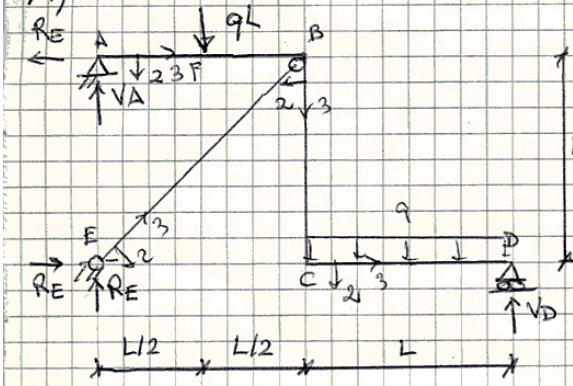
A)



Eq. ni cardinali della Statica:

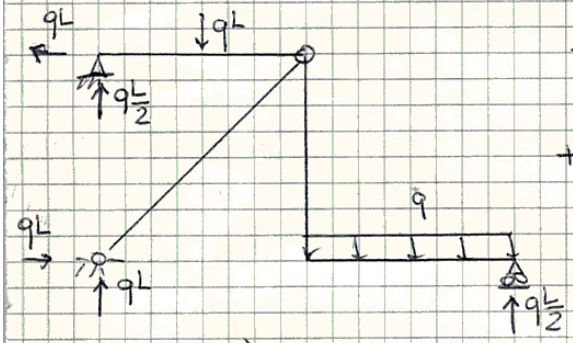
$(\rightarrow)\ R_E - R_E = 0$

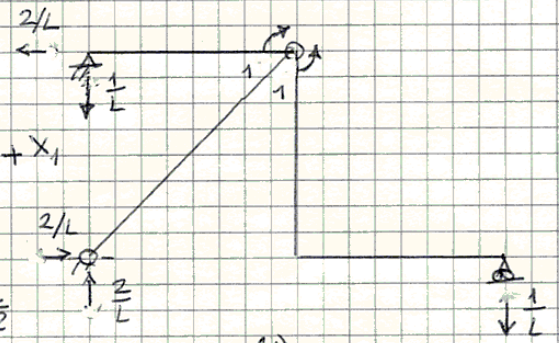
$(\uparrow)\ V_A + R_E + V_D = 2qL$

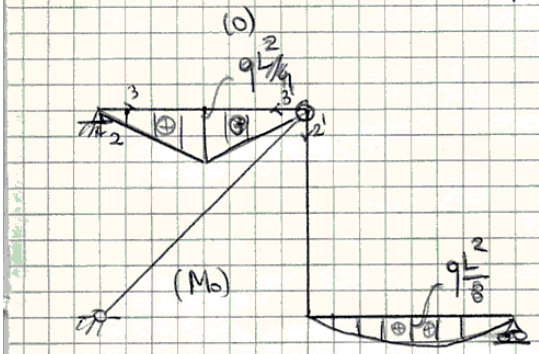
$(\curvearrowleft B)\ V_D L - q\frac{L^2}{2} + qL\frac{L}{3} - V_A L = 0$

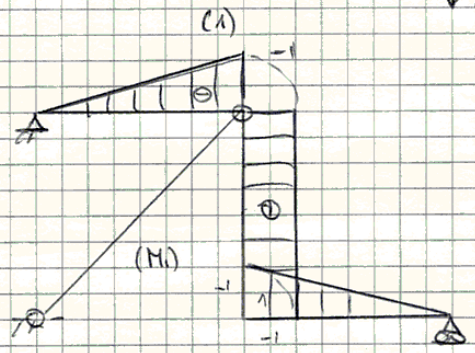
La trave è 1 volta iperstatica

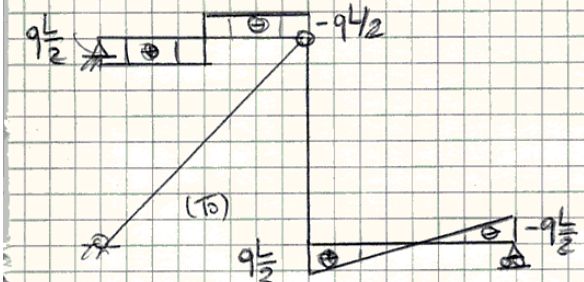
Incognita iperstatica $X_1 = M_B$

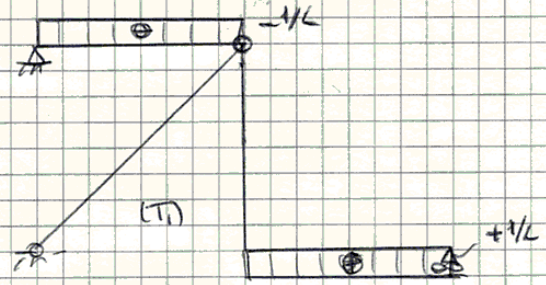


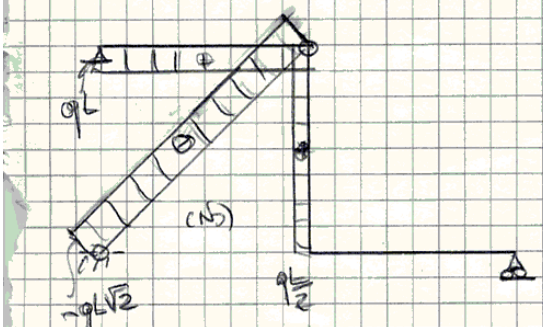


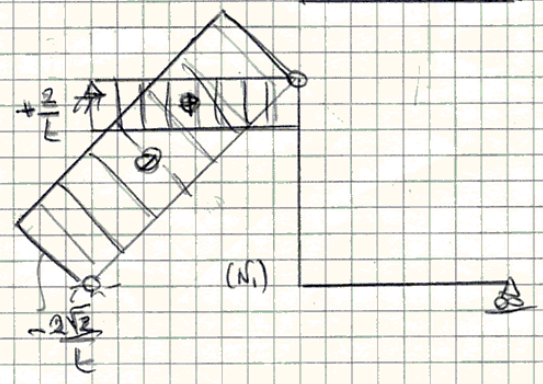












$$EI_1 \eta_{10} = \int_0^{L/2} \left(-\frac{x_3}{L}\right)\left(q\frac{L}{2}x_3\right)dx_3 + \int_0^{L/2} \left(-1+\frac{x_3'}{L}\right)\left(q\frac{L}{2}x_{3'}\right)dx_{3'} - \frac{qL^3}{24}$$

$$= -\frac{q}{2}\left[\frac{x_3^3}{3}\right]_0^{L/2} + q\frac{L}{2}\left[-\frac{x_{3'}^2}{2L} + \frac{x_{3'}^3}{3L}\right]_0^{L/2} - \frac{qL^3}{24} = -\frac{qL^3}{48} - \frac{qL^3}{16} + \frac{qL^3}{48} - \frac{qL^3}{24} = -\frac{qL^3}{4}\left(\frac{1}{4}+\frac{1}{6}\right)$$

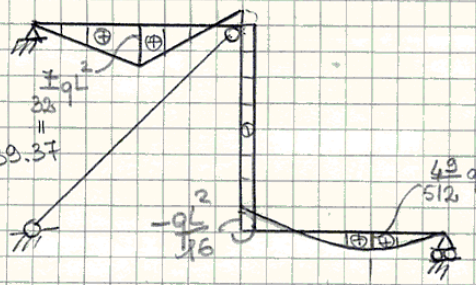
$$= -\frac{5}{48}qL^3$$

$$EI_1 \eta_{11} = \frac{2}{3}L + L = \frac{5}{3}L$$

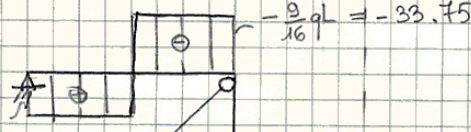
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{5}{48}qL^3\frac{3}{5L} = \frac{qL^2}{16} = +11.25 \text{ kNm}$$

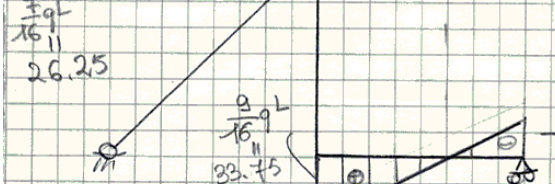
Diagrammi quotati:

$-qL^2/16 = -11.25$

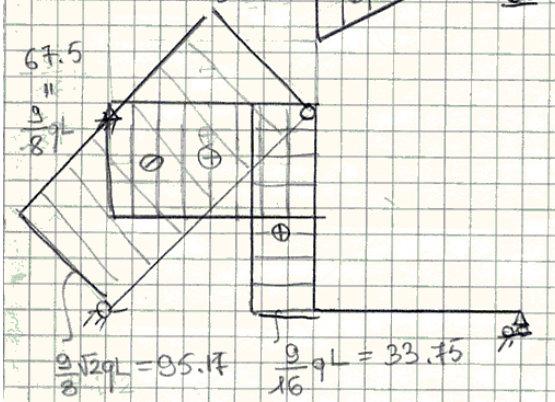


(M) (kNm)





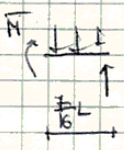
(T) (kN)



(N) (kN)

Calcoli:

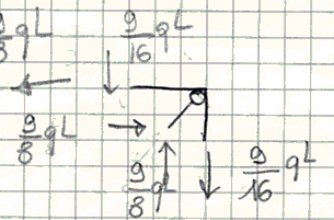
- $\frac{qL^2}{4} - \frac{qL^2}{32} = \frac{7}{32}qL^2$
- $q\frac{L}{2}\cdot\frac{6}{8} - q\frac{L}{16} = \frac{7}{16}qL$
- $q\frac{L}{2} + q\frac{L}{16} = \frac{9}{16}qL$



$\bar{M} = \frac{49}{256}\cdot\frac{1}{2}qL^2$

- $qL - q\frac{L}{8}^{*} = \frac{7}{8}qL$

Equilibrio del modo B:

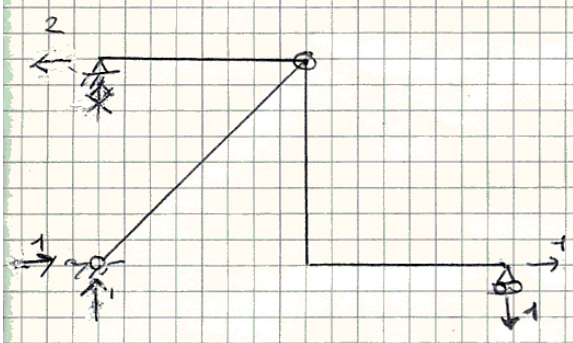


Dimensionamento:

$$W_1 \geq \frac{\frac{7}{32}qL^2}{\sigma_{AMM}} = \frac{39,37\cdot 10^{6}\cdot 10^{3}}{240\cdot 10^{6}}\ cm^3 < 164\ cm^3$$

IPE 200 $\begin{cases} W_1 = 194,3\ cm^3 \\ I_1 = 1943\ cm^4 \\ A = 28,48\ cm^2 \end{cases}$

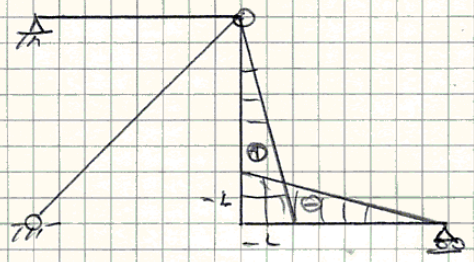
Spostamento in D.



$$1\cdot\delta_D = \frac{1}{EI_1}\int_0^L (-x_3')\left(\frac{7}{16}qLx_3' - q\frac{x_3'^2}{2}\right)dx_3'$$

$$+\frac{1}{EI_1}\left(-\frac{qL^2}{16}\right)\left(-\frac{L}{2}\right)$$

$$= -\frac{qL^4}{48EI_1} + \frac{qL^4}{32EI_1} = +\frac{qL^4}{96EI_1}$$



Carico termico:

$$M_{1t} = \frac{L}{2}\sqrt{2}\left(-\frac{2\sqrt{2}}{L}\right)\alpha\Delta T = -4\alpha\Delta T$$

$$X_1 = -\frac{M_{1p}}{M_{11}} - \frac{M_{1t}}{M_{11}} = \frac{qL^2}{16} + 4\alpha\Delta T\cdot\frac{3EI_1}{5L} = \frac{qL^2}{16} + \frac{12\alpha\Delta T EI_1}{5L}$$

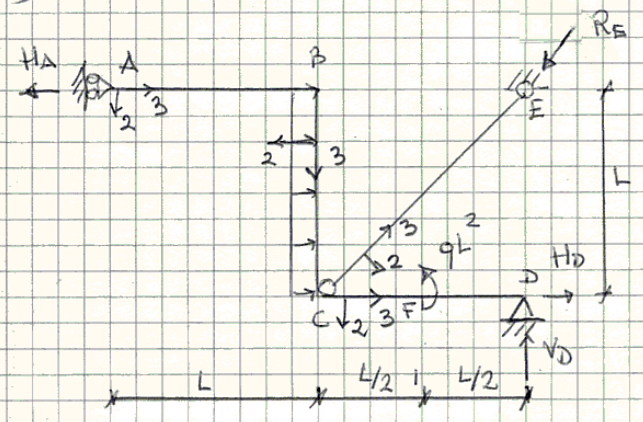
$$= \left(11{,}25 + \frac{12\cdot10^{-5}\cdot 21\cdot 21\cdot 10^{3}\cdot 1943\cdot 10^{-3}}{15}\right)\text{kNm}$$

$$= \left(11{,}25 + \frac{2\cdot 12\cdot 21\cdot 1943\cdot 10^{-5}}{15}\right)\text{kNm}$$

$$= (11{,}25 + 0{,}74)\,\text{kNm} = 11{,}99\ \text{kNm}$$

I diagrammi restano sostanzialmente invariati.

B)



Eq.ni cardinali:

$(\rightarrow)\ H_D - H_A - R_E\frac{\sqrt{2}}{2} + qL = 0$

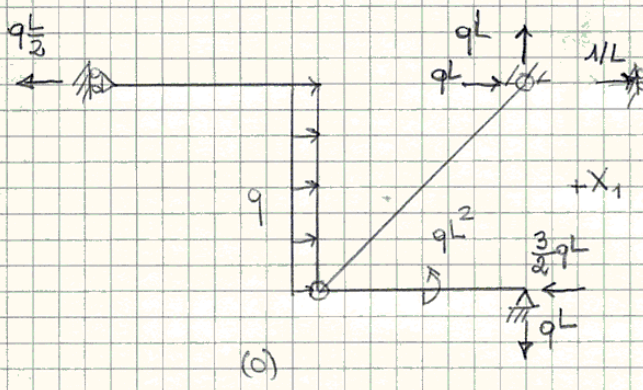
$(\uparrow)\ V_D - R_E\frac{\sqrt{2}}{2} = 0$

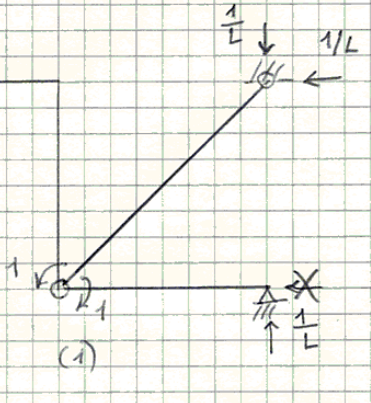
$(C\circlearrowleft)\ V_D L + qL^2 + H_A L = qL^2/2$

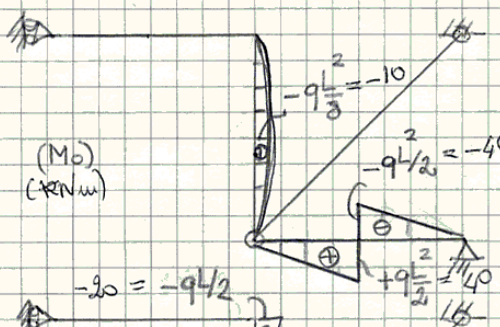
La travatura è una volta iperstatica.

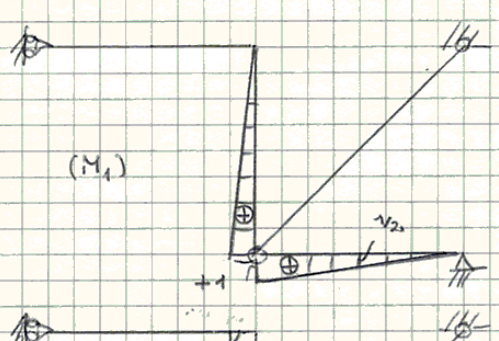
Incognita iperstatica $X_1 = M_C$.

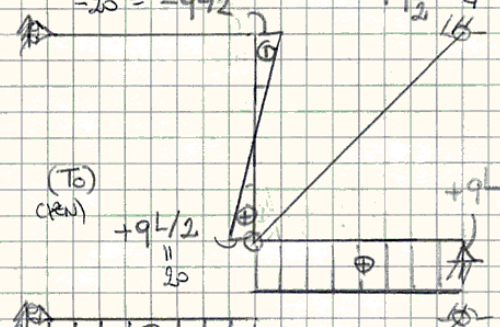
Si trascurano le deformazioni assiali.

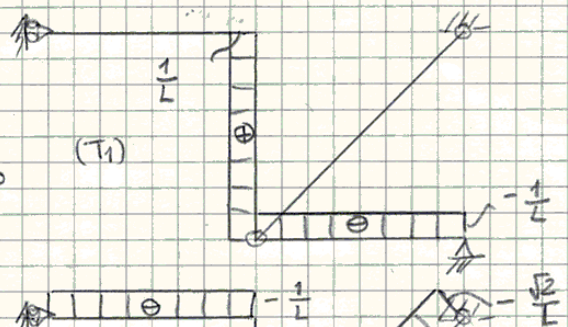


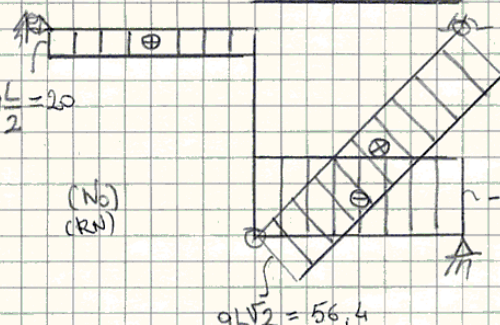


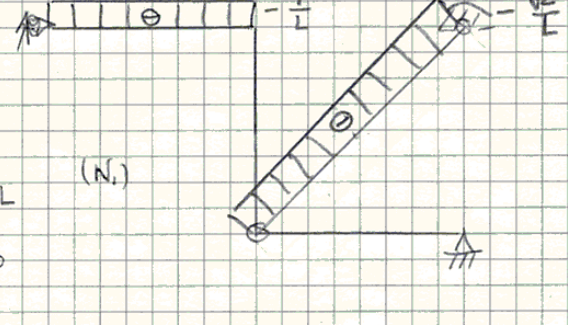












$$EI_1 \eta_{10} = -\frac{qL^3}{24} + \int_0^{L/2} (qLx_3)\left(1 - \frac{x_3}{L}\right) dx_3 + \int_0^{L/2} \left(-q\frac{L}{2}x_3'\right)\left(\frac{x_3'}{L}\right) dx_3'$$

$$= -\frac{qL^3}{24} + qL\left[\frac{x_3^2}{2} - \frac{x_3^3}{3L}\right]_0^{L/2} - q\left[\frac{x_3'^3}{3}\right]_0^{L/2}$$

$$= -\frac{qL^3}{24} + qL^3\left[\frac{1}{8} - \frac{1}{24}\right] - \frac{qL^3}{24} = -\frac{qL^3}{12} + \frac{2}{3}\frac{1}{8}qL^3 = 0$$

$$EI_1 \eta_{11} = 2\,\frac{1}{3}L$$

$X_1 = 0$

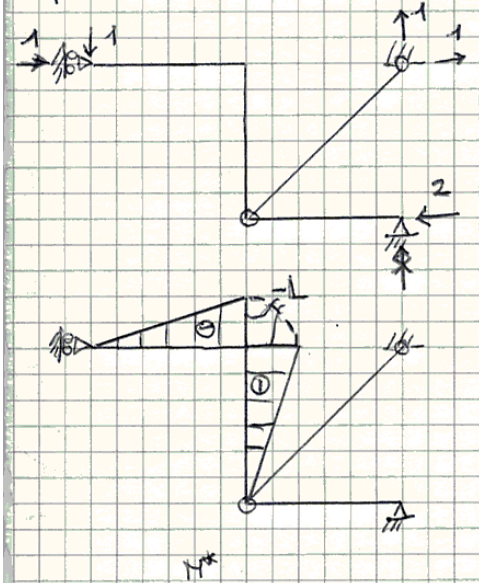
I diagrammi quotati sono quelli del sistema (0).

Dimensionamento (a flessione):

$$W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{40\ kNm}{240\ MPa} = \frac{40 \cdot 10^3\ Nm}{240 \cdot 10^6\ N/m^2} = \frac{10^{-3}}{6}\ m^3 = \frac{10^{-3+6}}{6}\ cm^3 = 167\ cm^3$$

IPE 200 $\begin{cases} W_1 = 194{,}3\ cm^3 \\ I_1 = 1943\ cm^4 = 1943 \cdot 10^{-8}\ m^4 \\ A = 28{,}48\ cm^2 \end{cases}$

Spostamento verticale in A:



$$1 \cdot \delta_A = \frac{1}{EI_1}\int_L MM^*\,dx_3 = \frac{1}{EI_1}\frac{qL^4}{24}$$

$$= \frac{20 \cdot 10^3 \cdot 4^4}{\frac{24}{6} \cdot 210 \cdot 10^9 \cdot 1943 \cdot 10^{-8}}\ m$$

$$= \frac{1000 \cdot 8}{6 \cdot 210 \cdot 1943} = 0{,}0033\ m$$

$$= 0{,}33\ cm$$

Carico termico: $\eta_{1t} + \eta_{10}^{\,0} + \eta_{11} X_1 = 0$

$$\eta_{1t} = -\frac{\sqrt{2}}{2} L\sqrt{2}\,\Delta T\,\alpha = -2\alpha\Delta T \qquad (NB: \Delta T < 0)$$

$$X_1 = -\frac{\eta_{1t}}{\eta_{11}} = \frac{2\alpha\Delta T\, 3EI_1}{2L} = \frac{10^{-5}(-20) \cdot 3 \cdot 210 \cdot 10^9 \cdot 1943 \cdot 10^{-8}}{2}\ 10^{-3}\ kNm = 1{,}22\ kNm$$

Il contributo del carico termico è trascurabile.

B.2