

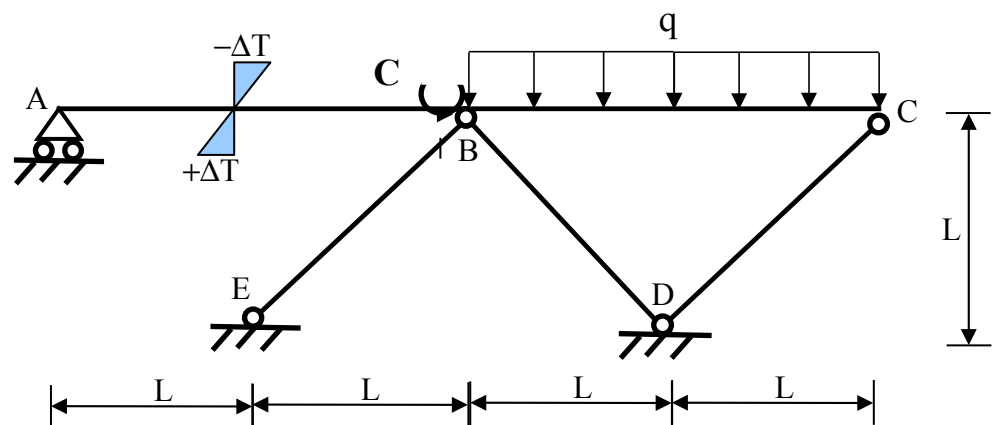
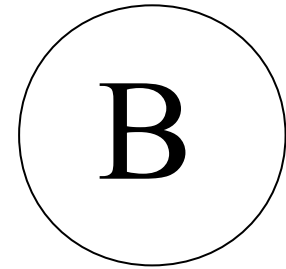
$$L = 2 \text{ m}, q = 20 \text{ kN/m}, C = 80 \text{ kNm}$$

$$\sigma_{\text{AMM}} = 240 \text{ MPa}, E = 210 \text{ GPa}$$

$$\Delta T = 20 \text{ }^{\circ}\text{C}, \alpha = 10^{-5} \text{ }^{\circ}\text{C}^{-1}$$

La trave ABC in figura deve essere realizzata con profilati IPE, mentre le bielle sono da realizzare con tondi in acciaio.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in presenza dei carichi q e C . In questa fase è consentito trascurare la deformabilità assiale delle bielle.
- Dimensionare la trave e le bielle.
- Calcolare la rotazione nel punto B.
- Disegnare nuovamente i diagrammi quotati considerando, in aggiunta ai carichi C e q , anche il carico termico sul tratto AB.

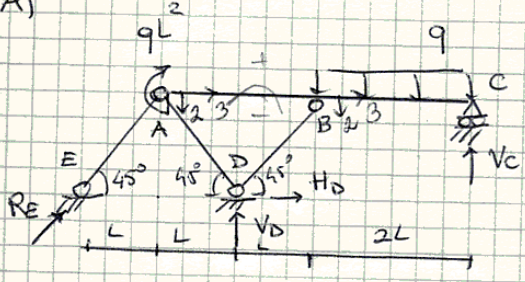


$$L = 1 \text{ m}, q = 25 \text{ kN/m}, C = 25 \text{ kNm}$$
$$\sigma_{\text{AMM}} = 240 \text{ MPa}, E = 210 \text{ GPa}$$
$$\Delta T = 20 \text{ }^{\circ}\text{C}, \alpha = 10^{-5} \text{ }^{\circ}\text{C}^{-1}$$

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A)



Analisi statica:

Tratto ABC:

$$(\rightarrow) -N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} - N_3 \frac{\sqrt{2}}{2} = 0$$

$$(\uparrow) -N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{2}}{2} - N_3 \frac{\sqrt{2}}{2} + V_c = 2qL$$

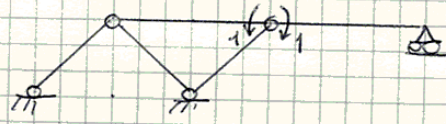
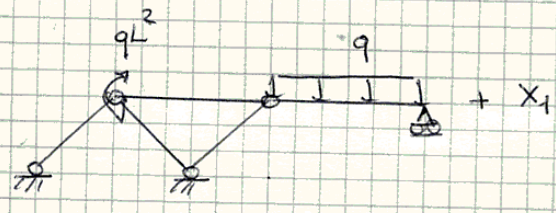
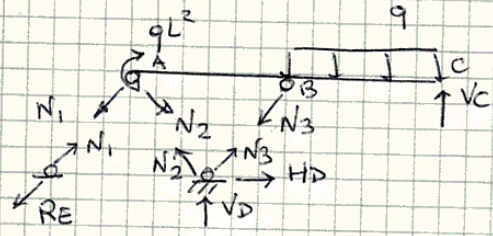
$$(A) -N_3 \frac{\sqrt{2}}{2} 2L + V_c 4L - qL^2 - 2qL 3L = 0$$

Incognite: N_1, N_2, N_3 e V_c (4)

Equazioni: 3

La truss è 1 volta iperstatica

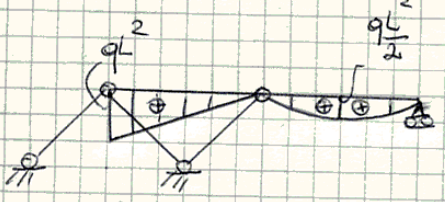
Incognita iperstatica: $X_1 = M_B$



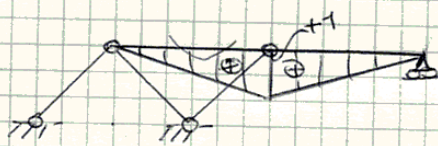
(0)

(1)

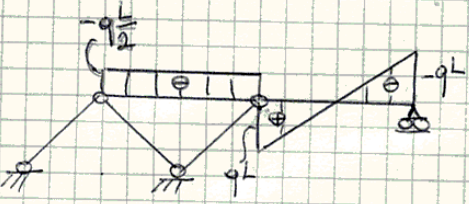
(M0)



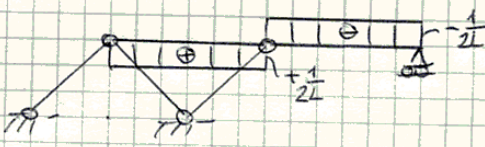
(M1)



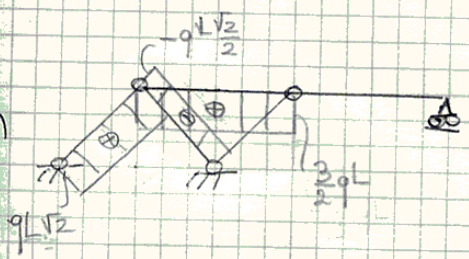
(Q0)



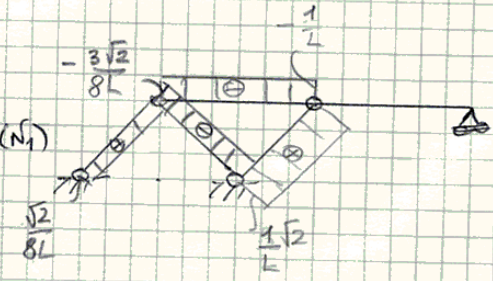
(Q1)



(N1)



(N1)



$$\begin{cases} N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} = q \frac{L}{2} \\ N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{2}}{2} + N_3 \frac{\sqrt{2}}{2} = -\frac{3}{2} qL \end{cases} \Rightarrow \begin{cases} N_1 = -N_3 \frac{\sqrt{2}}{2} = \frac{3}{2} qL \\ N_3 \frac{\sqrt{2}}{2} = -\frac{3}{2} qL \end{cases}$$

$$\begin{cases} -N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} = +\frac{q}{2L} \\ -N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{2}}{2} = -\frac{1}{2L} \end{cases}$$

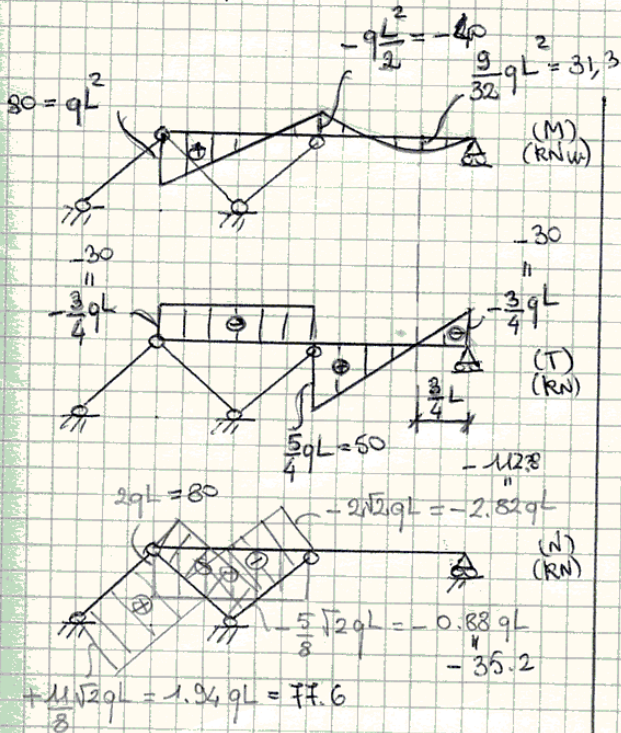
$$\begin{cases} N_3 \frac{\sqrt{2}}{2} = \frac{3}{2} \sqrt{2} \\ N = -N_3 \frac{\sqrt{2}}{2} = -\frac{3}{2L} \end{cases} \quad \text{A}$$

$$EI_1 \eta_{10} = \frac{1}{8} qL^2 + \frac{q(2L)^3}{24} = qL^3 \left(\frac{1}{8} + \frac{1}{3} \right) = \frac{8}{3} qL^3$$

$$EI_1 \eta_{11} = \frac{2 \cdot 2L}{3} = \frac{4L}{3}$$

$$x_1 = - \frac{\eta_{10}}{\eta_{11}} = - \frac{\frac{8}{3} qL^3}{\frac{4L}{3}} = - \frac{2qL^2}{1} = - \frac{20 \cdot 4}{2} \text{ kNm} = -40 \text{ kNm}$$

Diagrammi quotati:



Calcoli:

$$-qL/2 - \frac{1}{2L} qL^2 = -\frac{3}{4} qL$$

$$qL - \frac{1}{2L} (-qL^2) = \frac{5}{4} qL$$

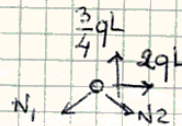
$$-qL - \frac{1}{2L} (-qL^2) = -qL + \frac{qL}{4} = -\frac{3}{4} qL$$

$$\bar{M} \uparrow \downarrow \downarrow \downarrow \downarrow \uparrow \quad \bar{M} = \frac{9}{32} qL^2$$

$$N_3 \frac{\sqrt{2}}{2} = -\frac{8}{4} qL = -2qL$$

$$\rightarrow N_3 = -2\sqrt{2}qL$$

$$N = -N_3 \frac{\sqrt{2}}{2} = 2qL$$



$$-N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} = -2qL$$

$$N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} = \frac{3}{4} qL$$

$$\rightarrow N_2 \sqrt{2} = -\frac{5}{4} qL$$

$$\rightarrow N_2 = -\frac{5\sqrt{2}}{8} qL$$

$$N_1 \frac{\sqrt{2}}{2} = \frac{3}{4} qL + \frac{5\sqrt{2}}{8} qL \frac{\sqrt{2}}{2}$$

$$= \frac{11}{8} qL$$

$$\rightarrow N_1 = \frac{11\sqrt{2}}{8} qL$$

Dimensionamento:

• Trave (a flessione) $W_x \geq \frac{qL^2}{8MM} = \frac{20 \cdot 10^3 \cdot 4 \cdot 10^4}{240 \cdot 10^6} \text{ cm}^3 = 333 \text{ cm}^3$

• Trave (a trazione) $A \geq \frac{2\sqrt{2}qL}{8MM} = \frac{2 \cdot \sqrt{2} \cdot 20 \cdot 10^3 \cdot 2 \cdot 10^4}{240 \cdot 10^6} \text{ cm}^2 = 4,71 \text{ cm}^2 = 471 \text{ mm}^2$

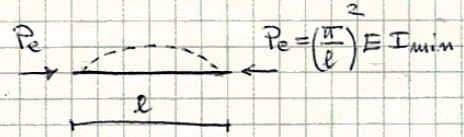
Trave IPE 270 $\left\{ \begin{array}{l} W_x = 428,9 \text{ cm}^3 \\ I_x = 5790 \text{ cm}^4 \\ A = 45,95 \text{ cm}^2 \\ H = 270 \text{ mm} \end{array} \right.$

Trave $\phi 25$ ($A = 490,87 \text{ mm}^2$)

Trave $R \geq \sqrt{\frac{471}{\pi}} = 12,24 \text{ mm}$

Entonces, considerando el caso crítico elevado, se ha de

$$2\sqrt{2}qL \leq \underbrace{\left(\frac{\pi}{2LV^2}\right)^2 EI_{\min}}_{P_e} = \frac{\pi^2}{8L^2} E \frac{\pi R^4}{4} = \frac{\pi^3 ER^4}{32L^2}$$

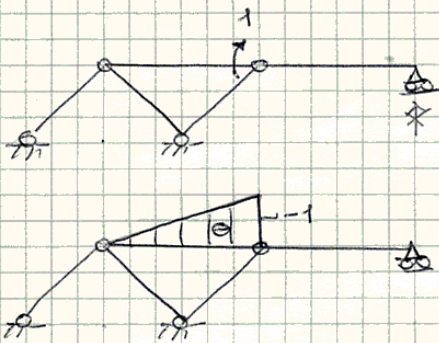


o sea

$$R \geq \sqrt[4]{\frac{64\sqrt{2}qL^3}{\pi^3 E}} = \sqrt[4]{\frac{64\sqrt{2} \cdot 20 \cdot 10^3 \cdot 8}{\pi^3 \cdot 210 \cdot 10^9}} \quad m = 38,62 \text{ mm} \quad (x_2 = 77,23 \text{ mm})$$

Por tanto se le hace el diámetro de 78 mm como resultado: $\phi 78$

Rotación en B:



$$\begin{aligned} 1 \cdot \varphi_B &= \frac{1}{EI_1} \int_0^{2L} \left(-\frac{x}{2L}\right) \left(qL^2 - \frac{3}{4}qLx\right) dx \\ &= \frac{1}{2EI_1} \int_0^{2L} \left(\frac{3}{4}qLx^2 - qL^2x\right) dx \\ &= \frac{1}{2EI_1} \left[\frac{3}{4}qL \frac{1}{3} 8L^3 - qL^2 \frac{1}{2} 4L^2\right] = 0 \end{aligned}$$

Carga térmica en AB:

$$M_{10} + M_{1e} + \eta_{11} X_1 = 0$$

$$\eta_{10} = \frac{2qL^3}{3EI_1}$$

$$\eta_{11} = \frac{4L}{3EI_1}$$

$$M_{1e} = -X_1 \cdot 2L \cdot \frac{1}{2} = -2\alpha\Delta T \frac{L}{H}$$

$$X_1 = \frac{2\alpha\Delta T}{H}$$

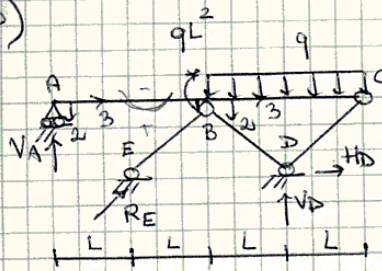
$$X_1 = -\frac{qL^2}{2} + \frac{2\alpha\Delta T L}{H} \frac{3EI_1}{2L}$$

$$= \left(-40 + \frac{10^{-5} \cdot 20 \cdot 3 \cdot 210 \cdot 10^9 \cdot 5730 \cdot 10^{-8} \cdot 10^{-3}}{2 \cdot 27 \cdot 10^2}\right) \text{ KNm}$$

$$= (-40 + 13,51) \text{ KNm} = -26,49 \text{ KNm}$$

Ocurren redistribuir los diagramas
quodati.

B)



Equilibrio del tratto ABC:

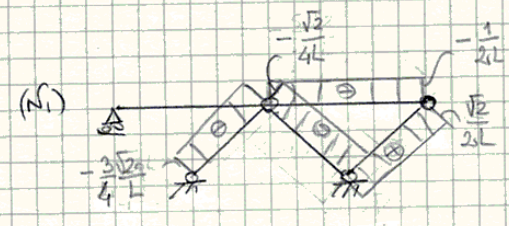
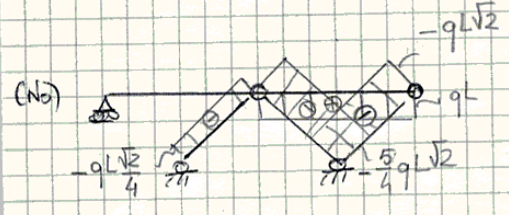
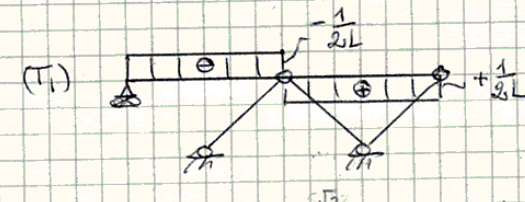
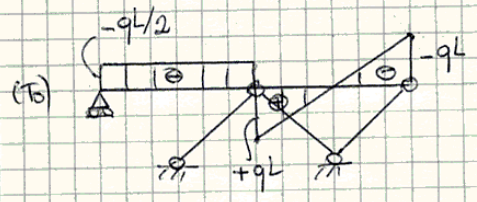
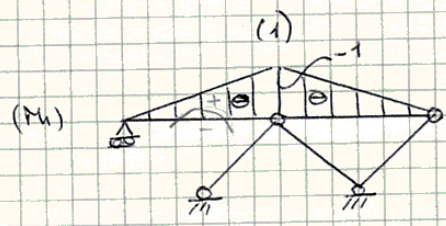
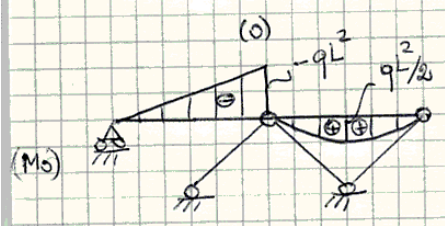
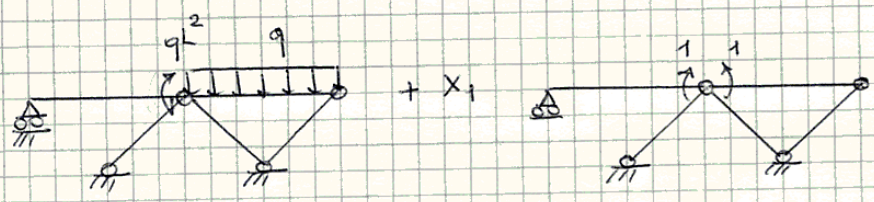
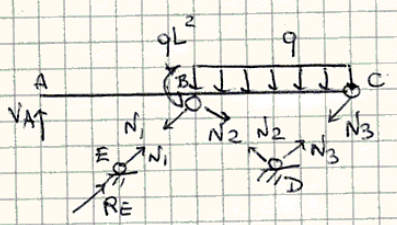
$$\begin{aligned} (-\rightarrow) -N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} - N_3 \frac{\sqrt{2}}{2} &= 0 \\ (\uparrow) N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} + N_3 \frac{\sqrt{2}}{2} + 2qL - V_A &= 0 \\ (B') V_A 2L - N_3 \frac{\sqrt{2}}{2} 2L + qL^2 - 2qL^2 &= 0 \end{aligned}$$

Incongnite: N_1, N_2, N_3 e V_A

Eq. in 3.

La trussatura è una beta iperstatica

Incongnita iperstatica $X_1 = M_B^+$



$$\begin{aligned} N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{2}}{2} &= qL \\ N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} &= -qL \frac{3}{2} \\ N_1 \frac{\sqrt{2}}{2} &= -qL \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{qL\sqrt{2}}{4} \\ \sqrt{2} N_2 &= -3qL + qL \frac{\sqrt{2}}{2} = -\frac{5qL\sqrt{2}}{4} \\ N_2 &= -\frac{5qL}{4} \end{aligned}$$

$$\begin{aligned} +N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{2}}{2} &= -\frac{1}{2L} \\ N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} &= \frac{1}{L} \\ N_1 \frac{\sqrt{2}}{2} &= -\frac{3}{2L\sqrt{2}} = -\frac{3\sqrt{2}}{4L} \\ N_2 \frac{\sqrt{2}}{2} &= \frac{1}{-L} + \frac{\sqrt{3}\sqrt{2}}{2 \cdot 2L} = -\frac{\sqrt{2}}{4L} \end{aligned}$$

$$\begin{aligned} \frac{1}{2L} \uparrow &= C \\ \frac{1}{\sqrt{2}} \leftarrow &= N_3 \\ N_3 \frac{\sqrt{2}}{2} &= \frac{1}{2L} \\ \rightarrow N_3 &= \frac{\sqrt{2}}{2L} \\ N &= -N_3 \frac{\sqrt{2}}{2} = -\frac{1}{2L} \end{aligned}$$

(1) B

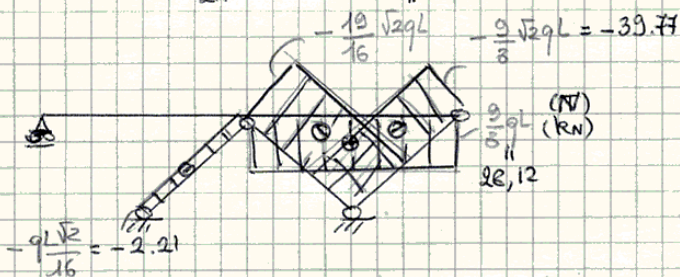
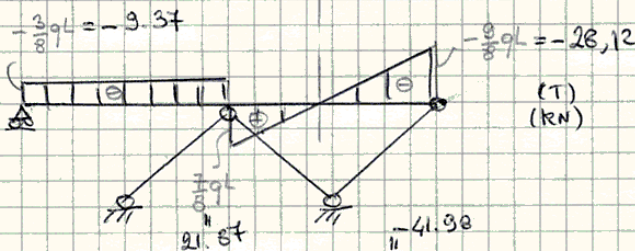
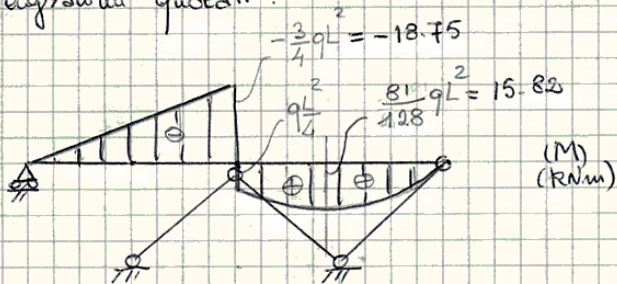
$$EI_1 M_{10} = \frac{1}{3} 2L (-qL^2) (-1) - \frac{1}{24} q (2L)^3$$

$$= \frac{2}{3} qL^3 - \frac{1}{3} qL^3 = \frac{1}{3} qL^3$$

$$EI_1 M_{11} = \frac{2}{3} 2L (-1) (-1) = \frac{4}{3} L$$

$$X_1 = - \frac{M_{10}}{M_{11}} = - \frac{1}{3} qL^3 \frac{3}{4L} = - \frac{qL^2}{4} = -6,25 \text{ KNm}$$

Diagrammi quotati:



Calcoli:

$$\bullet -q \frac{L^2}{2} + \frac{1}{24} q L^2 = -\frac{3}{8} qL$$

$$\bullet qL - q \frac{L}{8} = \frac{7}{8} qL$$

$$\bullet -qL - q \frac{L}{8} = -\frac{9}{8} qL$$

$$\bullet -qL \frac{\sqrt{2}}{4} + \frac{3}{4} \frac{\sqrt{2}}{2} q \frac{L}{4} = -\frac{1}{4} qL \frac{\sqrt{2}}{4} = -\frac{qL\sqrt{2}}{16} \checkmark$$

$$\bullet -\frac{5}{4} qL \frac{\sqrt{2}}{2} + q \frac{L}{4} \frac{\sqrt{2}}{4} = -\frac{19}{16} qL \frac{\sqrt{2}}{2} \checkmark$$

$$\bullet -qL \frac{\sqrt{2}}{2} - q \frac{L}{4} \frac{\sqrt{2}}{2} = -\frac{9}{8} qL \frac{\sqrt{2}}{2}$$

$$\bullet qL + q \frac{L}{4} \frac{1}{2L} = \frac{9}{8} qL$$

$$\frac{9}{8} qL \left(\frac{9}{8} qL \right) \rightarrow \left(\frac{9}{8} qL \right)^2 \rightarrow \left(\frac{9}{8} qL \right) \rightarrow \left(\frac{9}{8} qL \right)$$

$$\frac{9}{8} qL \left\{ \begin{array}{l} N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{2}}{2} = +\frac{9}{8} qL \\ N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{2}}{2} = \frac{19}{8} qL \end{array} \right.$$

$$\left. \begin{array}{l} N_1 = -\frac{qL\sqrt{2}}{16} \checkmark \\ N_2 = -\frac{19}{16} \sqrt{2} qL \checkmark \end{array} \right\}$$

Dimensionamento:

• trave $W_x \geq \frac{3/4 qL^2}{6AM} = \frac{3 \cdot 25 \cdot 10^3}{4 \cdot 240 \cdot 10^8} \cdot 10^6 \text{ cm}^3 = 78,12 \text{ cm}^3$ IPE 160 $\left\{ \begin{array}{l} W_1 = 108,7 \text{ cm}^3 \\ A = 20,08 \text{ cm}^2 \\ I_1 = 869,3 \text{ cm}^4 \\ H = 160 \text{ mm} \end{array} \right.$

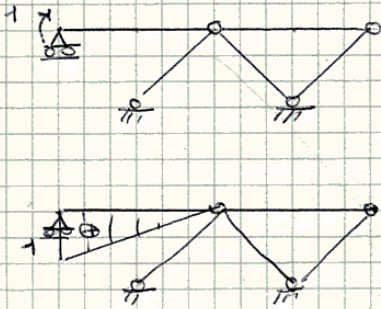
• brella $A \geq \frac{19 \sqrt{2} qL}{16 \cdot 6AM} = \frac{19 \sqrt{2} \cdot 25 \cdot 10^3}{16 \cdot 240 \cdot 10^8} \cdot 10^6 \text{ cm}^2 = 1,75 \text{ cm}^2 = 175 \text{ mm}^2 \rightarrow \phi 15$
 $(A = 201,08 \text{ mm}^2)$

Verifica a carico di punta (vedi fine A):

$$\frac{19 \sqrt{2} qL}{16} \leq \left(\frac{\pi}{2L\sqrt{2}} \right)^2 E \frac{\pi R^4}{4} = \frac{\pi^3}{32} \frac{ER^4}{L^2}$$

$$\hookrightarrow R \geq \sqrt[4]{\frac{32 \cdot 19 \sqrt{2} qL^3}{\pi^3 E}} = \sqrt[4]{\frac{38 \sqrt{2} qL^3}{\pi^3 E}} = \sqrt[4]{\frac{38 \sqrt{2} \cdot 25 \cdot 10^3}{\pi^3 \cdot 210 \cdot 10^9}} \cdot 10^3 \text{ mm} = 44 \text{ mm} \rightarrow \phi 44$$

Rotazione in A:



$$1 \cdot \varphi_A = \frac{1}{EI_1} \cdot 2L \cdot \frac{1}{8} \cdot 1 \cdot \left(-\frac{q}{4} L^2\right) = -\frac{qL^3}{4EI_1}$$

$$= -\frac{25 \cdot 10^3}{4 \cdot 210 \cdot 10^8 \cdot 869,3 \cdot 10^{-8}}$$

$$= -0,0034$$

$$= -0,19^\circ$$

Carico termico

$$M_{10} + M_{1t} + M_{11} X_1 = 0$$

$$M_{10} = \frac{qL^3}{3EI_1}$$

$$M_{11} = \frac{qL}{3EI_1}$$

$$M_{1t} = -\frac{2\alpha\Delta T}{H} \cdot 1 \cdot \frac{L}{2} = -\frac{2\alpha\Delta TL}{H}$$

$$X_1 = -\frac{M_{10}}{M_{11}} - \frac{M_{1t}}{M_{11}} = +\frac{2\alpha\Delta TL}{4} \cdot \frac{3EI_1}{4} - \frac{qL^3}{4}$$

$$= \left(\frac{2 \cdot 10^{-5} \cdot 20 \cdot 3 \cdot 210 \cdot 10^8 \cdot 869,3 \cdot 10^{-8} \cdot 10^{-3} - 6,25}{4 \cdot 0,16} \right) \text{ kNm}$$

$$= (3,42 - 6,25) \text{ kNm} = -2,82 \text{ kNm} \rightarrow \text{i diagrammi sono da rifare}$$