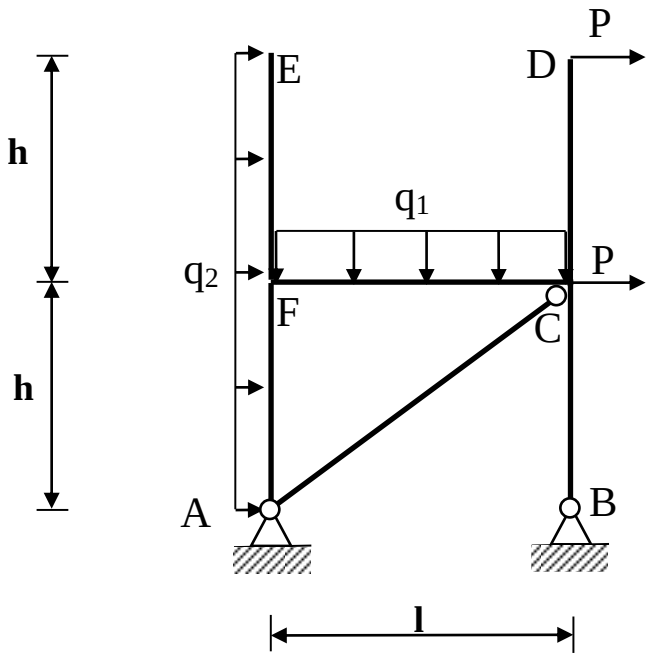
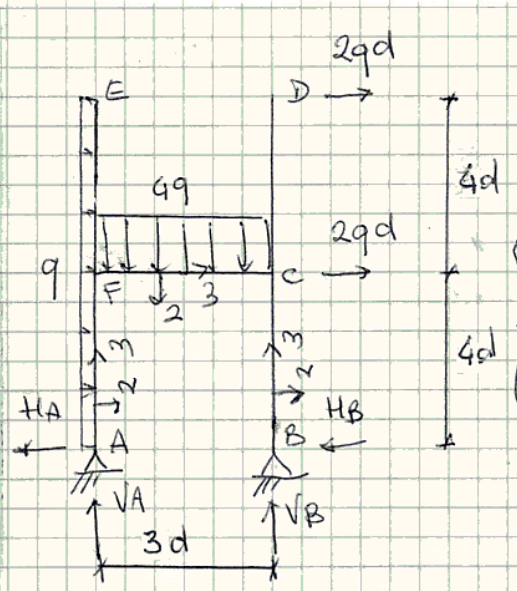
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
13/01/2016



l= 3 m, h = 4 m
q_1 = 20 kN/m, q_2 = 5 kN/m, P = 10 kN,
E = 210 GPa, σ_{amm} = 240 MPa

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in assenza del controvento AC.
- Dimensionare la travatura.
- Calcolare lo spostamento orizzontale del nodo C.
- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione anche in presenza del controvento AC.



$q = 5\ kN/m$

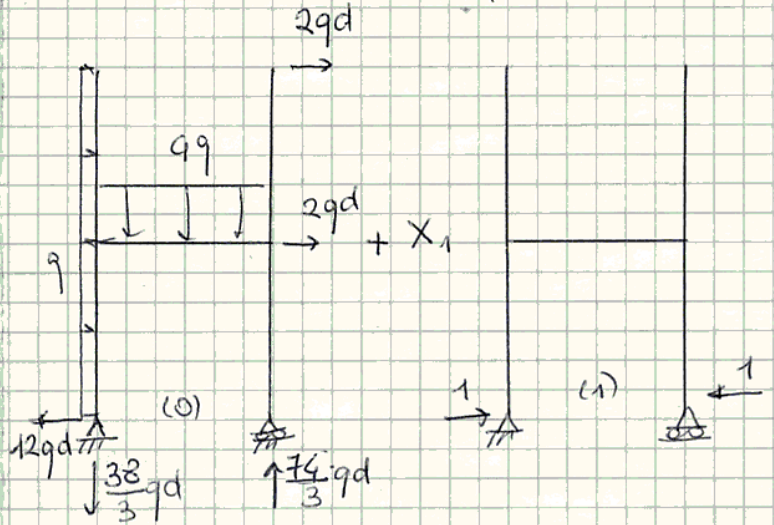
$d = 1\ m$

$$\begin{cases} H_A + H_B = 12qd \\ V_A + V_B = 12qd \\ (A)\quad V_B 3d = 3\cdot 4q\frac{3}{2}d^2 + 2qd\,4d + 2qd\,8d + 8qd\,4d \end{cases}$$

$$= qd^2(18 + 8 + 16 + 32) = 74\,qd^2$$

$$\begin{cases} V_B = 74/3\ qd = 123{,}3\ kN \\ V_A = (12 - 74/3)qd = -\frac{38}{3}qd = -63{,}3\ kN \\ H_A + H_B = 12qd \end{cases}$$

La trave è 1 volta iperstatica - Incognita iperstatica: $X_1 = H_B$

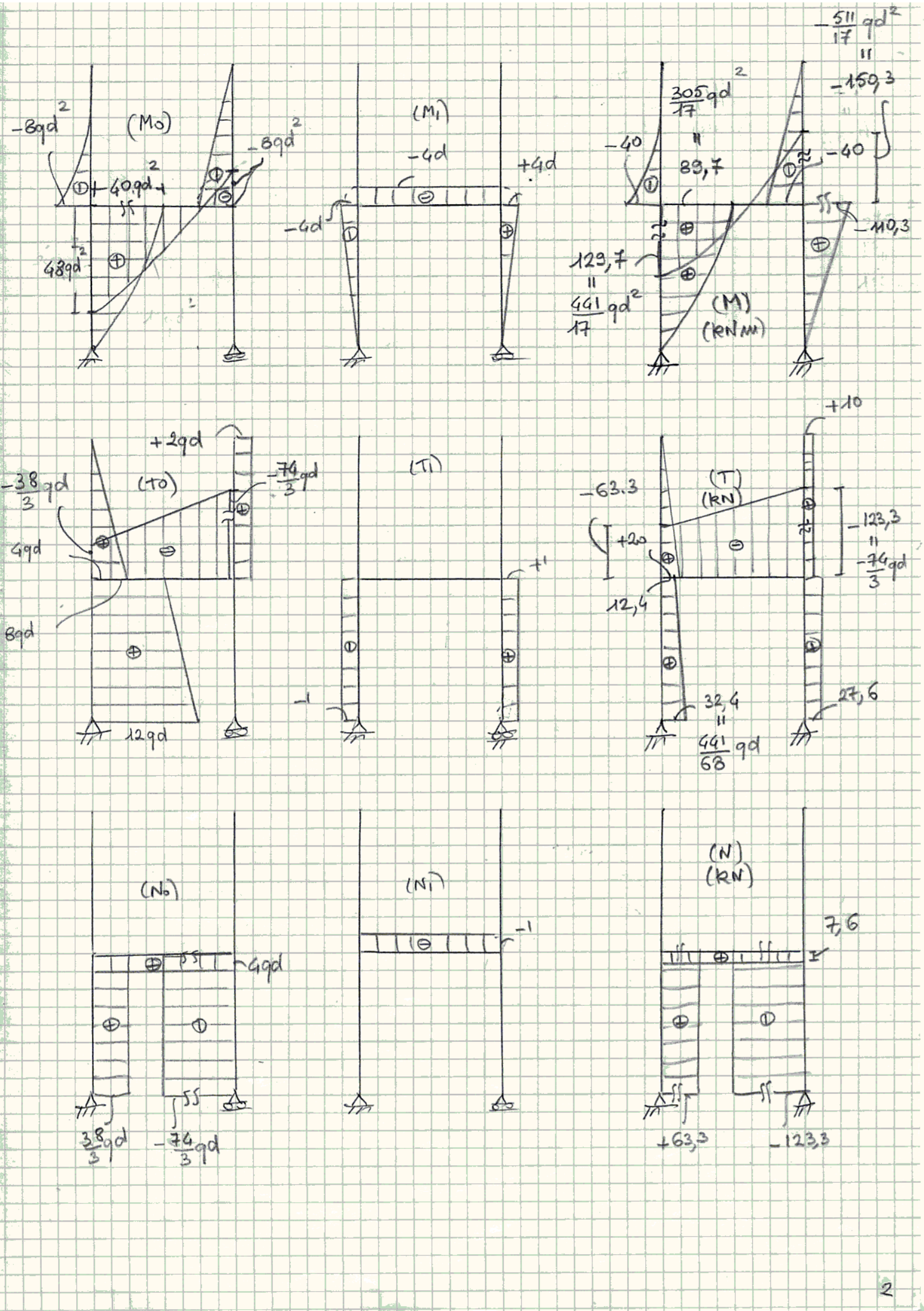


$$EI_1 M_{10} = \int_0^{4d} (-x)\left(-q\frac{x^2}{2} + 12qdx\right)dx + \int_0^{3d} (-4d)\left(-4q\frac{x^2}{2} - \frac{38}{3}qdx + 48qd^2\right)dx$$

$$= -224\,qd^4 - 276\,qd^4 = -500\,qd^4$$

$$EI_1 \gamma_{11} = 2\cdot\frac{1}{3}\cdot 4d\cdot 16d^2 + 3d\cdot 16d^2 = 16d^3\left(3 + \frac{8}{3}\right) = \frac{272}{3}d^3$$

$$X_1 = \frac{3\cdot 500}{272}qd^4 = \frac{375}{68}qd^4 = 27{,}57\ kN$$


$-8qd^2$
(M_0)
$40qd^2$
$48qd^2$
$-8qd^2$
(M_1)
$-4d$
$+4d$
$-4d$
$-\frac{511}{17}qd^2$
$=$
$-150,3$
$\frac{305}{17}qd^2$
$=$
$89,7$
-40
-40
$-110,3$
$129,7$
$=$
$\frac{441}{17}qd^2$
(M)
(kNm)
$+2qd$
$-\frac{38}{3}qd$
(T_0)
$-\frac{76}{3}qd$
$4qd$
$8qd$
$12qd$
(T_1)
$+1$
-1
$+10$
(T)
(kN)
$-63,3$
$+20$
$-123,3$
$=$
$-\frac{76}{3}qd$
$12,4$
$32,4$
$=$
$\frac{441}{68}qd$
$27,6$
(N_0)
$-4qd$
$\frac{38}{3}qd$
$-\frac{76}{3}qd$
(N_1)
-1
(N)
(kN)
$7,6$
$+63,3$
$-123,3$

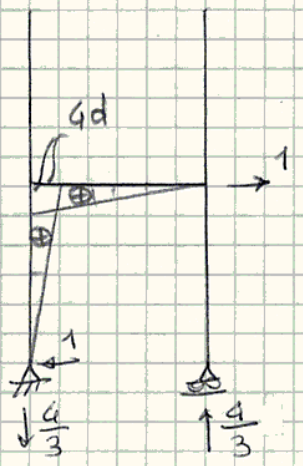
Dimensionamento:

$$W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{150{,}3 \cdot 10^3}{240 \cdot 10^6} \cdot 10^6 \ cm^3 = \frac{15030}{24} cm^3 = 626{,}25 \ cm^4$$

↳ IPE 330

$$\begin{cases} W_1 = 713{,}1 \ cm^3 \\ I_1 = 11770 \ cm^4 \\ A = 62{,}61 \ cm^2 \end{cases}$$

Spostamento orizzontale in C:



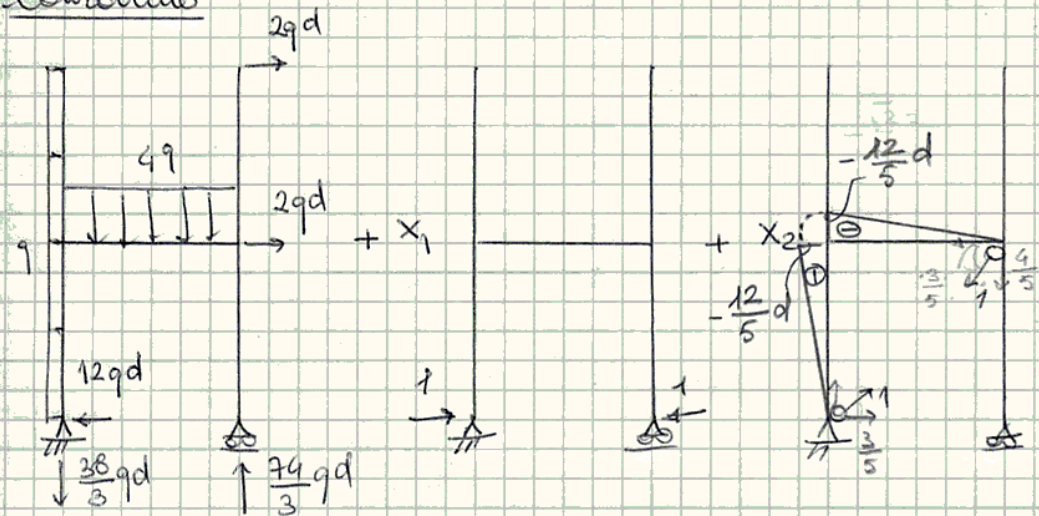
$$1 \cdot v_C = \frac{1}{EI_1}\left[\int_0^{4d} (x)\left(-9\frac{x^2}{2} + \frac{441}{68} q dx\right)dx + \int_0^{3d}\left(\frac{4}{3}x\right)\left(-49\frac{x^2}{2} + \frac{74}{3} q dx - \frac{511}{17} q d^2\right)dx\right]$$

$$= \frac{1808}{17}\frac{qd^4}{EI} + \frac{1048}{17}\frac{qd^4}{EI} = \frac{2856}{17}\frac{qd^4}{EI} = 168\frac{qd^4}{EI}$$

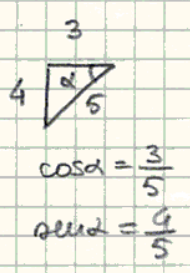
$$= \frac{168 \cdot 5{,}0 \cdot 10^3}{21 \cdot 10^6 \cdot 1177 \cdot 10^{-8}} \cdot 10^2 \ cm$$

$$= \frac{168 \cdot 500}{21 \cdot 1177} cm = 3{,}4 \ cm$$

Cedimento



(0) (1) (2)



$\cos\alpha = \frac{3}{5}$

$\sin\alpha = \frac{4}{5}$

$$EI_1 \eta_{10} = -500 \ qd^4$$

$$EI_1 \eta_{20} = \int_0^{4d}\left(-\frac{3}{5}x\right)\left(-9\frac{x^2}{2} + 12qdx\right)dx + \int_0^{3d}\left(-\frac{4}{5}x\right)\left(-49\frac{x^2}{2} + \frac{74}{3}qdx - 8qd^2\right)dx$$

$$= -\frac{672}{5}qd^4 - \frac{582}{5}qd^4 = -\frac{1254}{5}qd^4$$

$$EI_1 \eta_{11} = \frac{272}{3}d^3 \ ; \ EI_1 \eta_{12} = \frac{1}{3} 4d \cdot 4d \cdot \frac{12}{5}d + (+4d)\,3d\,\frac{1}{2}\,\frac{12}{5}d = \frac{136}{5}d^3$$

$$EI_1 \gamma_{22} = \frac{1}{3} 4d \cdot \frac{144}{25} d^2 + \frac{1}{\beta} \cdot \beta d \cdot \frac{144}{25} d^2$$

$$= \frac{7}{3} \cdot \frac{144}{25} d^3 = \frac{336}{25} d^3$$

$$\begin{bmatrix} \frac{272}{3} & \frac{136}{5} \\ \frac{136}{5} & \frac{336}{25} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = qd \begin{bmatrix} 500 \\ \frac{1254}{5} \end{bmatrix}$$

$$\begin{cases} X_1 = -\frac{159}{748} qd = -1{,}06 \text{ kN} \\ X_2 = \frac{210}{11} qd = 95{,}45 \text{ kN} \end{cases}$$

Poiché $X_1 \simeq 1\%$ di X_2, si ha che il tratto BC è praticamente scarico a flessione, essendo il taglio privo risultante sul tratto BC ora assorbito dal corrente AC. I diagrammi finali si possono quindi ottenere con buona approssimazione mediante sovrapposizione dei risultati (0) e (2).