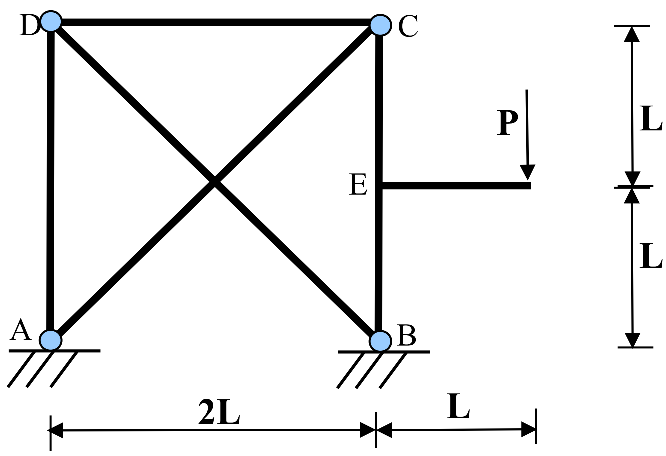
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA RISERVATA A LAUREANDI
16/11/2012



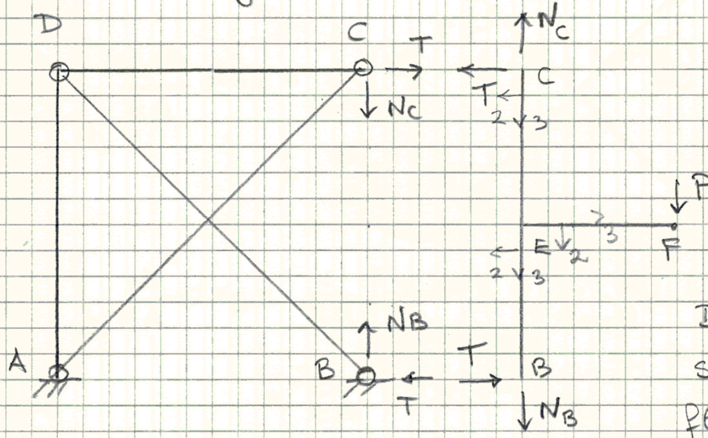
L= 2 m, P = 50 kN
E = 210 GPa, σ_{amm} = 240 MPa, $\alpha = 10^{-5}\,°C^{-1}$

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare il diagrammi quotati del momento flettente e dello sforzo di taglio.
- Dimensionare la travatura a flessione. Risolvere poi la travatura e disegnare il diagramma dello sforzo normale.
- Calcolare la rotazione del nodo E.
- Risolvere la travatura tenendo anche conto di una distorsione termica uniforme pari a +20°C della biella AC.

RISOLUZIONE dello SCRITTO DEL 16/11/12

PUNTO 1: diagrammi di M e T.



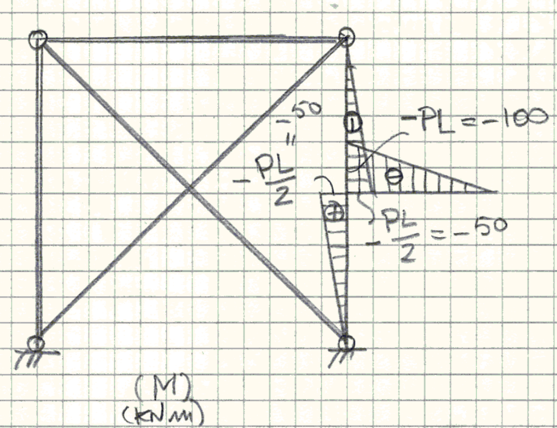
Equilibrio di CEBF:

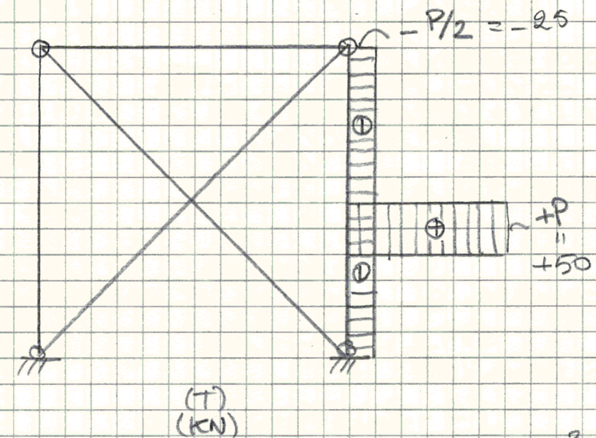
(↑) $N_C - N_B = +P$

(→) $T - T = 0$

(E↺) $T\,2\ell = P\ell$

Dunque il tratto CEBF è staticamente determinato a flessione. Si riportano di seguito i diagrammi di M e di T per la struttura.



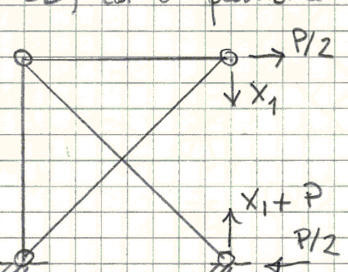


PUNTO 2: dimensionamento.

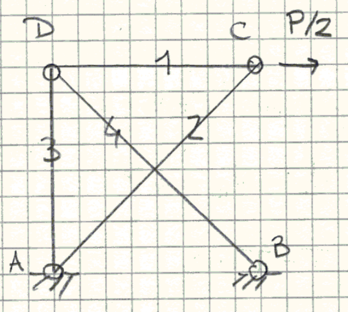
$$W_1 \geq \frac{PL}{\sigma_{AMM}} = \frac{50 \cdot 10^3 \cdot 2}{240 \cdot 10^6} \, 10^6 \; cm^3 = 416.7 \; cm^3 \rightarrow IPE\ 270$$

$W_1 = 429\ cm^3$
$I_1 = 5790\ cm^4$
$A = 46\ cm^2$

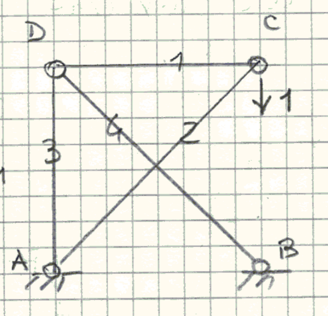
Per disegnare il diagramma di N occorre risolvere la travatura, che risulta una volta iperstatica. Infatti, se N_C è ad esempio noto, allora è possibile calcolare il valore dello sforzo normale in tutte le bielle (AD, CD, AB e BD) ed è possibile calcolare anche N_B (pari a $N_C - P$).



} Queste forze si scaricano sul vincolo in B



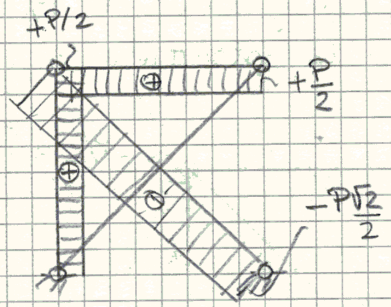
$+ \; X_1$



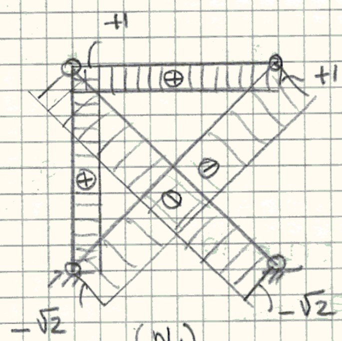
(0) (1)

$\left. \begin{array}{l} M_0 = 0 \\ T_0 = 0 \end{array} \right\}$ reticolare

$\left. \begin{array}{l} M_1 = 0 \\ T_1 = 0 \end{array} \right\}$ reticolare



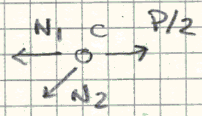
(N_0)



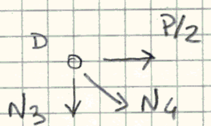
(N_1)

Equilibri ai nodi per il calcolo di N_0 e di N_1:

Sistema (0):



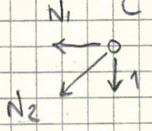
$$\begin{cases} N_2 \frac{\sqrt{2}}{2} = 0 \\ N_1 = P/2 \end{cases}$$



$$\begin{cases} N_4 \frac{\sqrt{2}}{2} = -P/2 \\ N_3 = -N_4 \frac{\sqrt{2}}{2} = P/2 \end{cases}$$

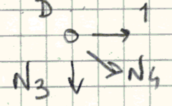
$$\begin{cases} N_3 = P/2 \\ N_4 = -\frac{P}{2}\sqrt{2} \end{cases}$$

Sistema (1):



$$\begin{cases} N_2 \frac{\sqrt{2}}{2} = -1 \\ N_1 = -N_2 \frac{\sqrt{2}}{2} = 1 \end{cases}$$

$$\begin{cases} N_1 = 1 \\ N_2 = -\sqrt{2} \end{cases}$$



$$\begin{cases} N_4 \frac{\sqrt{2}}{2} = -1 \\ N_3 = -N_4 \frac{\sqrt{2}}{2} = 1 \end{cases}$$

$$\begin{cases} N_3 = 1 \\ N_4 = -\sqrt{2} \end{cases}$$

Eq.ne di congruenza: $\eta_{10} + \eta_{11} X_1 = \eta_1$

$\eta_1 = 0$

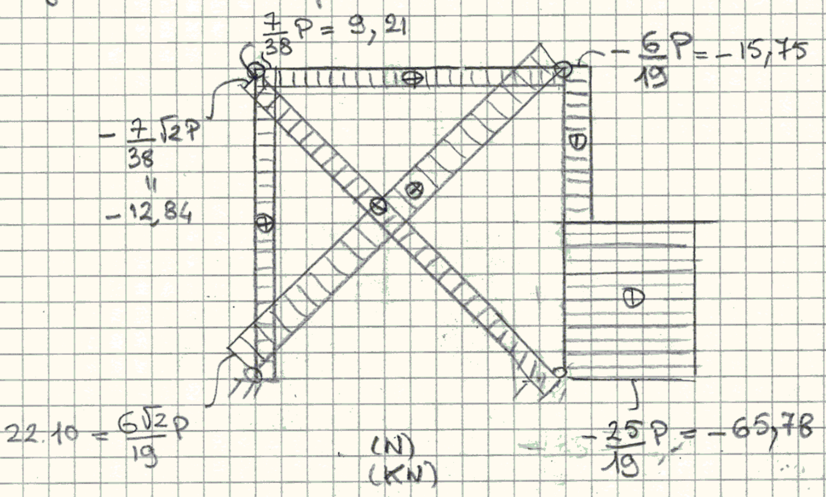
$$\eta_{10} = \int_S \frac{N_1 N_0}{EA} dx_3 = \frac{1}{EA}\left[\frac{P}{2}\cdot 1 \cdot L \cdot 2 + \left(-\frac{P\sqrt{2}}{2}\right)(-\sqrt{2})\, L\sqrt{2}\right] = \frac{PL}{EA}\left[1+\sqrt{2}\right]$$

$$\eta_{11} = \int_S \frac{N_1^2}{EA} dx_3 = \frac{1}{EA}\left[2\cdot 1\cdot L + 2\cdot(\sqrt{2})^2 L\sqrt{2}\right] = \frac{2L}{EA}\left[1+2\sqrt{2}\right]$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{PL}{EA}\frac{(1+\sqrt{2})EA}{2L(1+2\sqrt{2})} = -\frac{(1+\sqrt{2})}{2(1+2\sqrt{2})}P = -\frac{6}{19}P = -15,75 \text{ kN}$$

$\sqrt{2} \simeq \frac{7}{5}$

Diagramma dello sforzo normale



$N_B = X_1 - P = -\frac{25}{19}P = -65,78$ kN

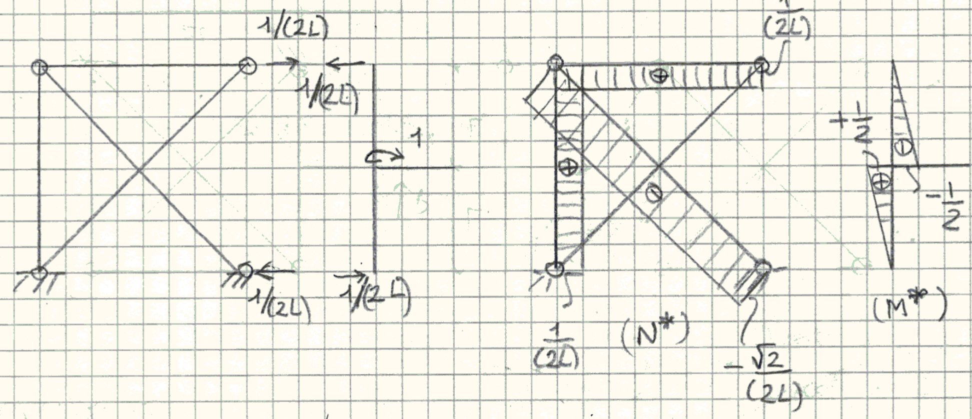
$N_{CD} = \frac{P}{2} - \frac{6}{19}P = \frac{7}{38}P = 9,21$ kN

$N_{AD} = N_{CD}$

$$N_{BD} = -\frac{P\sqrt{2}}{2} + \frac{6}{19}P\sqrt{2} = P\sqrt{2}\left(\frac{6}{19} - \frac{1}{2}\right) = -\frac{7}{38}\sqrt{2}P = -12,84 \text{ kN}$$

$N_{AC} = +\sqrt{2}\,\frac{6}{19}P = 22,10$ kN

PUNTO 3: rotazione in E.



$$1\cdot\varphi_E = 2\cdot\frac{1}{3}\left(-\frac{PL}{2}\right)\left(-\frac{1}{2}\right)\frac{L}{EI_1} + \frac{1}{EA}\left[2\cdot L\,\frac{1}{(2L)}\cdot\frac{7}{38}P + L\sqrt{2}\cdot\left(-\frac{\sqrt{2}}{2L}\right)\left(-\frac{7}{38}\sqrt{2}P\right)\right]$$

$$= \frac{PL^2}{6EI_1} + \frac{PL}{EA}\left[\frac{7}{38}+\frac{7}{38}\sqrt{2}\right] = \frac{PL^2}{6EI_1} + \frac{7}{38}\frac{PL}{EA}\frac{12}{5} = \frac{PL^2}{6EI_1} + \frac{42}{95}\frac{PL}{EA}$$

$\sqrt{2}\simeq\frac{7}{5}$

$$\varphi_E = \frac{50\cdot10^3\cdot4}{6\cdot210\cdot10^9\cdot5790\cdot10^{-8}} + \frac{42\cdot50\cdot10^3\cdot4}{95\cdot210\cdot10^9\cdot46\cdot10^{-4}}$$

$$= \frac{200}{6\cdot21\cdot579} + \frac{42}{95\cdot21\cdot46\cdot10} = 0.00274 + 0.00004 = 0.00278$$

$$= 0{,}16^\circ$$

PUNTO 4: carico termico ($\Delta T = +20°C$)

$$\eta_{1t} = (-\sqrt{2})\,L\sqrt{2}\,\alpha\Delta T = -2L\alpha\Delta T$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1t}}{\eta_{11}} = -\frac{6}{19}P + \frac{2L\alpha\Delta T\,EA}{2L(1+2\sqrt{2})} = -\frac{6}{19}P + \frac{5\alpha\Delta T\,EA}{19}$$

$1+2\sqrt{2} = 1+\frac{14}{5} = \frac{19}{5}$

$$X_1 = \left(-15.75 + \frac{5\cdot10^{-5}\cdot20\cdot210\cdot10^9\cdot46\cdot10^{-4}}{19\cdot10^3}\right)\text{kN}$$

$$= (-15.75 + 50.84)\,\text{kN} = 35{,}09\ \text{kN}$$

$N_B = X_1 - P = -14.91$ kN

$N_{CD} = \frac{P}{2} + X_1 = 60.09$ kN

$N_{AD} = N_{CD}$

$N_{BD} = -\frac{P\sqrt{2}}{2} - \sqrt{2}X_1 = -84.12$ kN

$N_{AC} = -\sqrt{2}X_1 = -49.12$ kN