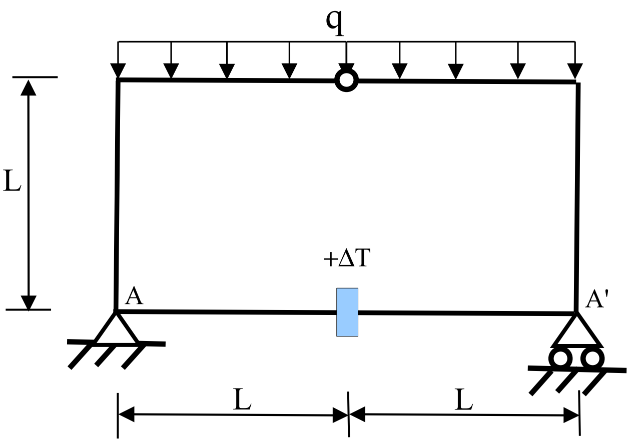
LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 15/02/2013

A



$L = 1\text{m}$, $q = 2$ t/m
$\sigma_{AMM} = 240$ MPa , $E = 210$ GPa
$\Delta T = +20$ °C, $\alpha = 10^{-5}$ °C^{-1}

Si consideri la travatura iperstatica di figura.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del solo carico q. Disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M).
2. Dimensionare la travatura con profilati IPE.
3. Calcolare lo spostamento verticale della cerniera.
4. Risolvere nuovamente la travatura considerando anche un riscaldamento uniforme del tratto AA': disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M) comprensivi sia di q che di ΔT.

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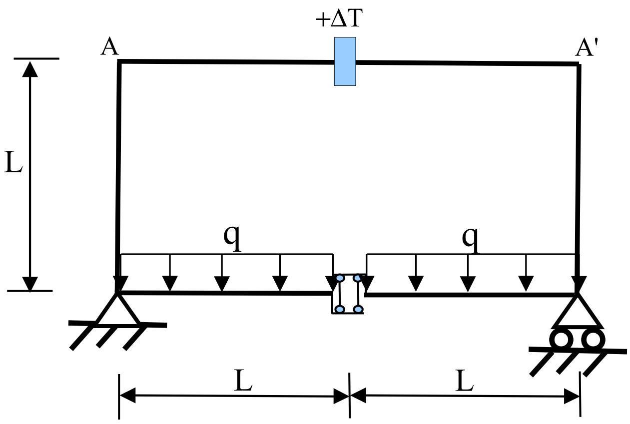
B



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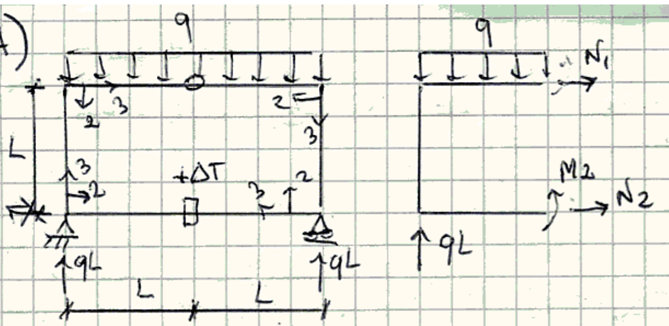
A)



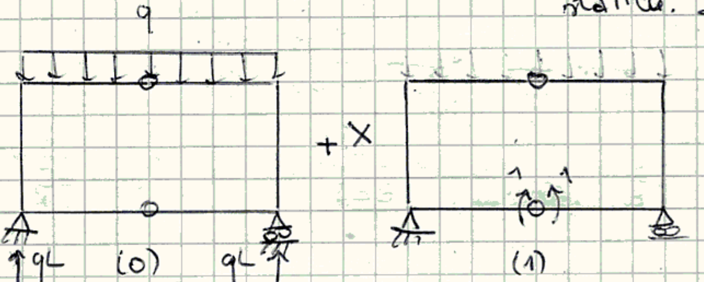
Esternamente la struttura è isostatica. Internamente si ha che:

$(\rightarrow)\quad N_1 + N_2 = 0$

$(\uparrow)\quad qL - qL = 0$

$(\circlearrowleft)\quad M_2 + N_2 L - q\frac{L^2}{2} = 0$

Internamente la travatura è una volta iperstatica. Incognita iperstatica $X = M_2$.



Diagrammi delle c.s.

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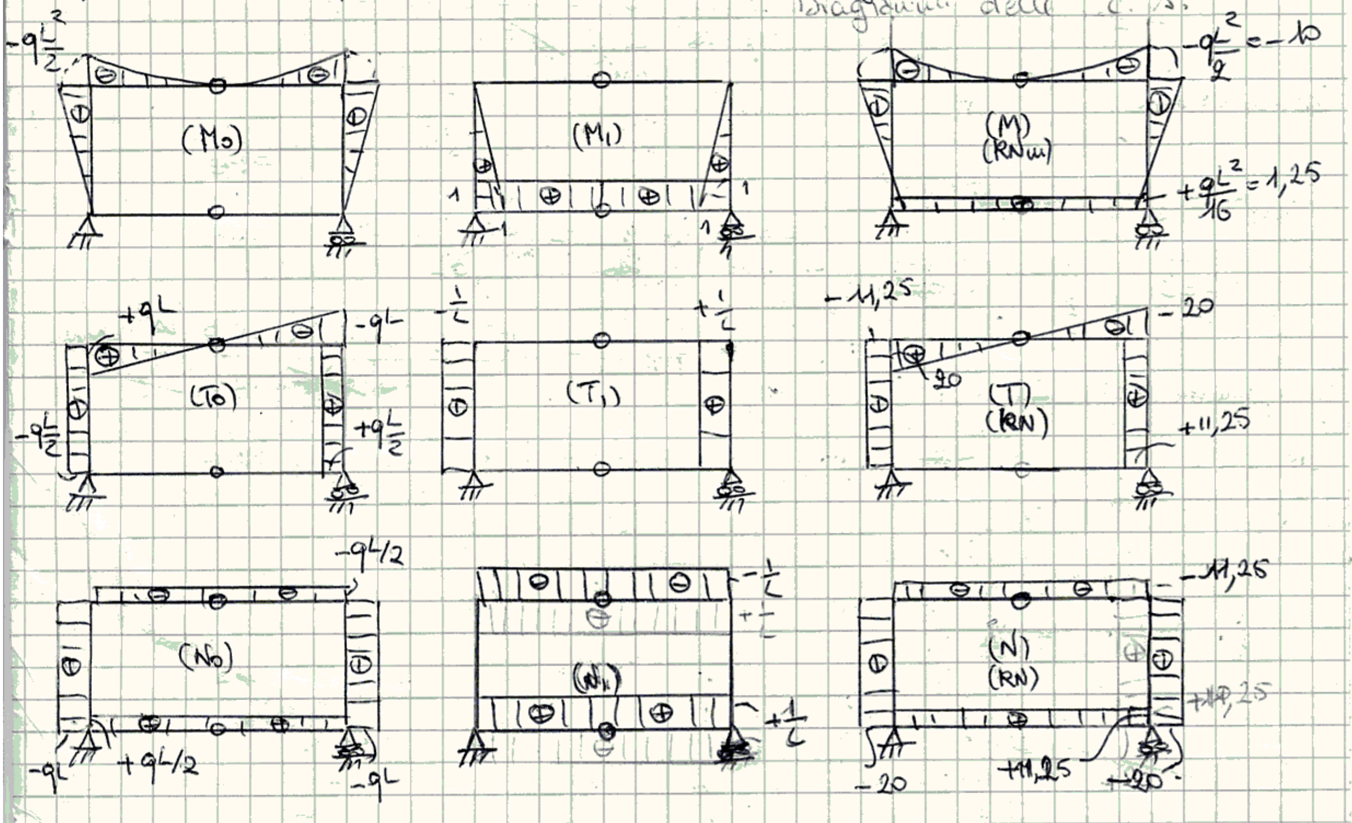


$$EI_1\eta_{10} = 2\cdot\frac{1}{6}L\left(-q\frac{L^2}{2}\right) = -q\frac{L^3}{6}$$

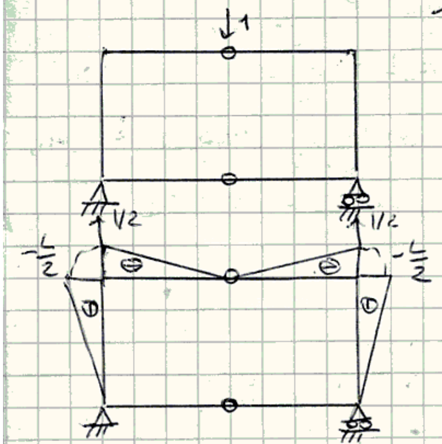
$$EI_1\eta_{11} = 2\cdot\frac{1}{3}L + 2L = \frac{8}{3}L$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{qL^3}{6}\cdot\frac{3}{8L} = \frac{qL^2}{16} = 1,25\ \text{kN m}$$

Dimensionamento:

$$W_1 \geq \frac{qL^2}{2}\frac{1}{\sigma_{AMM}} = \frac{10\cdot 10^3}{240}\ cm^3 = 41,7\ cm^3 \qquad \text{IPE 120} \quad \begin{cases} A = 13,21\ cm^2 \\ I_1 = 317,8\ cm^4 \end{cases}$$

Spostamento verticale:



$$1\cdot v = \frac{\cancel{2}}{EI_1}\left\{\int_0^L \left(-\frac{x}{\cancel{2}}\right)\left(-q\frac{x^2}{2}\right)dx + \int_0^L\left(-\frac{x}{\cancel{2}}\right)\left(q\frac{L^2}{16} - \frac{9}{16}qLx\right)dx\right\}$$

$$= \frac{1}{EI_1}\left\{\int_0^L \frac{q}{2}x^3\,dx + \int_0^L \frac{qL}{16}\left(9x^2 - Lx\right)dx\right\}$$

$$= \frac{1}{EI_1}\left\{q\frac{L^4}{8} + \frac{qL}{16}\left[\cancel{9}^{\,3}\frac{L^3}{\cancel{3}} - \frac{L^3}{2}\right]\right\}$$

$$= \frac{1}{EI_1}\left\{q\frac{L^4}{8} + \frac{qL^4}{16}\frac{5}{2}\right\} = \frac{qL^4}{8EI_1}\left(1 + \frac{5}{4}\right) = \frac{9}{32}\frac{qL^4}{EI_1}$$

$$= \frac{9\cdot 2\cancel{0}\cdot 10^3\cdot 10^{\cancel{3}\,2}}{32\cdot 21\cancel{0}\cdot \cancel{10^8}\cdot 317,8\cdot \cancel{10^{-8}}} = 8,42\ mm$$

Carico termico:

$$\eta_{1t} = \cancel{2h}\frac{1}{\cancel{h}}\varepsilon_t = 2\alpha\Delta T$$

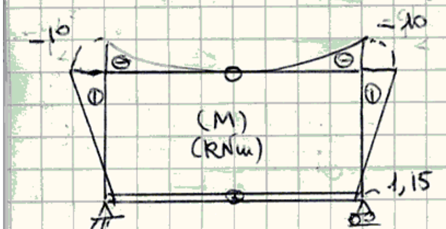
$$\eta_{11} = \frac{8L}{3EI_1}, \qquad \eta_{10} = -\frac{qL^3}{6EI_1}$$

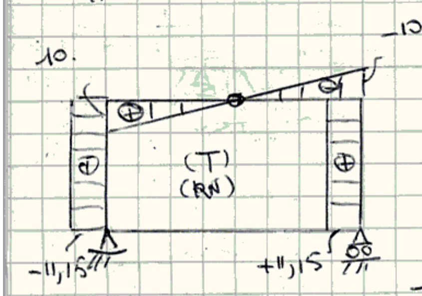
$$\rightarrow X_1 = \frac{qL^2}{16} - \frac{\cancel{2}\alpha\Delta T\,3EI_1}{{}_4\cancel{8}L}$$

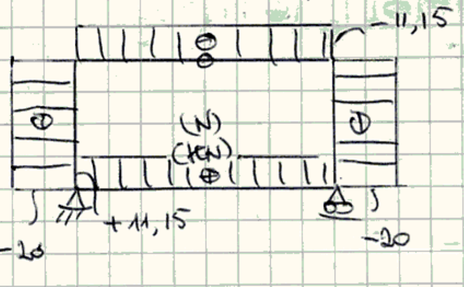
$$= \left(1,25 - \frac{10^{-5}\cdot 2\cancel{0}\cdot 3\cdot 21\cancel{0}\cdot 10^{\cancel{8}}\cdot 317,8\cdot \cancel{10^{-8}}}{4\cdot 10^{\cancel{3}\,2}}\right) kNm$$

$$= (1,25 - 0,1)\ kNm = 1,15\ kNm$$

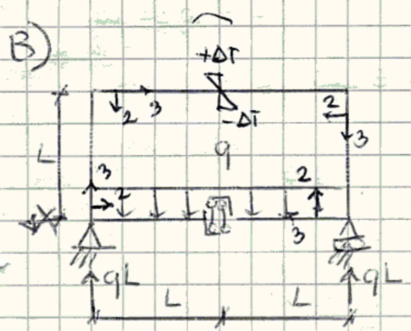
Diagrammi (q e ΔT):

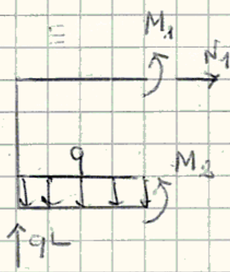






B)





Esternamente la struttura è isostatica. Internamente si ha che:

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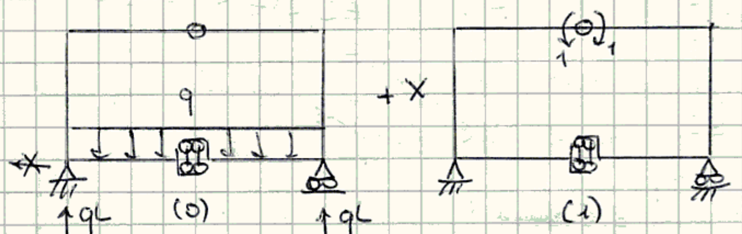
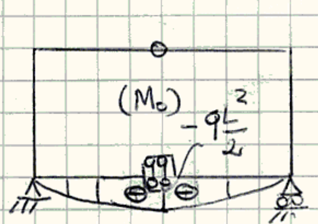
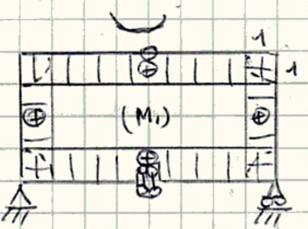
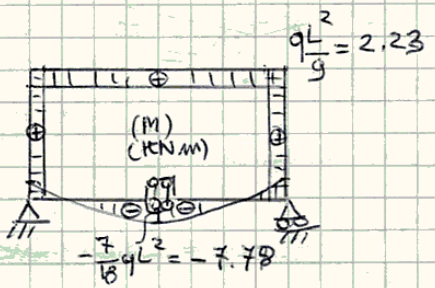
Internamente la struttura è una volta iperstatica. Incognita iperstatica $X_1 = M_1$.

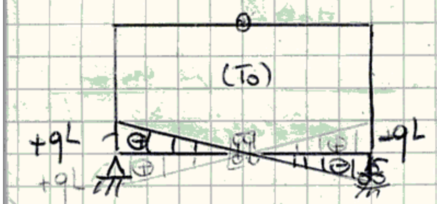


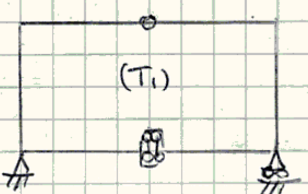
Diagrammi delle c. s.

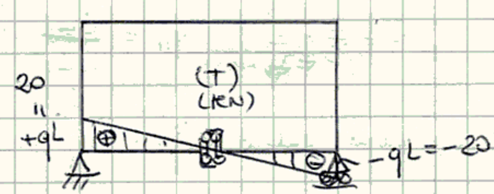


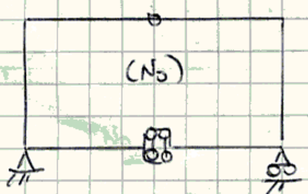


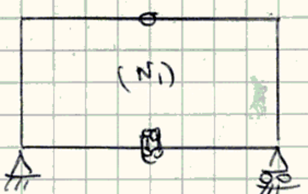


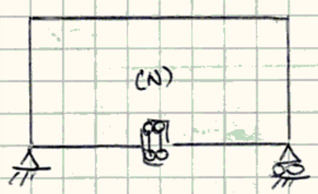












$$EI_1\eta_{10} = -\cancel{2}\left[q\frac{L^3}{\cancel{2}} - \frac{1}{3}L\,q\frac{L^2}{\cancel{2}}\right] = -\frac{2}{3}qL^3$$

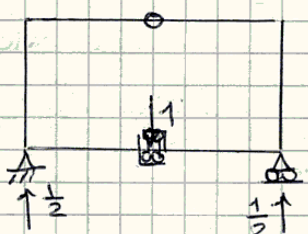
$$EI_1\eta_{11} = 6L$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{\cancel{2}}{3}qL^{\cancel{3}\,2}\frac{1}{\cancel{6}_3\cancel{L}} = \frac{qL^2}{9} = 2.23\ \text{kNm}$$

Dimensionamento:

$$W_1 \geq \frac{7}{18} q L^2 \frac{1}{\sigma_{AMM}} = \frac{7.78}{240} \cdot 10^3 \ cm^3 = 32,42 \ cm^3 \qquad \text{IPE } 100 \left\{ \begin{array}{l} A = 10,32 \ cm^2 \\ I_1 = 171 \ cm^4 \\ H = 100 \ mm \end{array} \right.$$

Spostamento verticale:

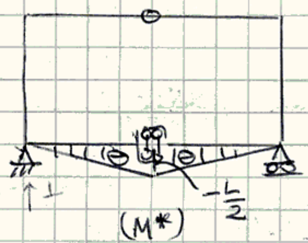


$$1 \cdot v = \frac{\cancel{2}}{EI_1} \int_0^L \left(q\frac{L^2}{9} - qLx + q\frac{x^2}{2}\right)\left(-\frac{x}{\cancel{2}}\right) dx$$

$$= \frac{1}{EI_1} \int_0^L \left(-q\frac{L^2}{9}x + qLx^2 - q\frac{x^3}{2}\right) dx = \frac{qL^4}{EI_1}\left[-\frac{1}{9}\frac{1}{2} + \frac{1}{3} - \frac{1}{8}\right]$$

$$= \frac{11}{72}\frac{qL^4}{EI_1} = \frac{11 \cdot 20 \cdot 10^3 \cdot \cancel{10^8}^{\,2}}{72 \cdot 210 \cdot \cancel{10^8} \cdot 171 \cdot \cancel{10^{-8}}} \ mm$$

$$= 8,5 \ mm$$



Carico termico:

$$\eta_{1t} = 2L\chi_t = -2L\left(\frac{2\Delta T\alpha}{H}\right) = -4\,\Delta T\frac{L\alpha}{H} \Rightarrow X_1 = q\frac{L^2}{9} + \frac{\cancel{4}^{\,2}\Delta T \cancel{L}\alpha E I_1}{H\,3\,\cancel{8}\cancel{L}}$$

$$\eta_{10} = -\frac{2}{3}\frac{qL^3}{EI_1}, \quad \eta_{11} = \frac{6L}{EI_1}$$

$$= \left(2.23 + \frac{2 \cdot 20 \cdot 10^{-5} \cdot 210 \cdot 10^{6} \cdot 171 \cdot 10^{-8}}{3 \cdot 0,1 \cdot 10^3 \cdot 2}\right) kN\,m$$

$$= (2.23 + 0,47) \ kNm$$

$$= 2,7 \ kNm$$

Diagrammi delle c.s. comprensivi di q e del carico termico: