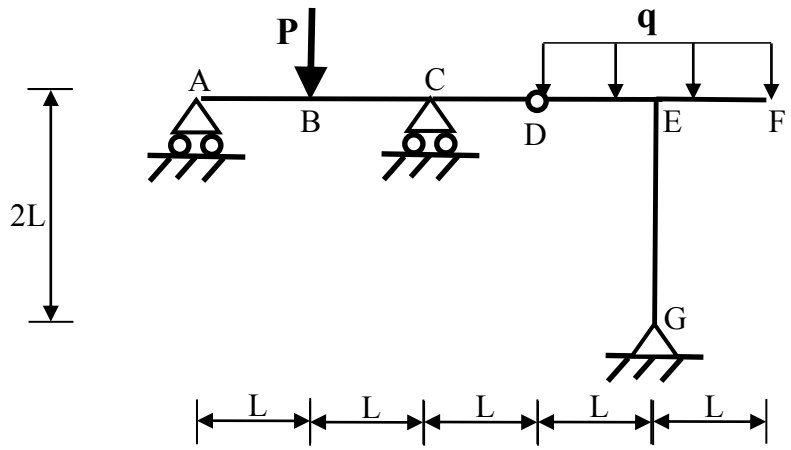
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
07/06/2016

A



L= 1 m, q = 20 kN/m, P = 40 kN
σ_{AMM} = 240 MPa , E = 210 GPa

La travatura isostatica in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare lo spostamento verticale del punto F.
- Calcolare nuovamente lo spostamento verticale di F considerando anche un abbassamento verticale del vincolo in G pari a 1 cm.

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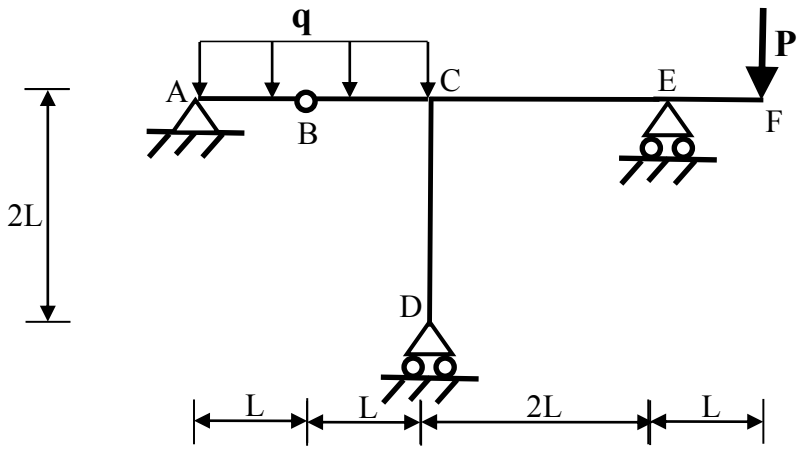
B



L= 1 m, q = 20 kN/m, P = 40 kN
σ_{AMM} = 240 MPa , E = 210 GPa

La travatura isostatica in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare la rotazione nel punto E.
- Calcolare nuovamente la rotazione in E considerando anche un abbassamento verticale del vincolo in D pari a 1 cm.

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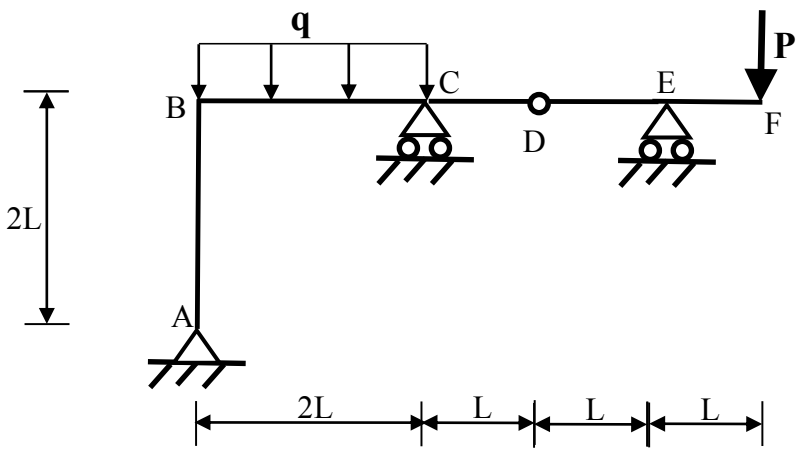
C



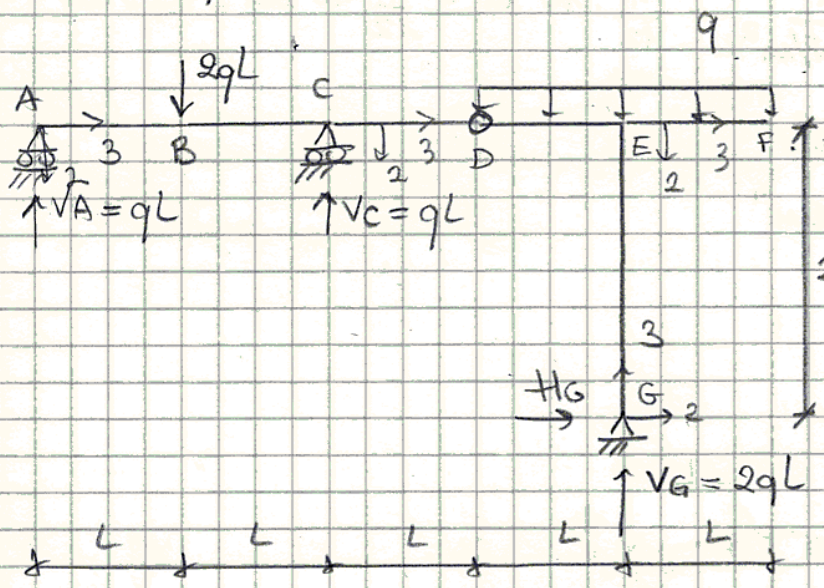
L= 1 m, q = 20 kN/m, P = 40 kN
σ_{AMM} = 240 MPa , E = 210 GPa

La travatura isostatica in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare lo spostamento orizzontale nel punto B.
- Calcolare nuovamente lo spostamento orizzontale in B considerando anche un abbassamento verticale del vincolo in A pari a 1 cm.

FILA A) $qL = 20\ kN$, $qL^2 = 20\ kN\,m$

Diagrammi quotati:



$(\rightarrow)\ H_G = 0$

$(\curvearrowright)_{DEFG}\ V_G \cancel{L} = 2qL^{\cancel{2}}$

$(\curvearrowright)_{ABCD}$

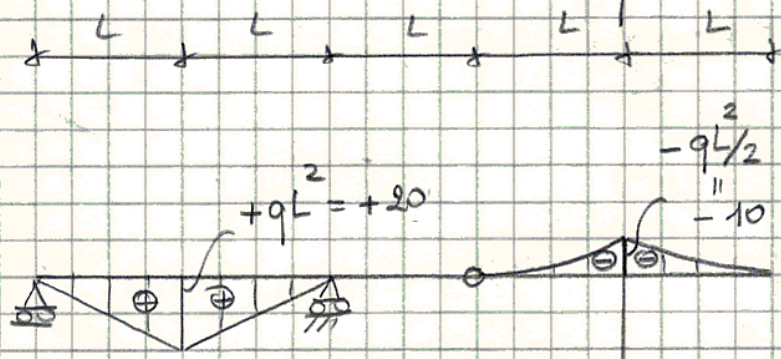
$V_A 3\cancel{L} + V_C \cancel{L} = 2qL \cdot 2\cancel{L}$

$(\uparrow)\ V_A + V_C = 4qL - 2qL$

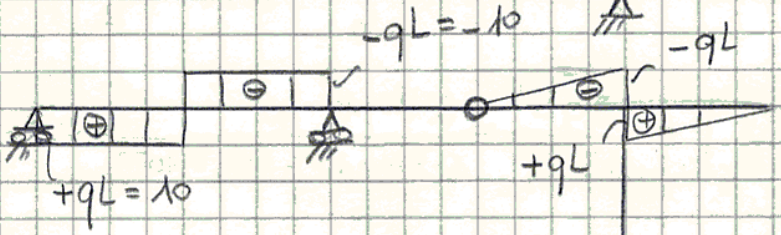
$\cancel{2} V_A = \cancel{2} qL$

$\hookrightarrow V_C = 2qL - V_A = qL$

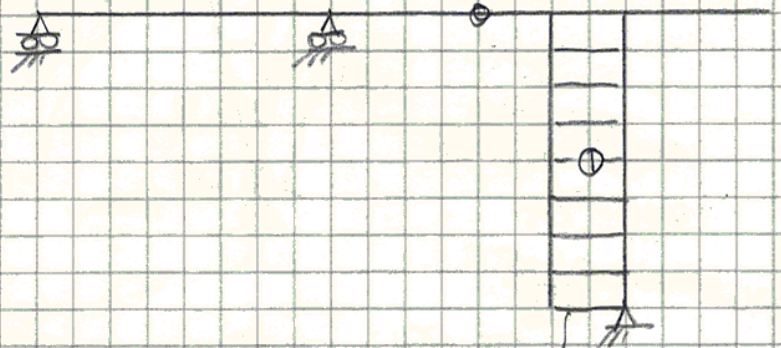
$$\begin{cases} V_A = qL \\ V_C = qL \\ V_G = 2qL \end{cases}$$



(M)
(kN m)



(T)
(kN)



(N)
(kN)

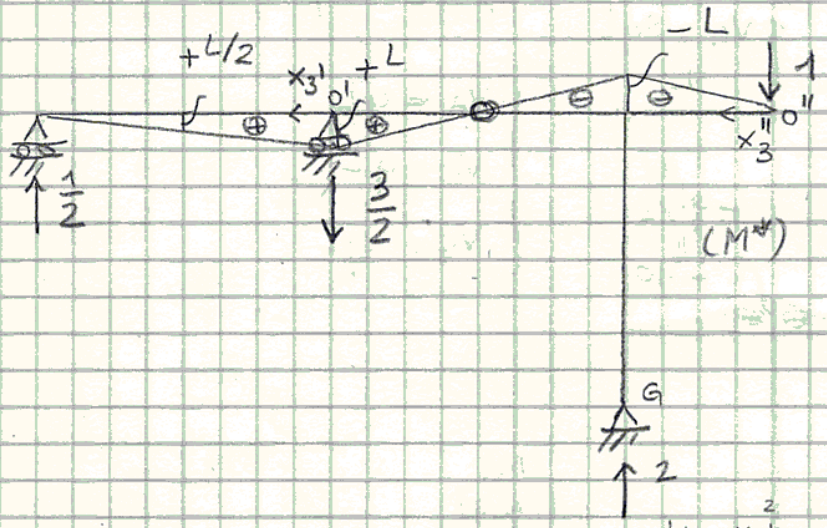
Dimensionamento:

$$W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{20 \cdot 10^3 \ Nm}{240 \cdot 10^6 \ N/m^2} = \frac{1}{12} \cdot 10^{-3} \ m^3 = \frac{1}{12} \cdot 10^3 \ cm^3 = 83{,}3 \ cm^3$$

IPE 160 $W_1 = 108{,}7 \ cm^3$

- $A = 20{,}09 \ cm^2$
- $I_1 = 869{,}3 \ cm^4$
- $H = 16 \ cm$

Spostamento verticale in F:



$$1 \cdot v_F = \frac{1}{EI_1}\left[\frac{1}{3} L \frac{L}{2} qL^2 + \int_0^L (qLx_3')\left(L - \frac{x_3'}{2}\right) dx_3' + 2\int_0^L (-x_3'')\left(-q\frac{x_3''}{2}\right) dx_3''\right]$$

$$= \frac{qL^4}{EI_1}\left[\frac{1}{6} + \frac{1}{2} - \frac{1}{6} + \frac{2 \cdot 1}{8}\right] = \frac{3qL^4}{4EI_1} = \frac{3 \cdot 20 \cdot 10^3 \cdot 10^2}{4 \cdot 210 \cdot 10^9 \cdot 869{,}3 \cdot 10^{-8}} \ cm = 0{,}82 \ cm$$

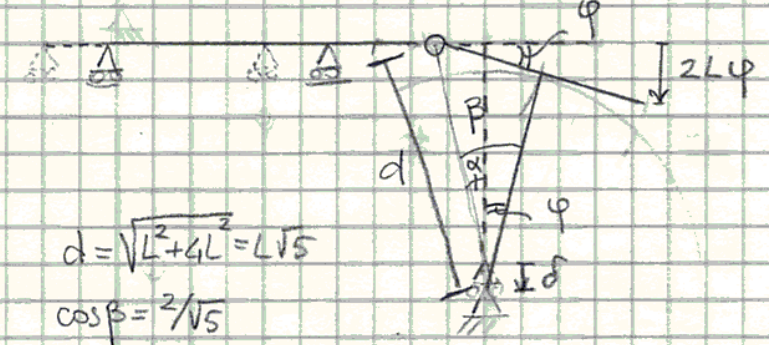
Spostamento in F con cedimento in G:

$$1 \cdot v_F - 2 \cdot \delta_G = \frac{3qL^4}{4EI_1}$$

$$\hookrightarrow v_F = \frac{3qL^4}{4EI_1} + 2\delta_G$$

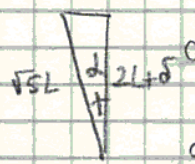
$$= (0{,}82 + 2) \ cm$$

$$= 2{,}82 \ cm$$



$$d = \sqrt{L^2 + 4L^2} = L\sqrt{5}$$

$$\cos\beta = 2/\sqrt{5}$$

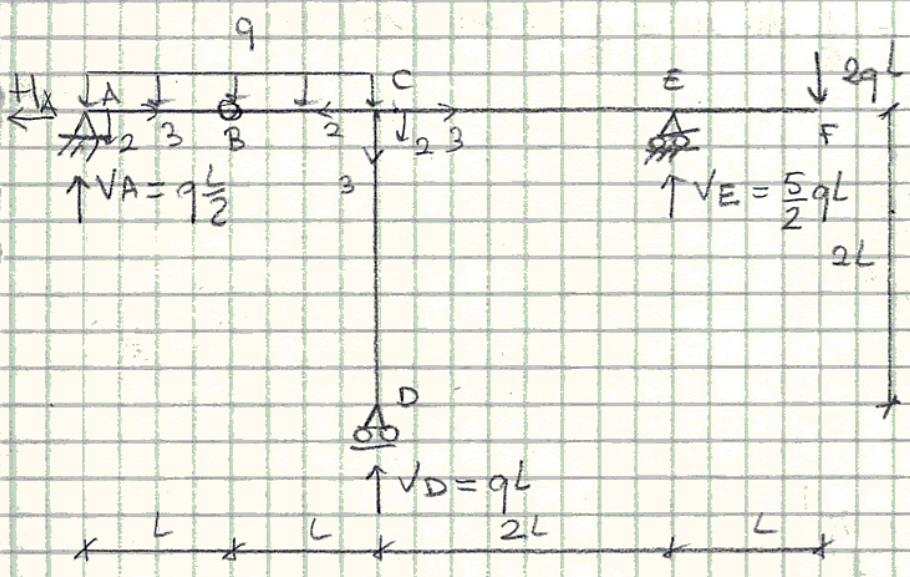


$$\cos\alpha = \frac{2L + \delta}{\sqrt{5}L}$$

$$\alpha = \arccos\left(\frac{2}{\sqrt{5}} + \frac{\delta/L}{\sqrt{5}}\right) \simeq \beta - \frac{\delta}{L} \rightarrow \varphi = \beta - \alpha \simeq \frac{\delta}{L}$$

A/2

FILA B) $qL = 20\ kN$, $qL^2 = 20\ kNm$, $P = 2qL$



$(\rightarrow)\ H_A = 0$

$(B\circlearrowleft)_{AB}\ V_A \not{2} = qL \frac{L}{2}$

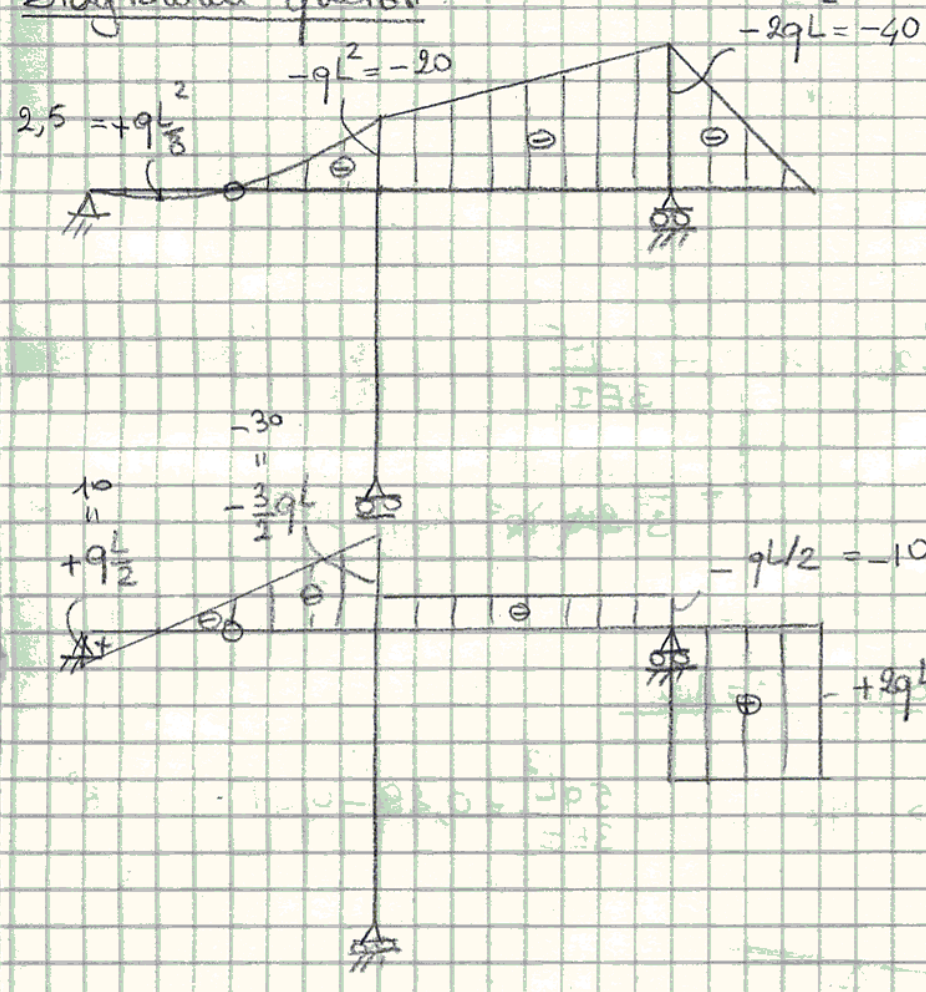
$(D\circlearrowleft)\ V_E\, 2\not{L} - qL\frac{L}{2} 2\not{L} + 2qL\not{L} - 2qL\, 3\not{L} = 0$

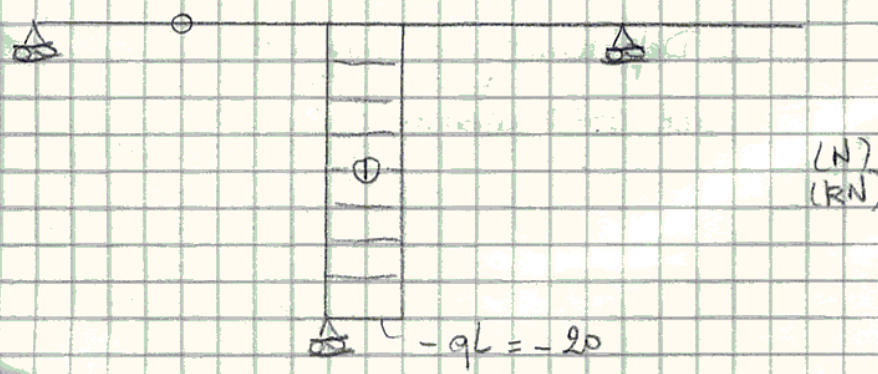
$\hookrightarrow V_E = 5qL/2$

$(\uparrow)\ V_D = 4qL - \frac{5}{2}qL - qL/2$

$= qL$

Diagrammi quotati:





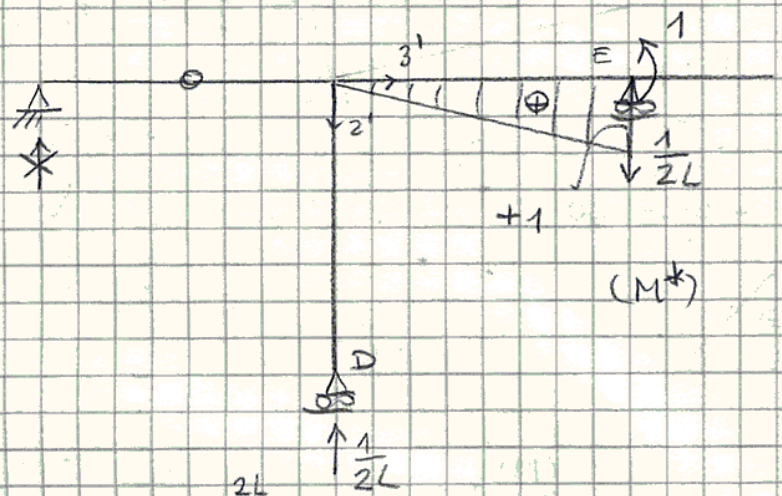
Dimensionamento

$$W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{40 \cdot 10^3 \ Nm}{240 \cdot 10^6 \ N/m^2} = \frac{1}{6} \cdot 10^{-3} \ m^3 = \frac{1}{6} \cdot 10^3 \ cm^3 = 167 \ cm^3$$

IPE 200

$$\begin{cases} W_1 = 194,3 \ cm^3 \\ I_1 = 1943 \ cm^4 \\ A = 28,48 \ cm^2 \\ H = 20 \ cm \end{cases}$$

Rotazione in E:



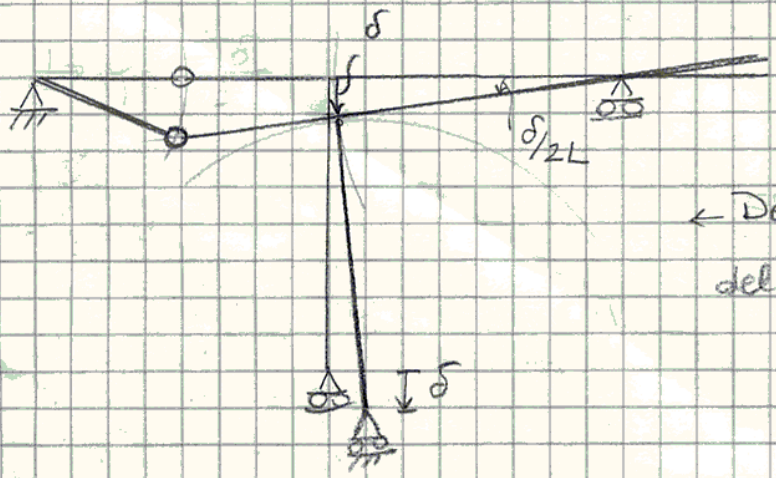
$$1 \cdot \varphi_E = \frac{1}{EI_1} \int_0^{2L} \left(\frac{x_3'}{2L}\right)\left(-qL^2 - q\frac{L}{2}x_3'\right) dx_3' = -\frac{5qL^3}{3EI_1}$$

$$= -\frac{5 \cdot 20 \cdot 10^3}{3 \cdot 210 \cdot 10^9 \cdot 1943 \cdot 10^{-8}}$$

$$= -0,47°$$

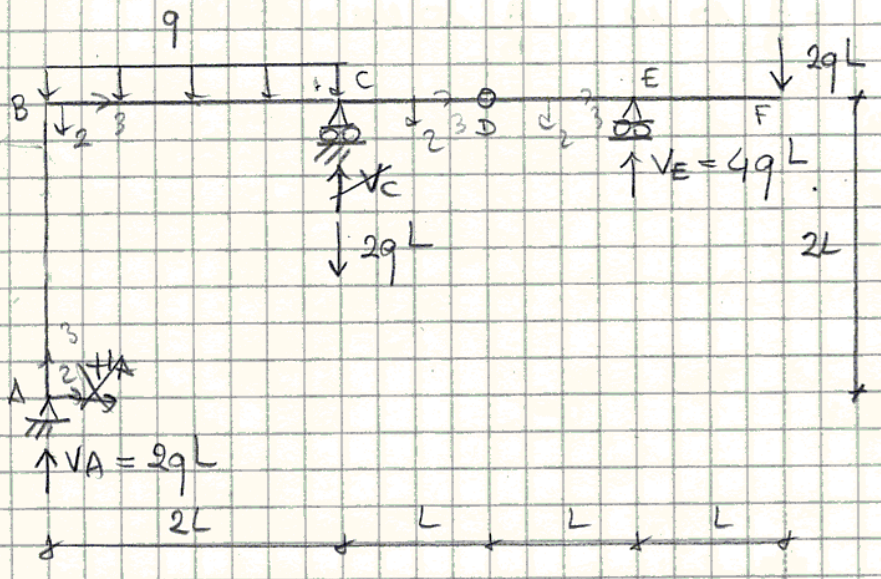
Rotazione in E con cedimento in D ($\delta = 1 cm$):

$$1 \cdot \varphi_E - \frac{1}{2L}\delta = -\frac{5qL^3}{3EI_1} \rightarrow \varphi_E = \frac{\delta}{2L} - \frac{5qL^3}{3EI_1} = 0,29° - 0,47° = -0,18°$$



← Deformata a seguito del solo cedimento in D

FILA C) $qL = 20\ kN$, $qL^2 = 20\ kNm$, $P = 2qL$



$(\rightarrow)\ H_A = 0$

$(\circlearrowleft)_{DEF}\ V_E \cancel{L} = 2q \cancel{L} 2L$

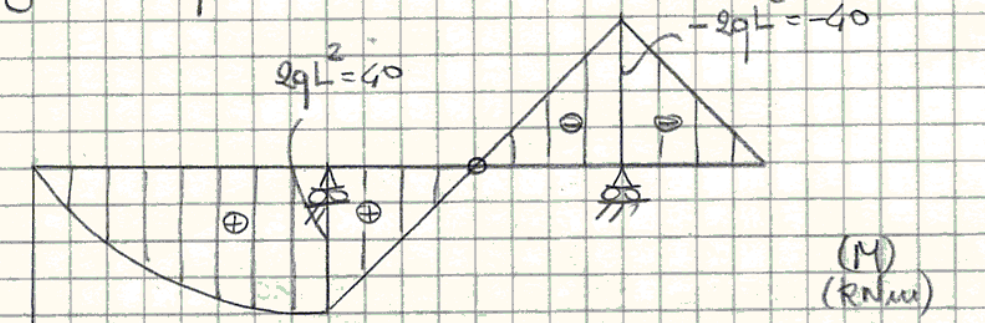
$\hookrightarrow V_E = 4qL$

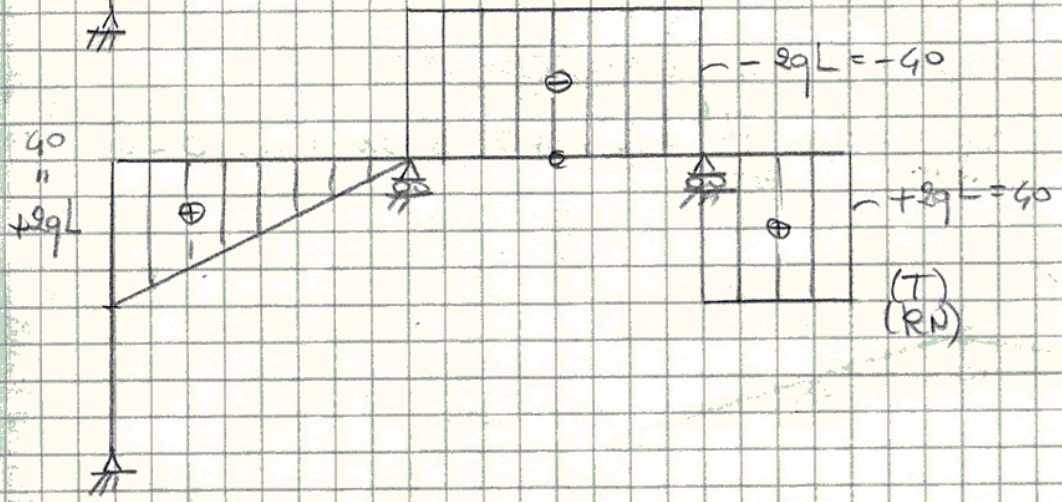
$(\circlearrowleft A)\ V_C 2\cancel{L} + 4q\cancel{L} 4L - 2q\cancel{L} 5L - 2qL^2 = 0$

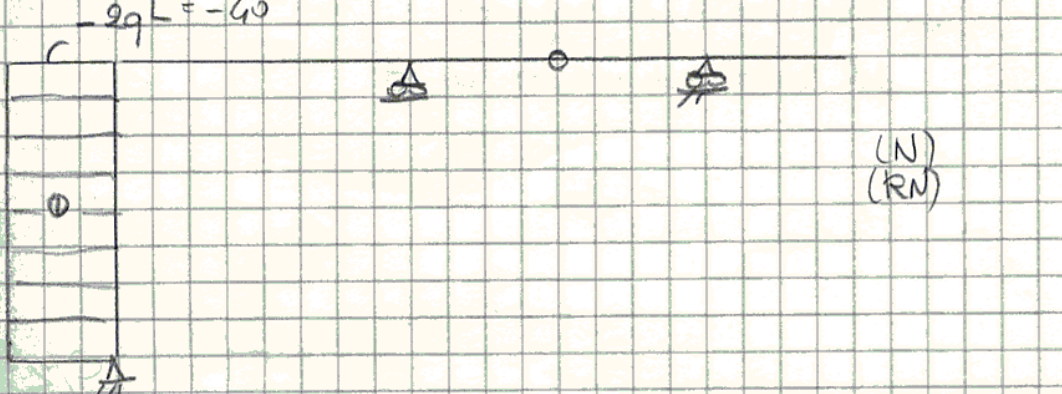
$\hookrightarrow V_C = -2qL$

$(\uparrow)\ V_A = \cancel{4qL} + 2qL - \cancel{4qL}$

Diagrammi quotati:







Dimensionamento:

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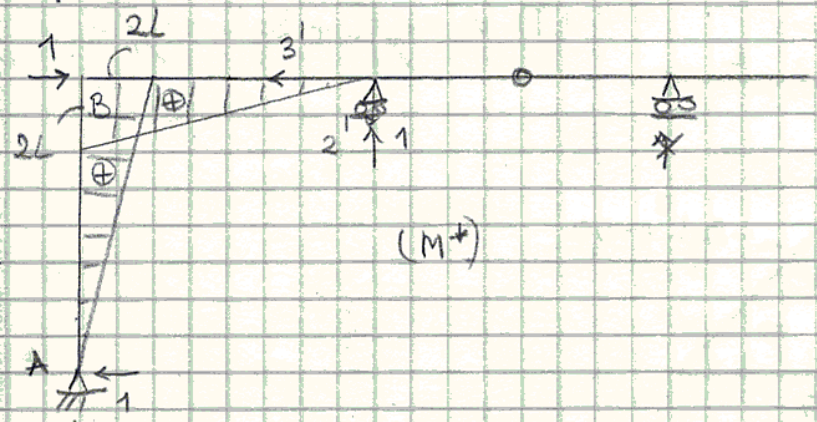
IPE 200
- $W_1 = 194{,}3 \ cm^3$
- $I_1 = 1943 \ cm^4$
- $A = 28{,}48 \ cm^2$
- $H = 20 \ cm$

Spostamento orizzontale in B:

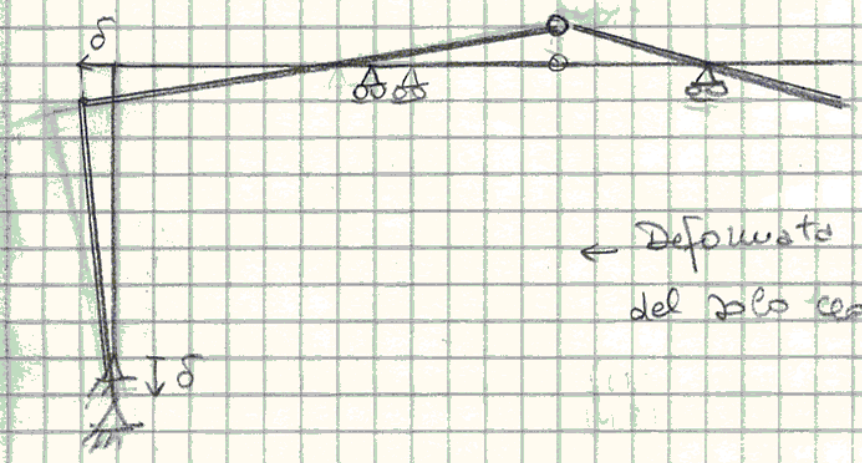


$$1 \cdot v_B = \frac{1}{EI_1} \int_0^{2L} (x_3') \left(2qL^2 - q\frac{x_3'^2}{2}\right) dx_3' = \frac{29L^4}{EI_1} = \frac{2 \cdot 20 \cdot 10^3 \cdot 10^2}{210 \cdot 10^8 \cdot 1943 \cdot 10^{-8}} \ cm$$

$$= 0{,}98 \ cm$$

Spostamento orizzontale in B con cedimento in A:

$$1 \cdot v_B + 1 \cdot \delta = \frac{29L^4}{EI_1} \rightarrow v_B = -\delta + \frac{29L^4}{EI_1} = (-1 + 0{,}98) \ cm = -0{,}02 \ cm.$$



← Deformata a seguito del solo cedimento in A