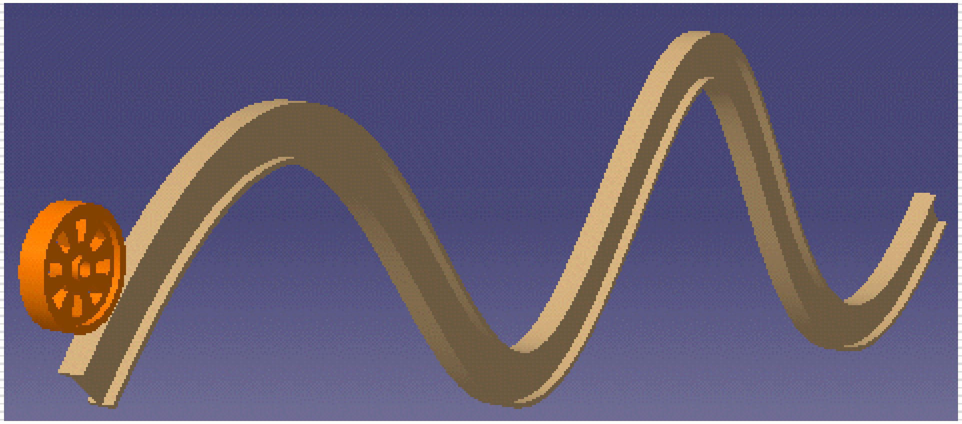




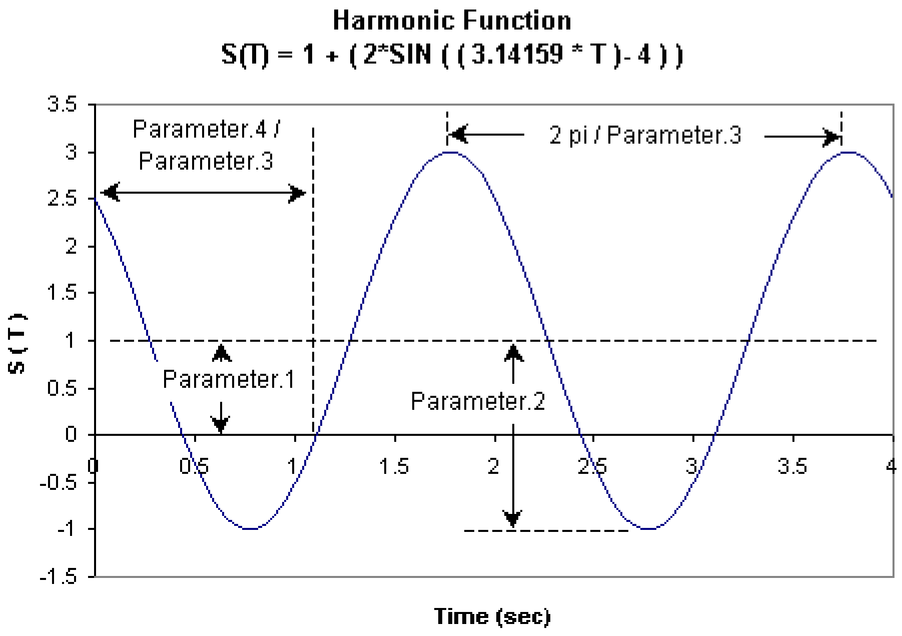






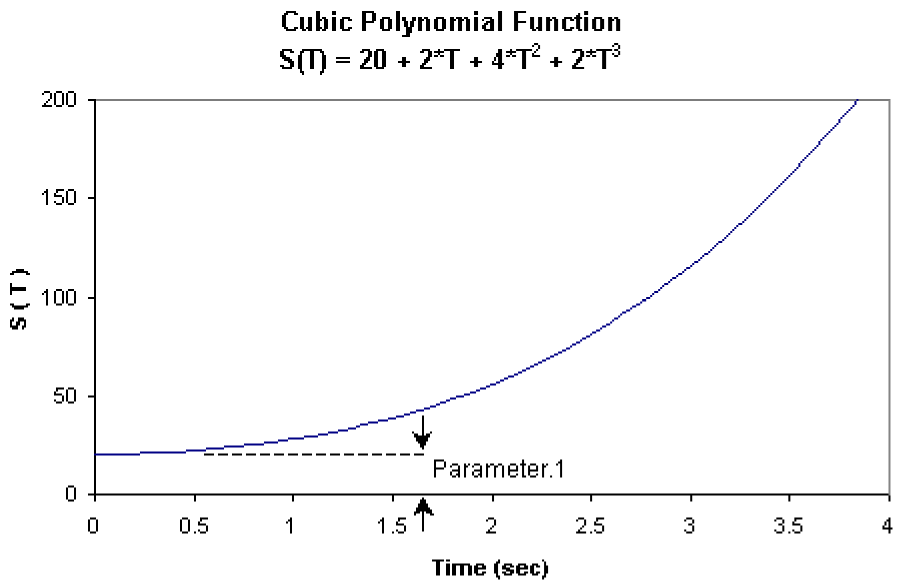









Drivers

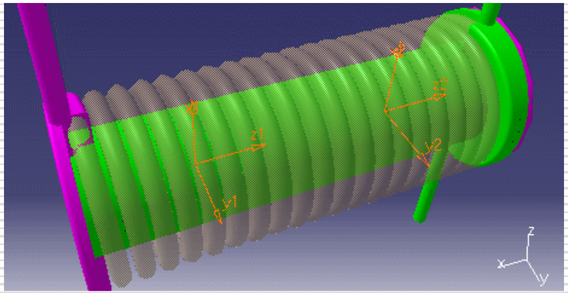




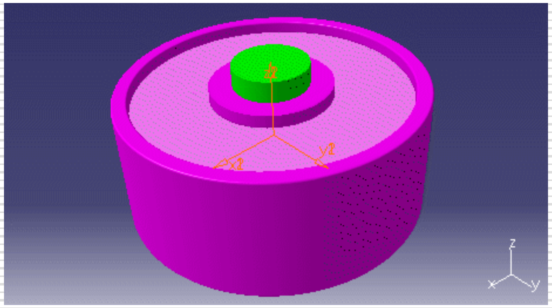
Creating Spline Curves

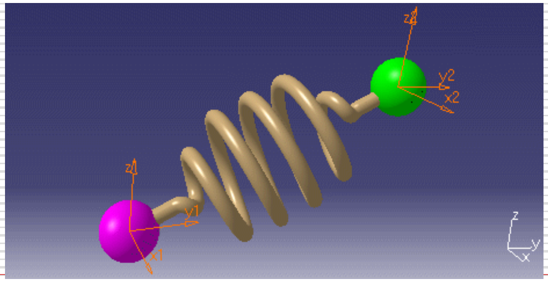
Forze

Rotational Spring-Damper-Actuator



- Forze e coppie scalari
- Forze viscose ed elastiche (RSDA)
- Forze di attrito
- Modello di Tire
- Modello di sospensioni
- Forze di contatto
- ....





Overview

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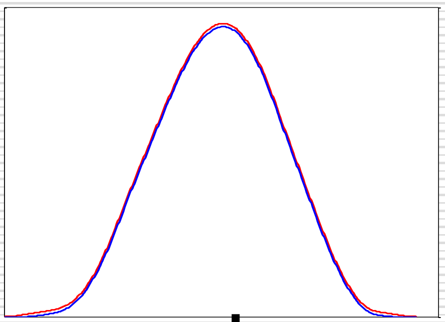
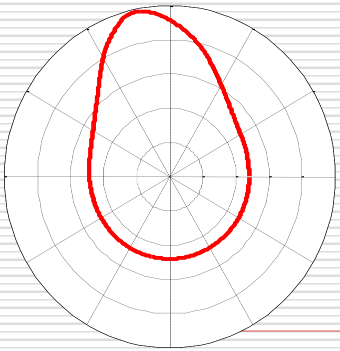
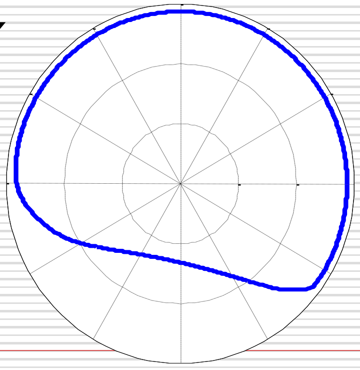
SINTESI CINEMATICA



INPUT

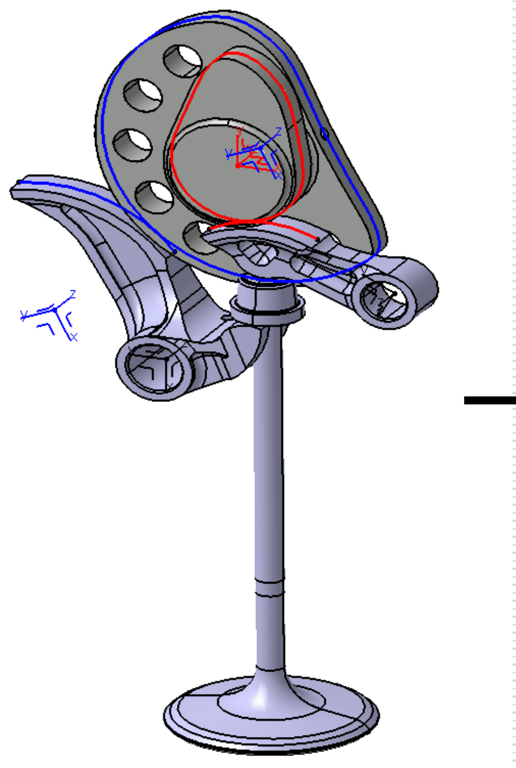
VALIDATI TRAMITE ANALISI **CINEMATICA DIRETTA**



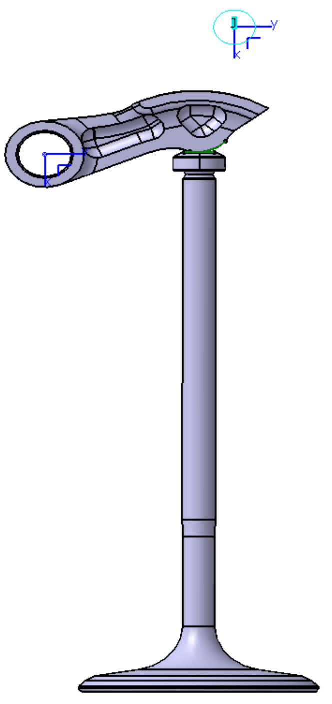


OUTPUT

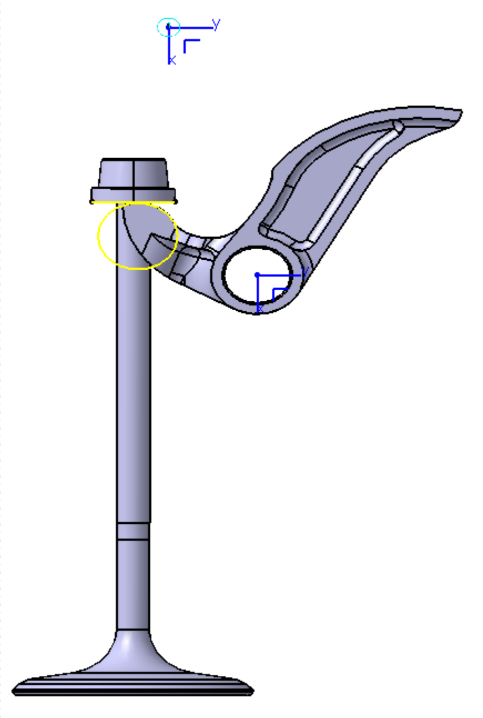
SINTESI CINEMATICA



DISTRIBUZIONE

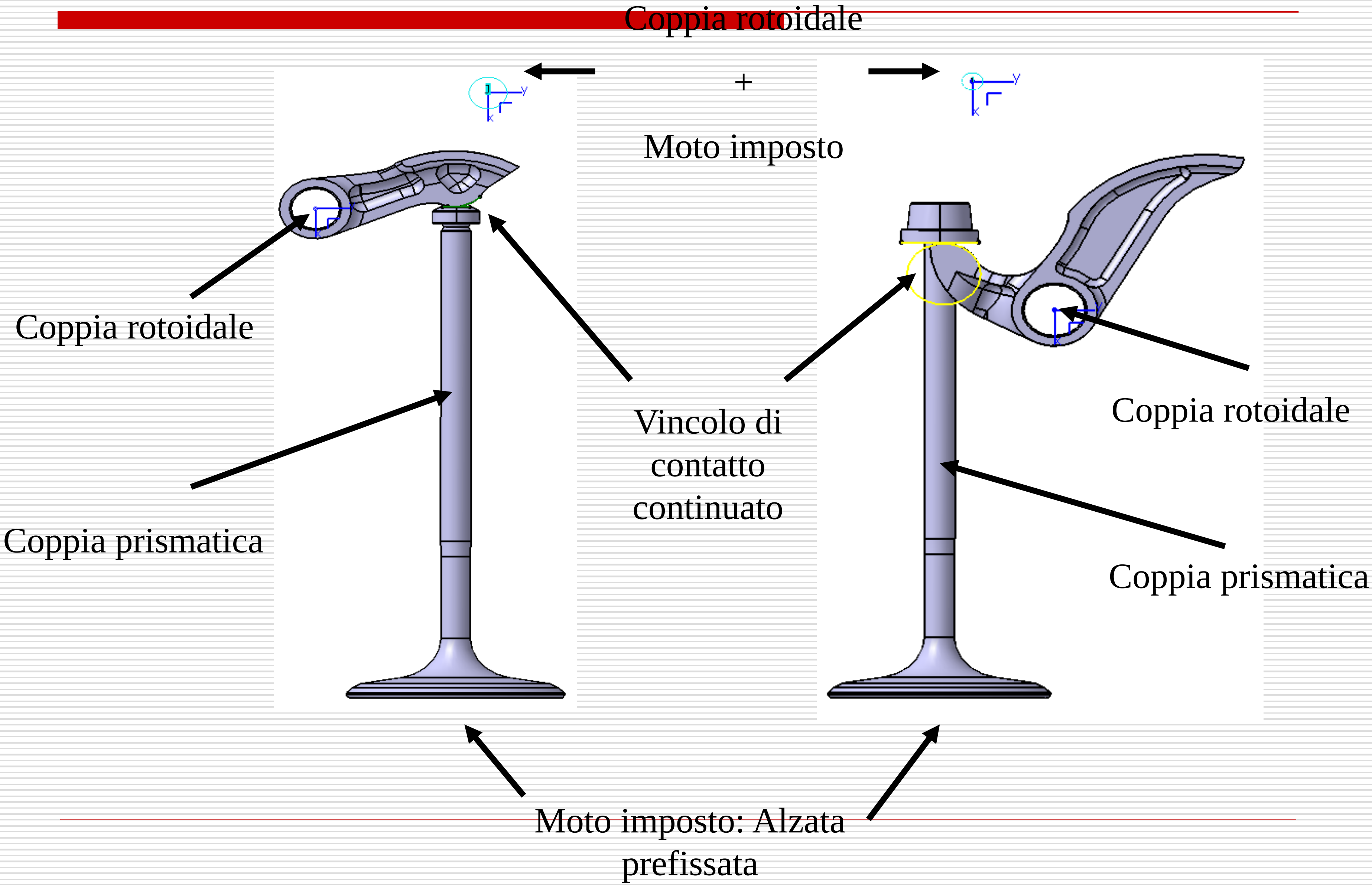


MECCANISMO
DI APERTURA



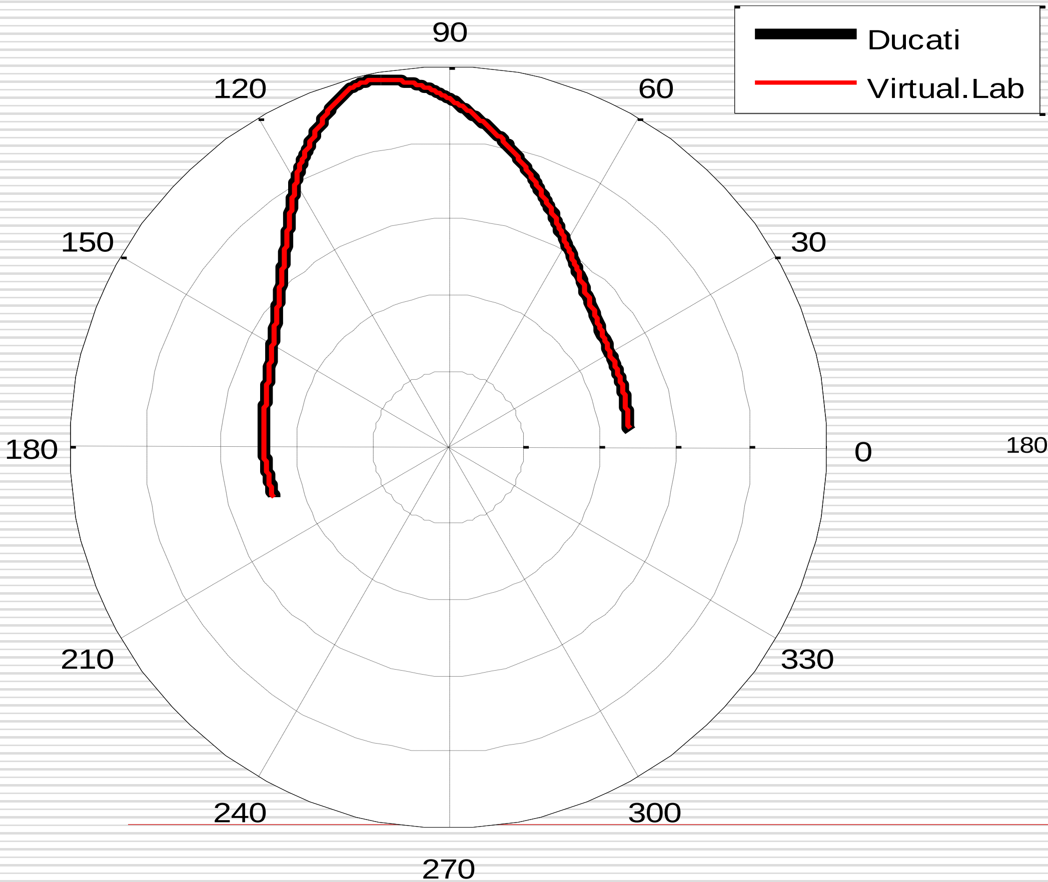
MECCANISMO
DI CHIUSURA

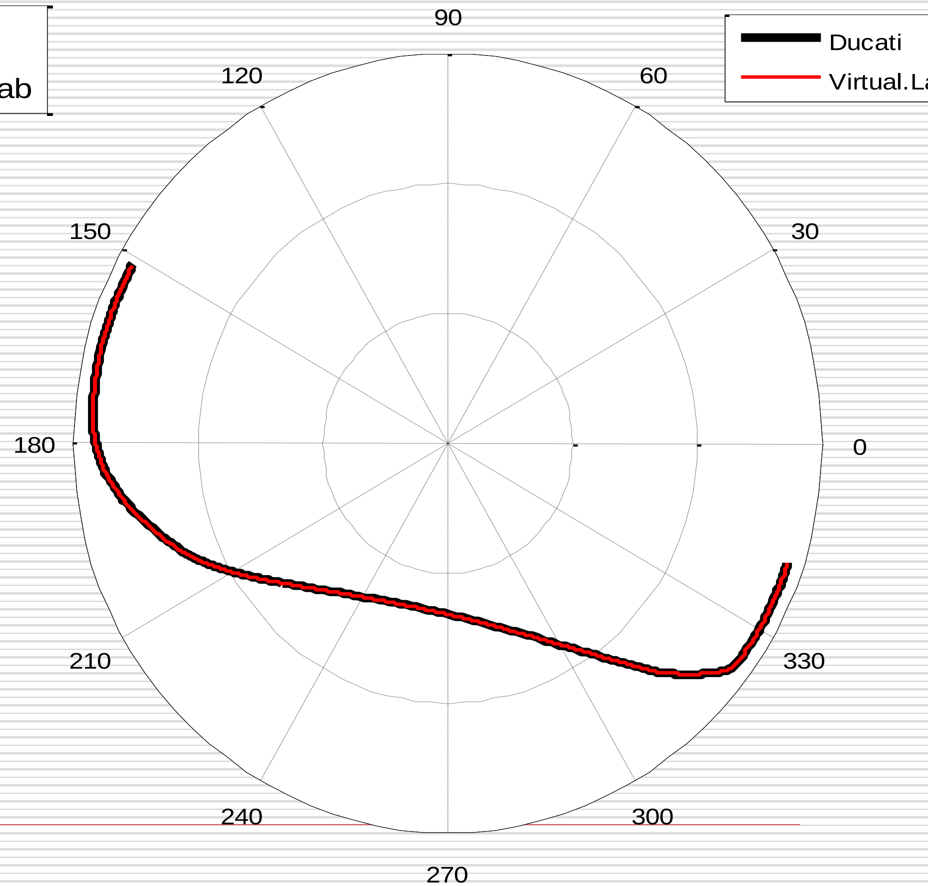
Modello - Sintesi cinematica



SINTESI CINEMATICA – risultati ottenuti

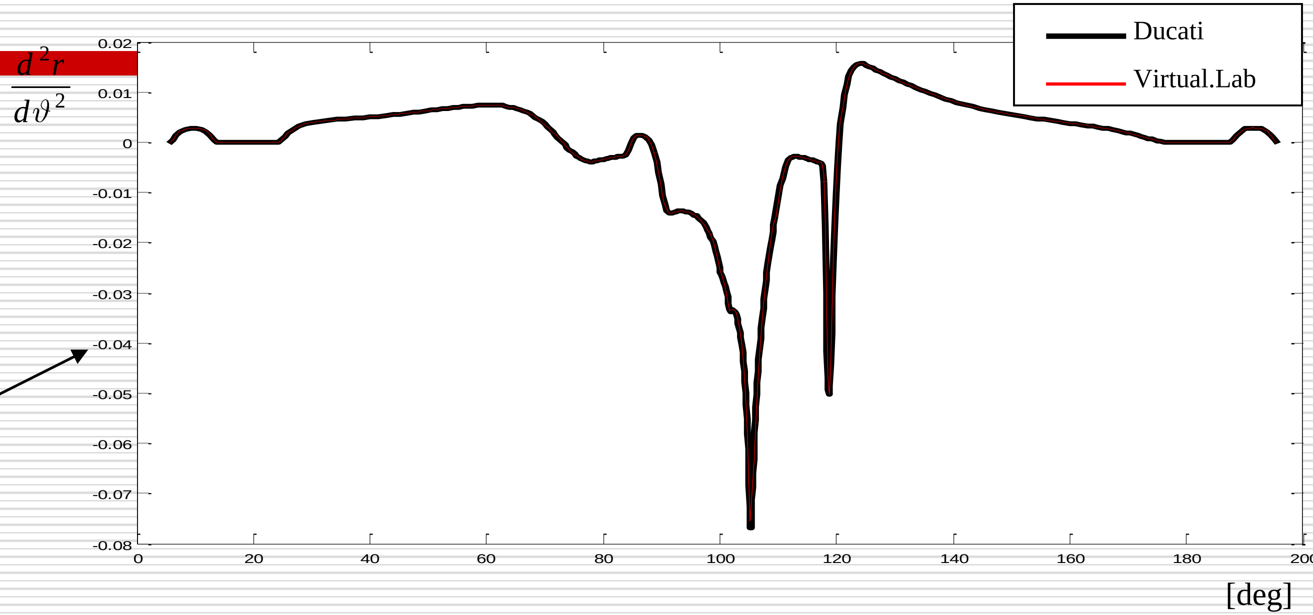
CAMMA DI APERTURA	CAMMA DI CHIUSURA





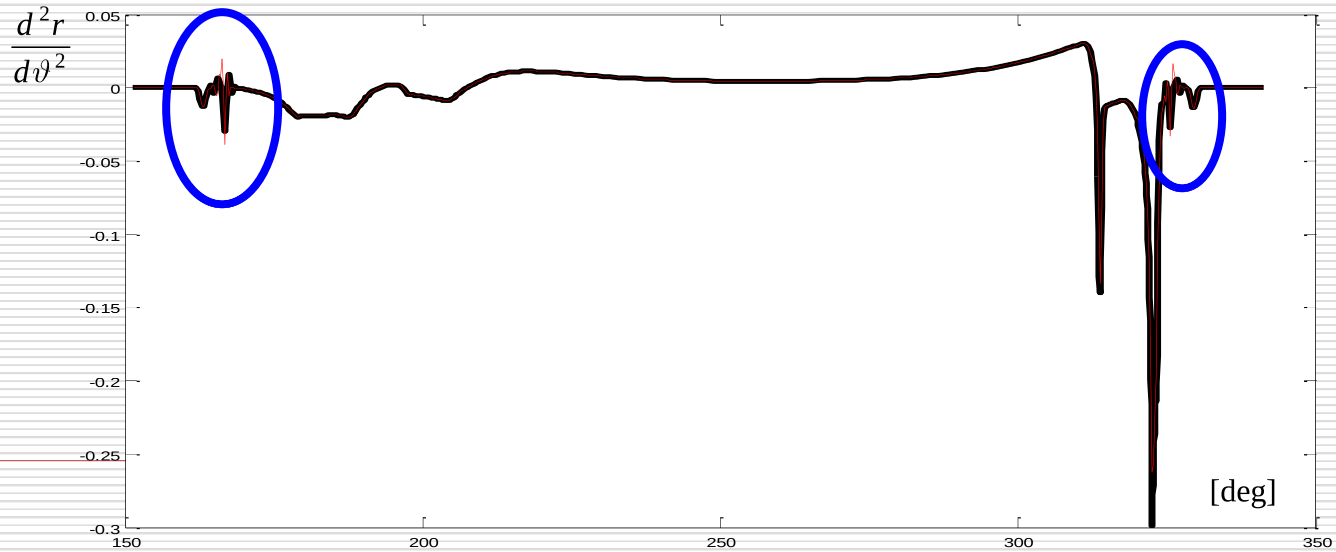
ANALISI APPROFONDITA DEI DUE PROFILI

CAMMA DI APERTURA

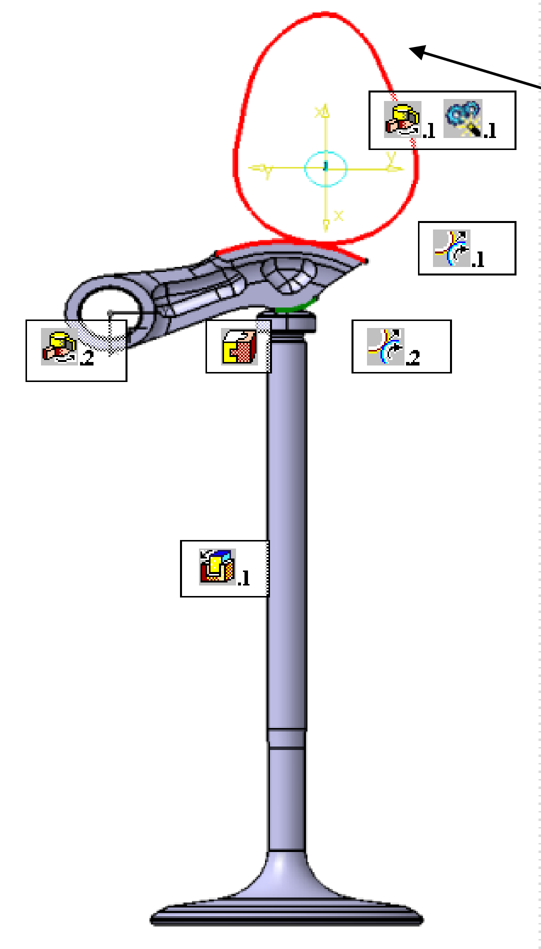


Derivata seconda del profilo della camma

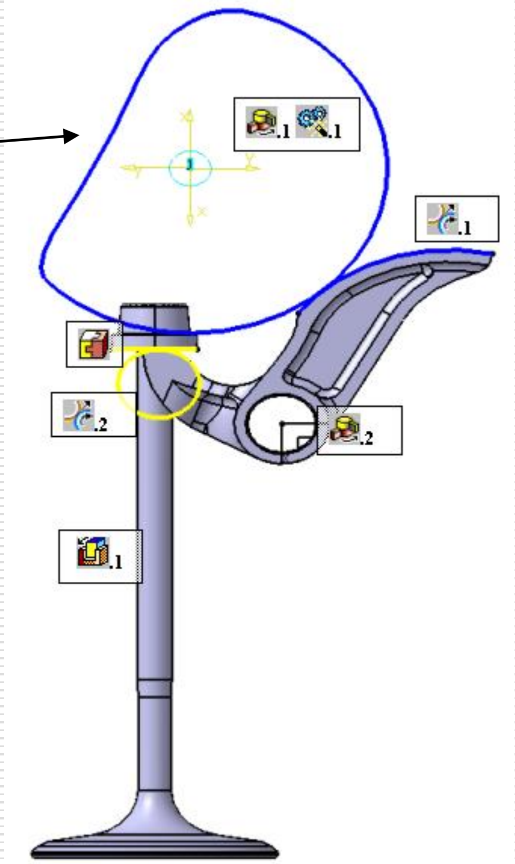
CAMMA DI CHIUSURA



Cinematica diretta



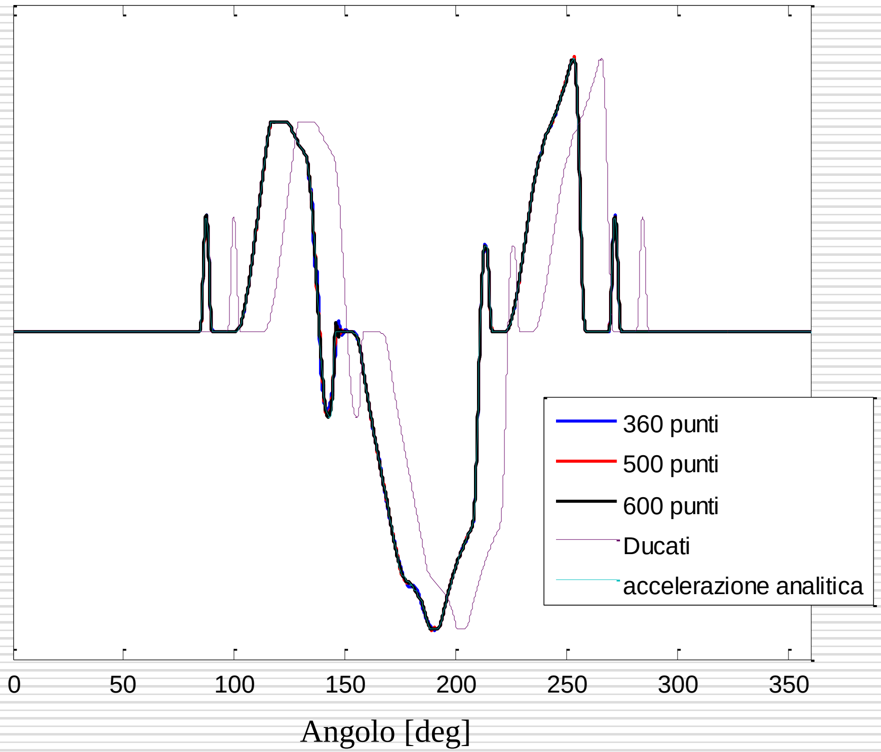
Camme appena ottenute in VL

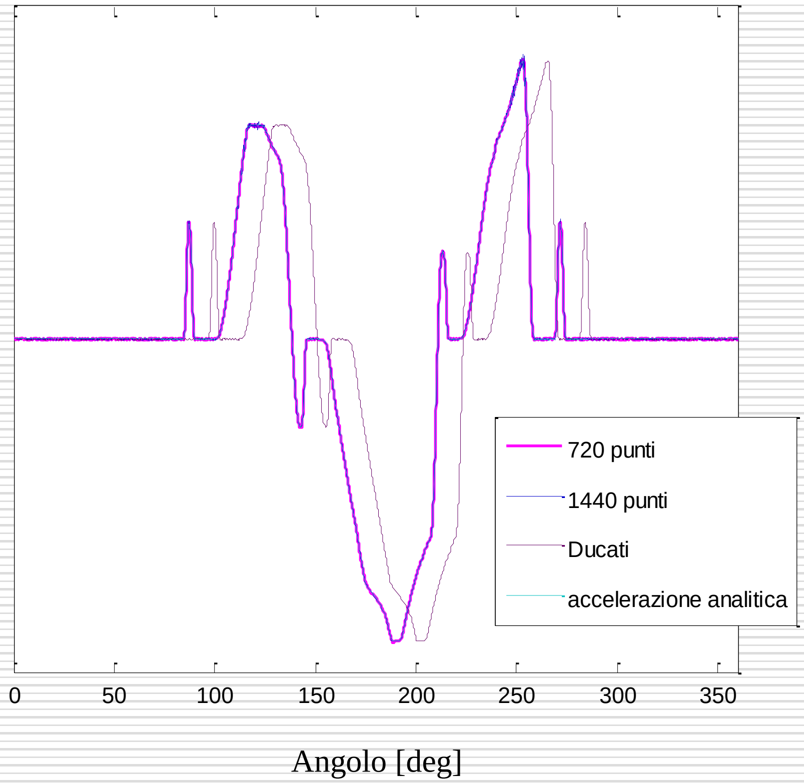


- No giochi
- Contatto permanente

Cinematica diretta-Apertura

accelerazione

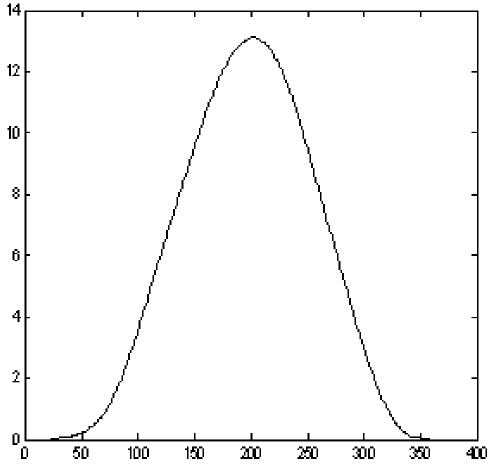




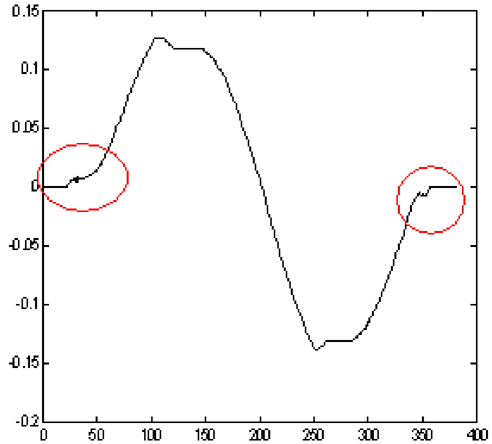
#punti: numero di punti del profilo camma

Cinematica diretta-Chiusura

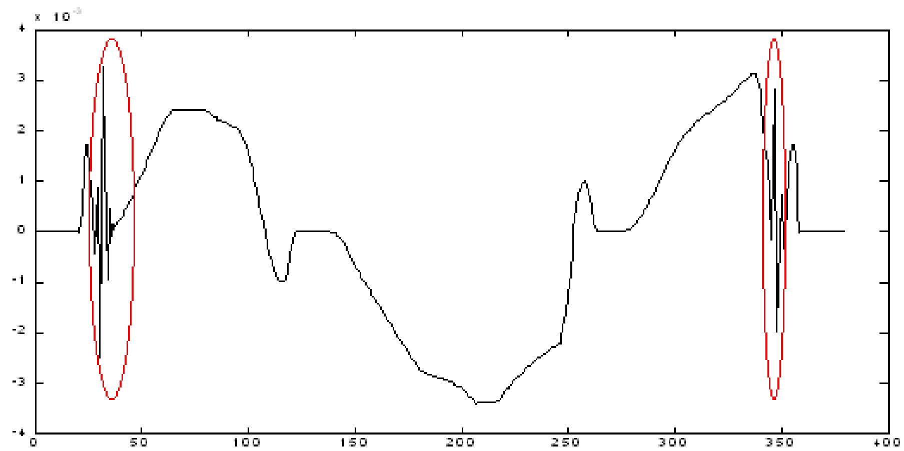
Legge di alzata Ducati per mecc chiusura



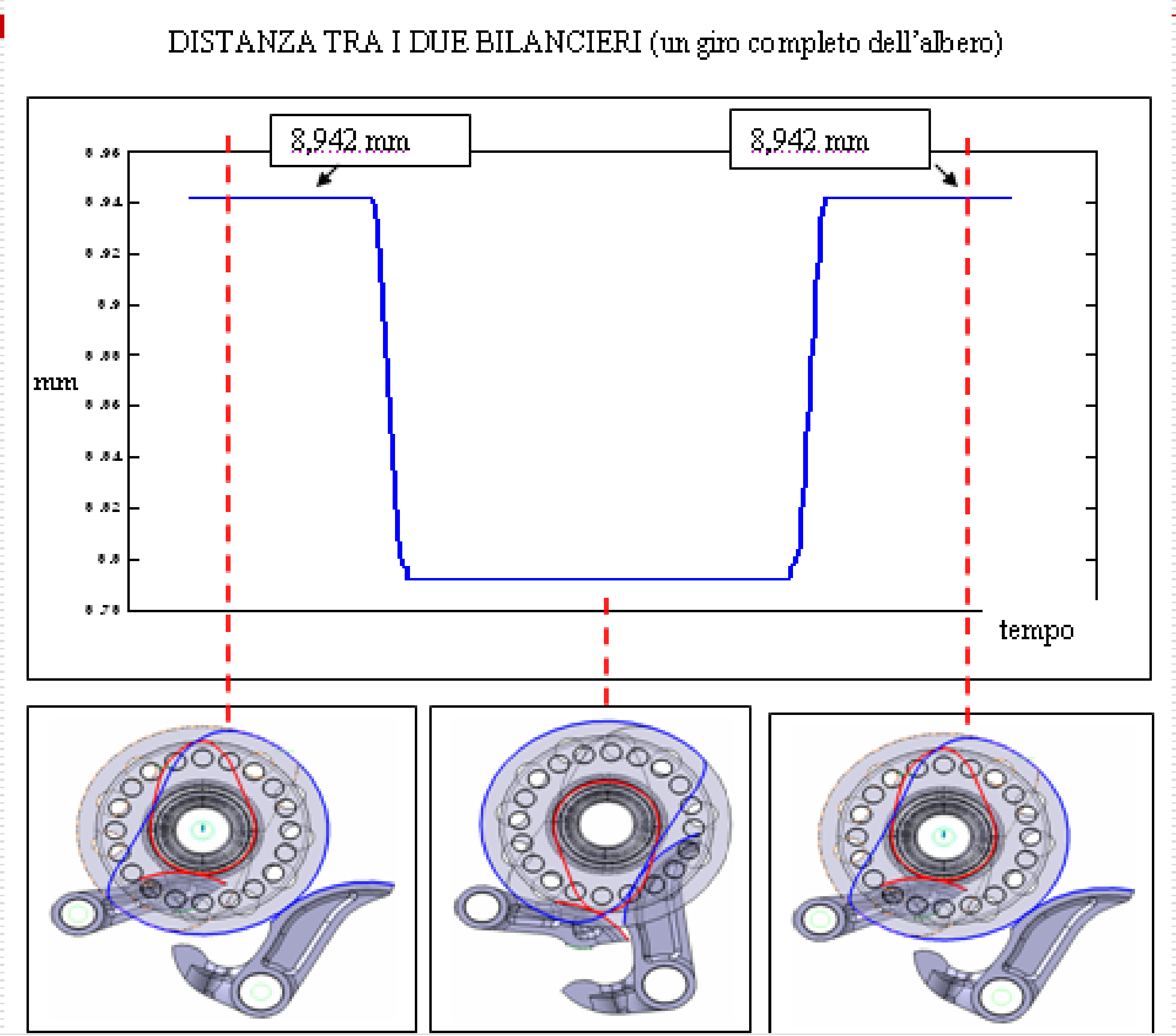
Derivata prima analitica



Derivata seconda analitica



DISTANZA TRA I BILANCIERI



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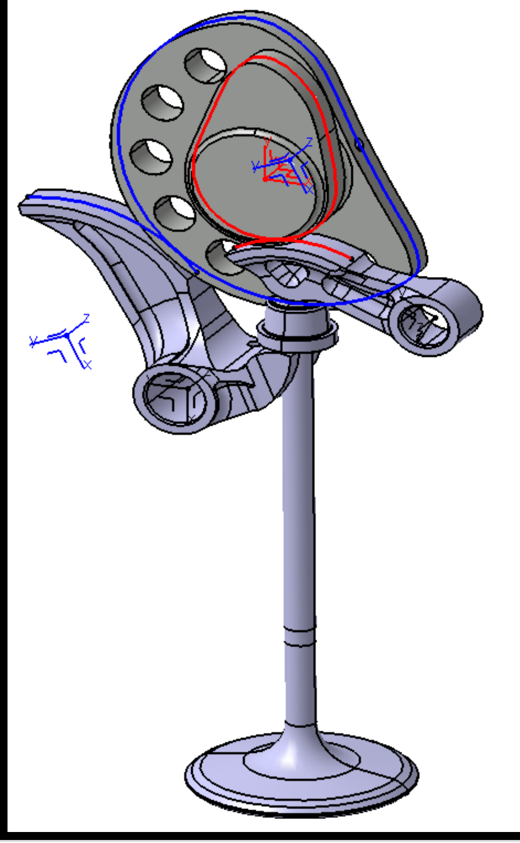
DINAMICA



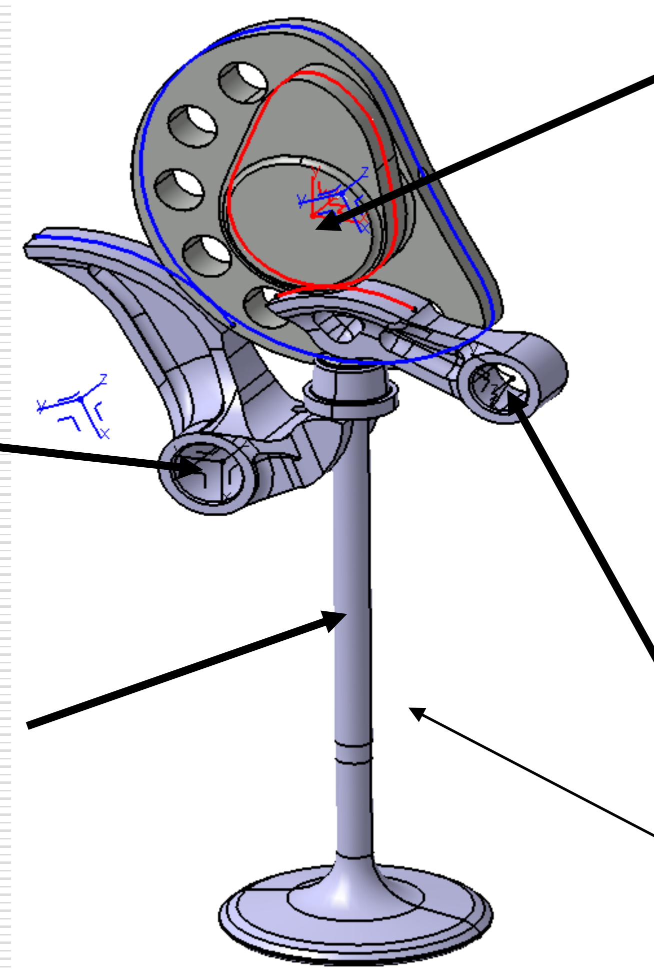
Funzioni di vincolo

$$\begin{bmatrix} M & \Psi_q^T \\ \Psi_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q_e \\ \gamma \end{bmatrix}$$

Moltiplicatore di Lagrange

$$\{\gamma\} = -\{\Psi_{tt}\} - \left(\left[\Psi_q\right]\{\dot{q}\}\right)_q \{\dot{q}\} - 2\left[\Psi_{qt}\right]\{\dot{q}\}$$

DINAMICA – VINCOLI



Coppia rotoidale

- Contatto camme -bilancieri
- Contatto bilancieri - registro
- Contatto valvola – sede
- Giochi

Coppia rotoidale

Coppia rotoidale

Coppia prismatica

Moto imposto

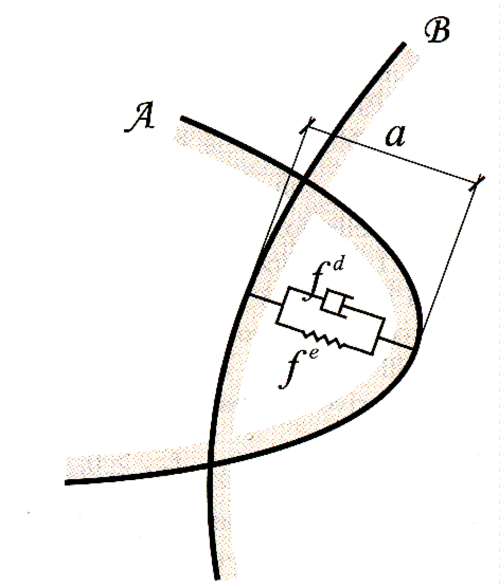
- Contatto camme -bilancieri

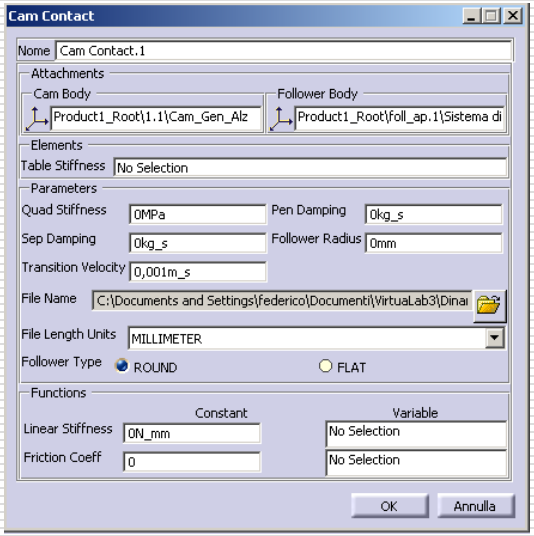
- Contatto bilancieri – registro

  (contatto punto-superficie)

- Contatto valvola – sede

  (contatto punto-superficie)

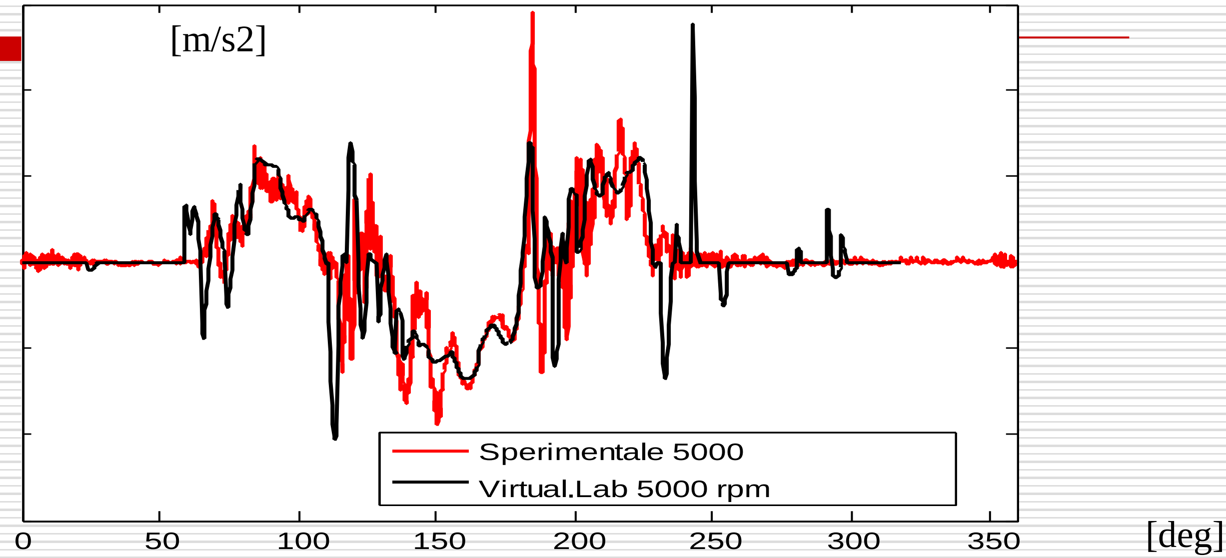




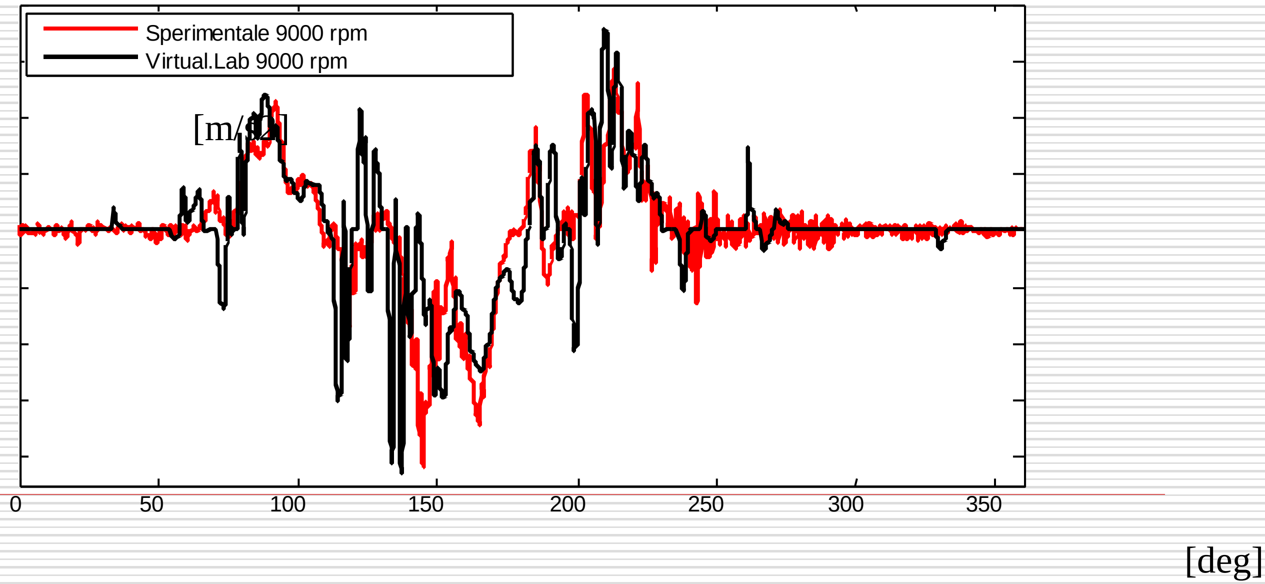
RISULTATI OTTENUTI – ACCELERAZIONE VALVOLA

CORPI RIGIDI

5000 RPM



9000 RPM



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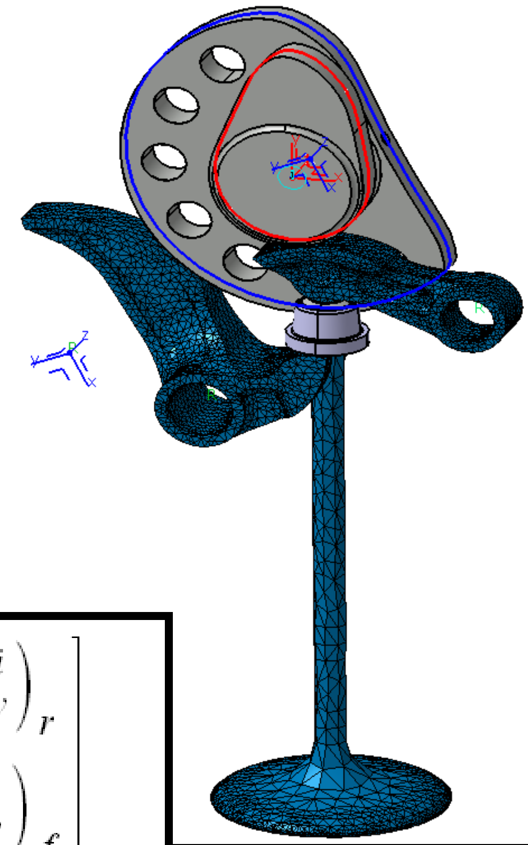
DINAMICA CON CORPI FLESSIBILI



$$\begin{bmatrix} m^{i}_{rr} & m^{i}_{rf} \\ m^{i}_{fr} & m^{i}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{q}^{i}_{r} \\ \ddot{q}^{i}_{f} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K^{i}_{ff} \end{bmatrix} \begin{bmatrix} q^{i}_{r} \\ q^{i}_{f} \end{bmatrix} + \begin{bmatrix} \Psi^{T}_{q^{i}_{r}} \\ \Psi^{T}_{q^{i}_{f}} \end{bmatrix} \lambda = \begin{bmatrix} \left(Q^{i}_{e}\right)_{r} \\ \left(Q^{i}_{e}\right)_{f} \end{bmatrix} + \begin{bmatrix} \left(Q^{i}_{v}\right)_{r} \\ \left(Q^{i}_{v}\right)_{f} \end{bmatrix}$$

$$i = 1, 2, \cdots, nb$$

$$\left[B^{i}\right]^{T} \begin{bmatrix} \boldsymbol{m}^{i}_{rr} & \boldsymbol{m}^{i}_{rf} \\ \boldsymbol{m}^{i}_{fr} & \boldsymbol{m}^{i}_{ff} \end{bmatrix} \left[B^{i}\right] \begin{bmatrix} \ddot{\boldsymbol{p}}^{i}_{r} \\ \ddot{\boldsymbol{p}}^{i}_{f} \end{bmatrix} + \left[B^{i}\right]^{T} \begin{bmatrix} 0 & 0 \\ 0 & K^{i}_{ff} \end{bmatrix} \left[B^{i}\right] \begin{bmatrix} \boldsymbol{p}^{i}_{r} \\ \boldsymbol{p}^{i}_{f} \end{bmatrix} = \left[B^{i}\right]^{T} \begin{bmatrix} \left(Q^{i}_{e}\right)_{r} \\ \left(Q^{i}_{e}\right)_{f} \end{bmatrix} + \left[B^{i}\right]^{T} \begin{bmatrix} \left(Q^{i}_{v}\right)_{r} \\ \left(Q^{i}_{v}\right)_{f} \end{bmatrix} - \left[B^{i}\right]^{T} \begin{bmatrix} \Psi^{T}_{q^{i}_{r}} \\ \Psi^{T}_{q^{i}_{f}} \end{bmatrix} \lambda$$

$$\boldsymbol{i} = 1, 2, \cdots, \boldsymbol{nb}$$

Modi di Graig-Bampton

Modi Statici:
(spostamento unitario nei nodi di interfaccia)

Sol 101 (Guyan reduction)

Modi Normali:
(nodi di interfaccia vincolati)

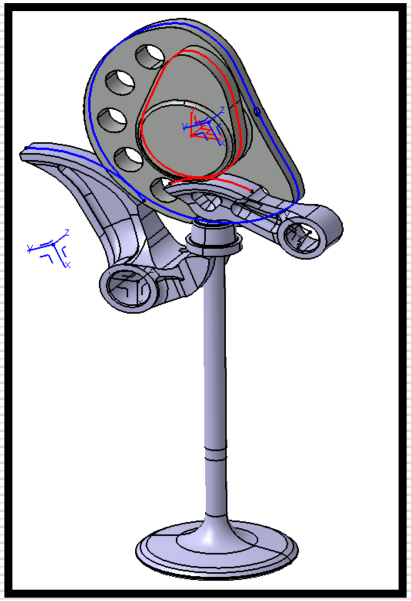
ortogonalizzazione

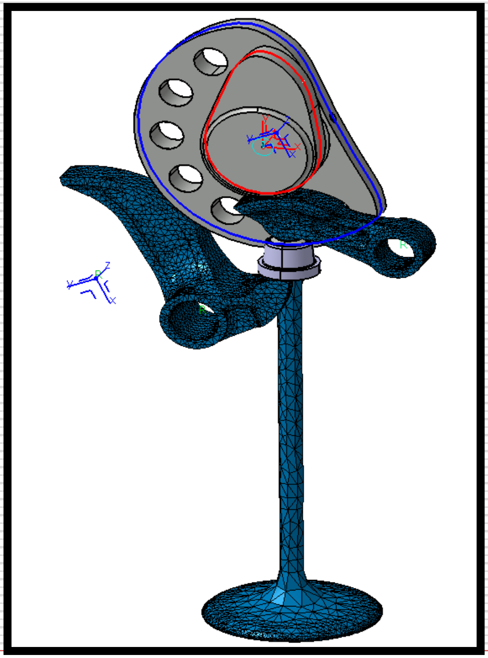
[B]

$$B = \begin{bmatrix} I_{bb} & 0_{bn} \\ \psi_{ib} & \Phi_{in} \end{bmatrix}$$

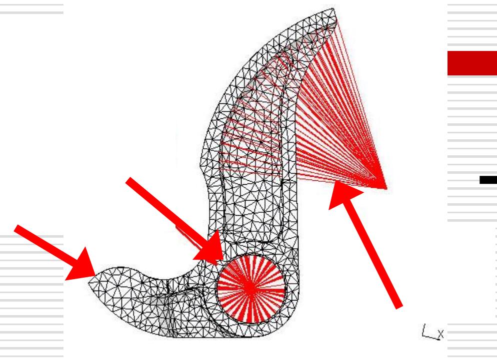
Procedura:

- Mesh
- Matrice B dei singoli componenti
- Uso di Dummy bodies per scambiare le forze

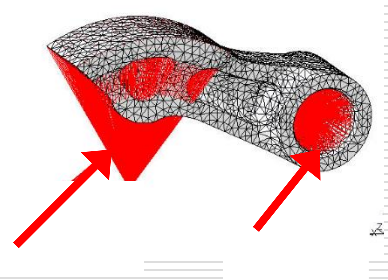




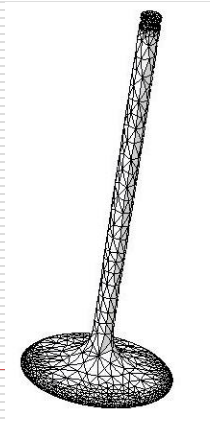
MESH – INTERFACCE – MODELLO MODALE



TET10 → 25000 NODI → 10 MODI



TET10 → 25000 NODI → 8 MODI

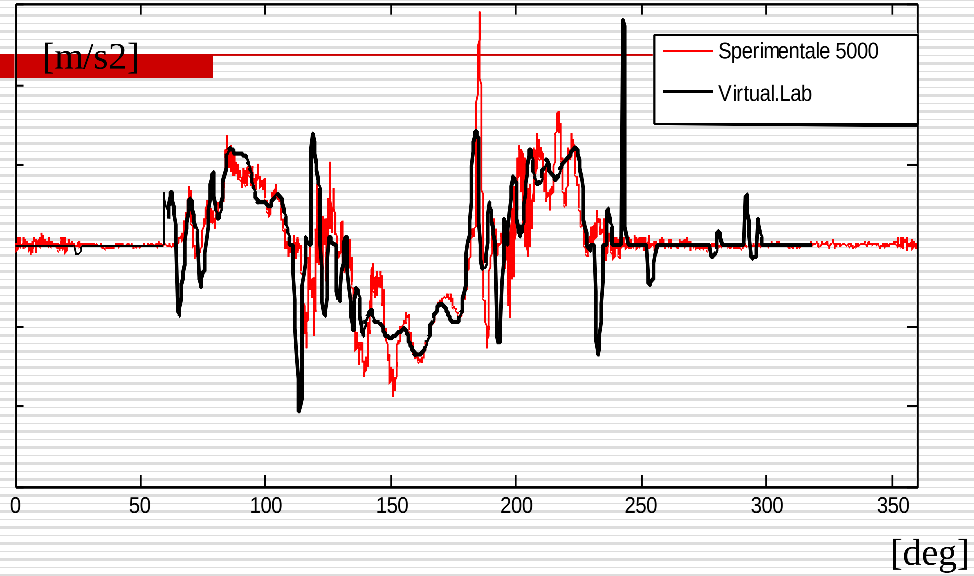


TET10 → 14000 NODI → 6 MODI

RISULTATI OTTENUTI - ACCELERAZIONE

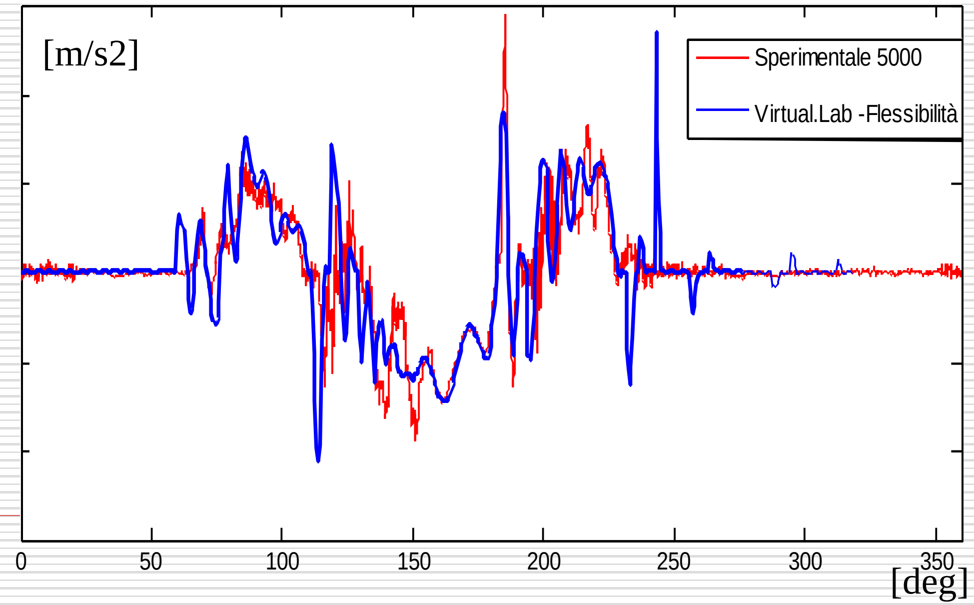
CORPI RIGIDI

CORPI RIGIDI –5000 RPM

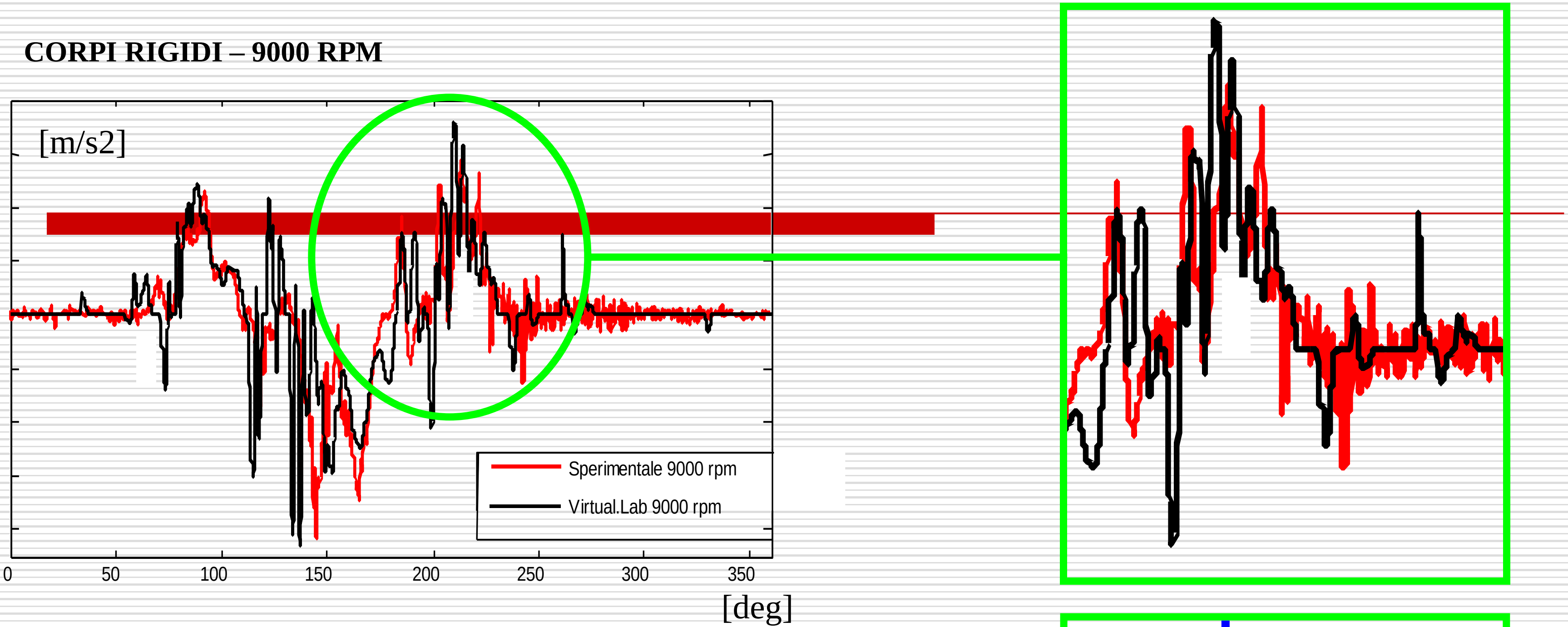


CORPI FLESSIBILI

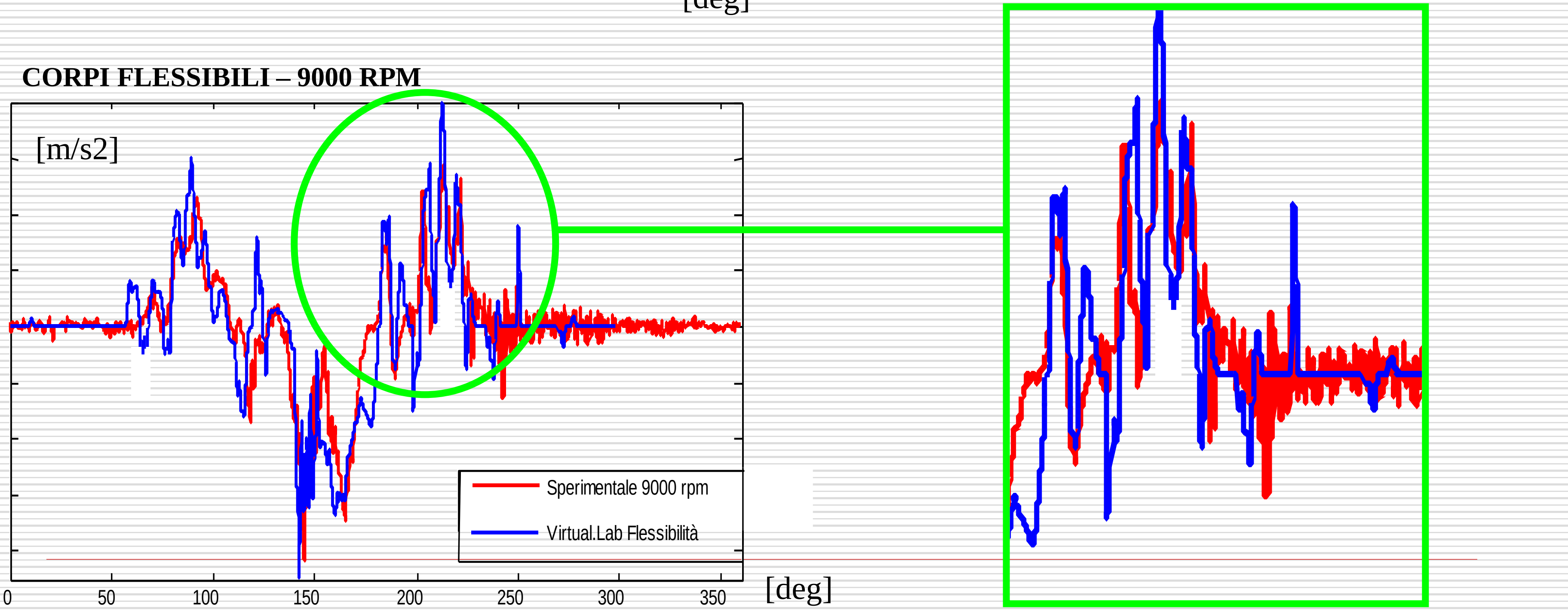
CORPI FLESSIBILI – 5000 RPM



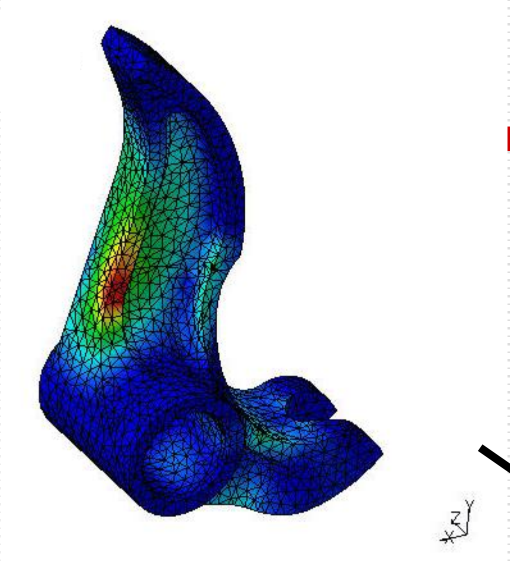
CORPI RIGIDI – 9000 RPM

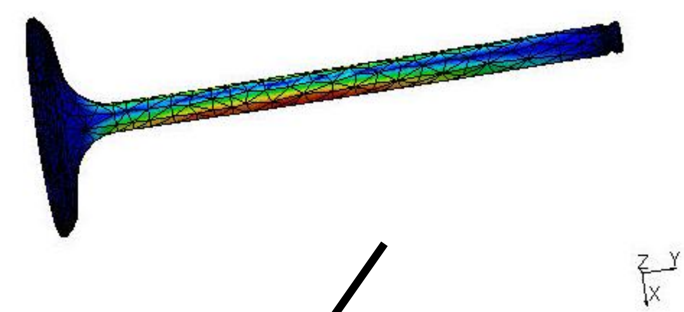


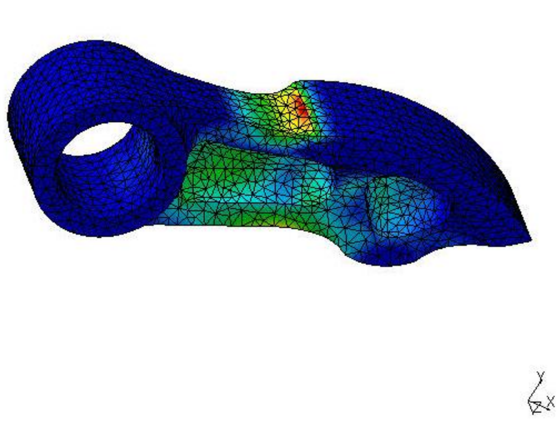
CORPI FLESSIBILI – 9000 RPM



ANALISI RESISTENZIALE







In che istante si verificano ?

5000 RPM

9000 RPM