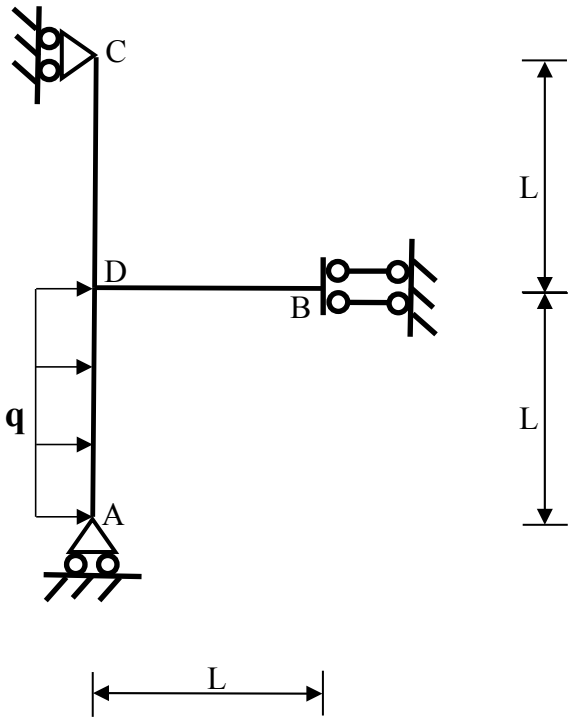
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
22/12/2014

A



L= 2 m, q = 25 kN/m
σ_{AMM} = 240 MPa , E = 210 GPa

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in presenza del carico q.
- Dimensionare la travatura.
- Calcolare lo spostamento orizzontale del punto A.
- Disegnare nuovamente i diagrammi quotati considerando, in aggiunta al carico q, anche un cedimento orizzontale del vincolo in C pari a 1 cm.

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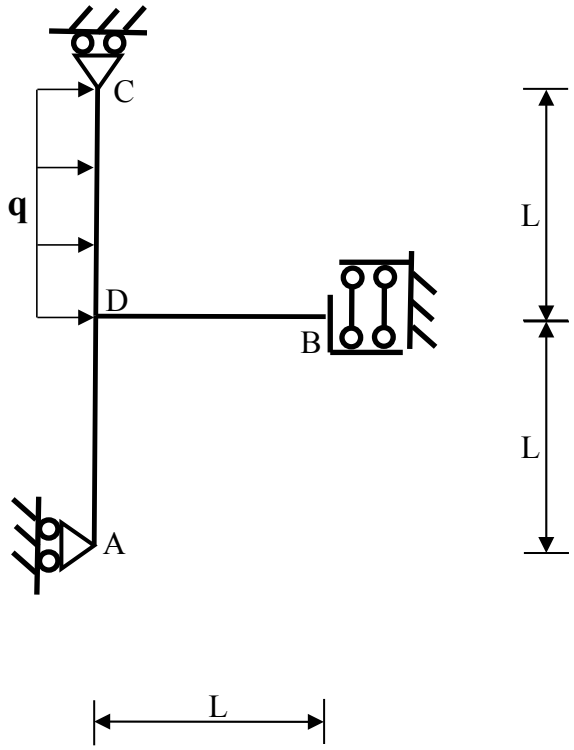
B

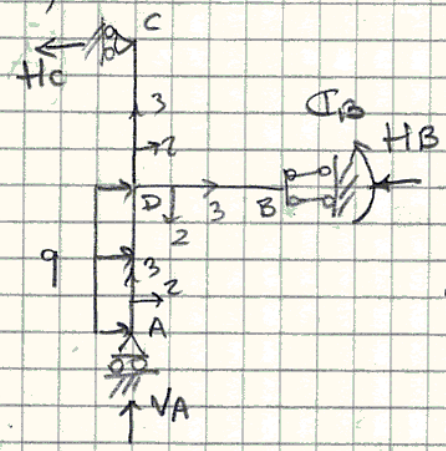


L= 2.5 m, q = 20 kN/m
σ_{AMM} = 240 MPa , E = 210 GPa

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in presenza de carico q.
- Dimensionare la travatura.
- Calcolare lo spostamento orizzontale in C.
- Disegnare nuovamente i diagrammi quotati considerando, in aggiunta al carico q, anche un cedimento orizzontale del vincolo in A pari a 1.5 cm.

A)

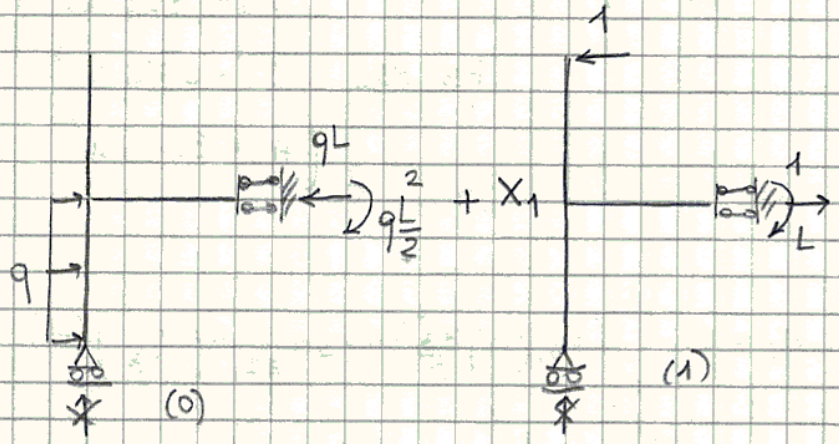


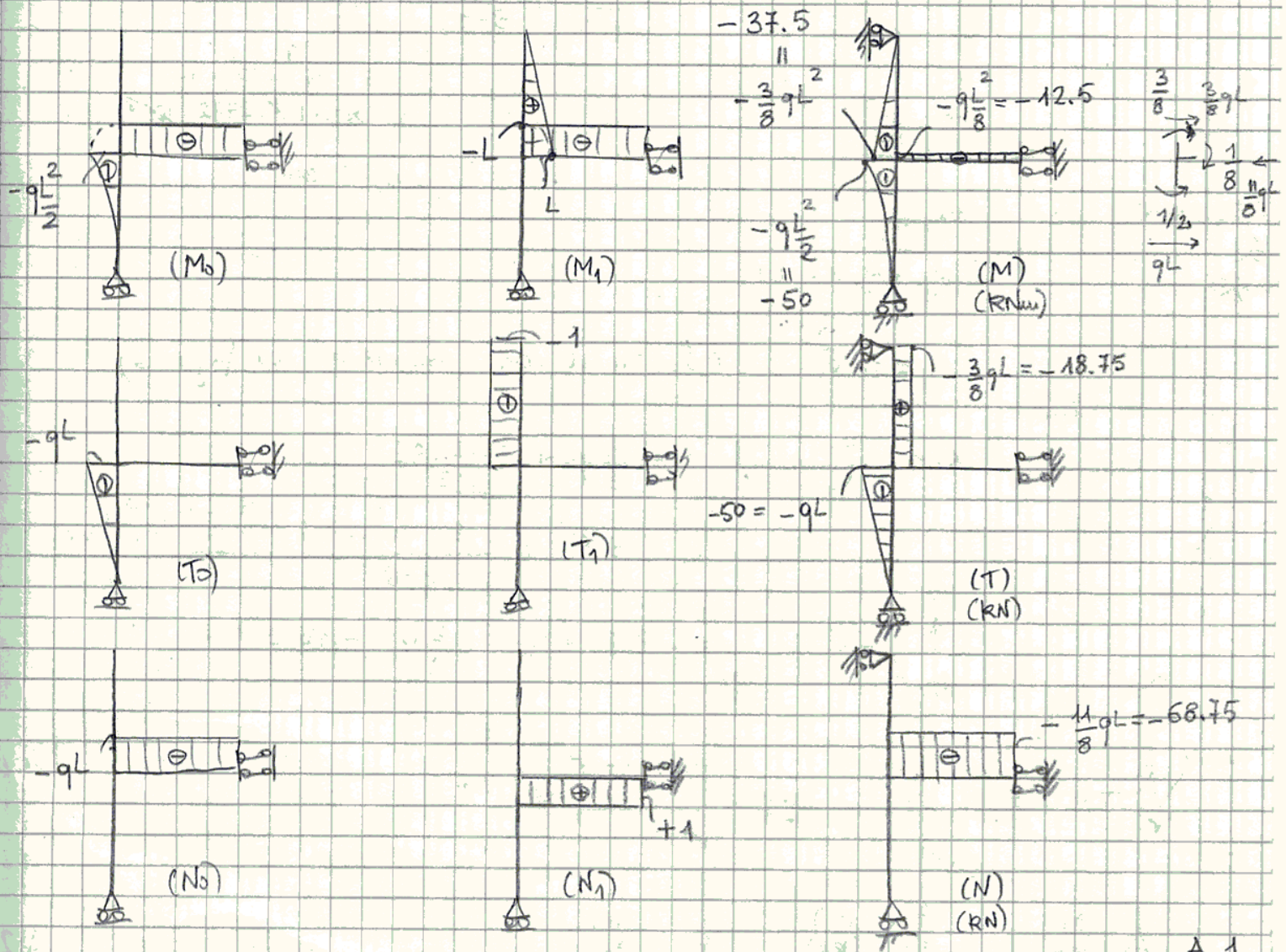
$(\rightarrow)\quad H_B + H_C = qL$

$(\uparrow)\quad V_A = 0$

$(C\curvearrowright)\quad C_B - H_B L + \frac{3}{2} qL^2 = 0$

La trave data è una volta iperstatica. $X_1 = H_C$.





$$EI_1\, M_{10} = L\left(-q\frac{L^2}{2}\right)(-L) = q\frac{L^4}{2}$$

$$EI_1\, M_{11} = \frac{L^3}{3} + L^3 = \frac{4}{3}L^3$$

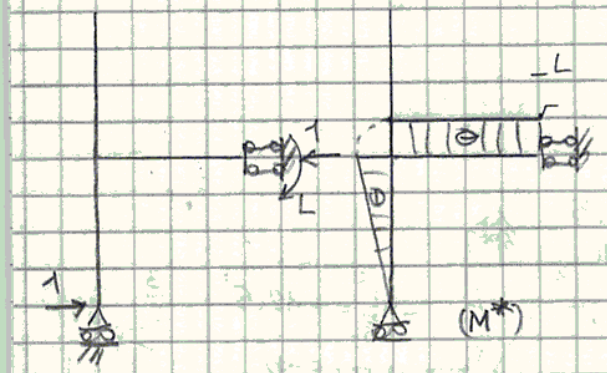
$$X_1 = -\frac{M_{10}}{M_{11}} = -q\frac{L^4}{2}\frac{3}{4L^3} = -\frac{3}{8}qL = -\frac{3}{8}\cdot 25\ \text{kN} = -18.75\ \text{kN}$$

DIMENSIONAMENTO:

$$W_1 \geq \frac{qL^2/2}{\sigma_{AMM}} = \frac{50\cdot 10^3}{240\cdot 10^6}\cdot 10^6\ \text{cm}^3 = \frac{5000}{24}\ \text{cm}^3 = 208{,}3\ \text{cm}^3$$

IPE 220 $\begin{cases} W_1 = 252\ \text{cm}^3 \\ I_1 = 2772\ \text{cm}^4 \\ A = 33{,}4\ \text{cm}^2 \end{cases}$

SPOSTAMENTO ORIZZONTALE:



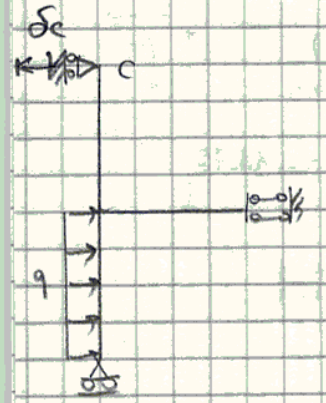
$$1\cdot v_A = \frac{1}{EI_1}\int_0^L (-x_3)\left(-q\frac{x_3^2}{2}\right)dx_3 + \frac{1}{EI_1}L\left(-q\frac{L^2}{8}\right)(-L)$$

$$= +\frac{qL^4}{8EI_1} + \frac{qL^4}{8EI_1} = \frac{qL^4}{4EI_1}$$

$$= \frac{25\cdot 16^4\cdot 10^3\cdot 10^2}{4\cdot 210\cdot 10^8\cdot 2772\cdot 10^{-8}}$$

$$= 1{,}71\ \text{cm}$$

CEDIMENTO $\delta_c = 1$ cm.



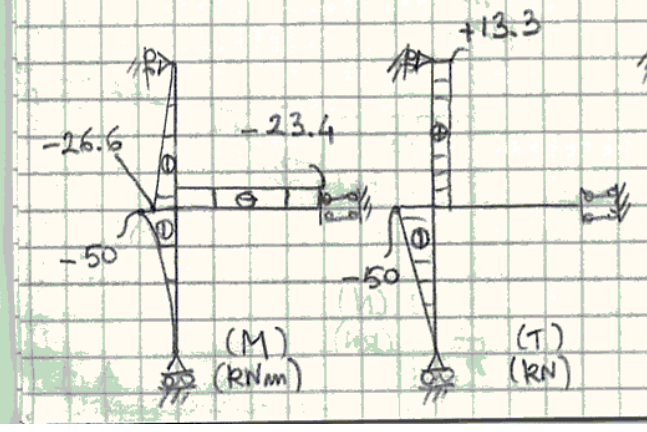
$$M_{10} + M_{11}X_1 = M_1$$

$M_1 = 1\cdot\delta_c$ (δ_c ha il verso concordato con f_{gund})

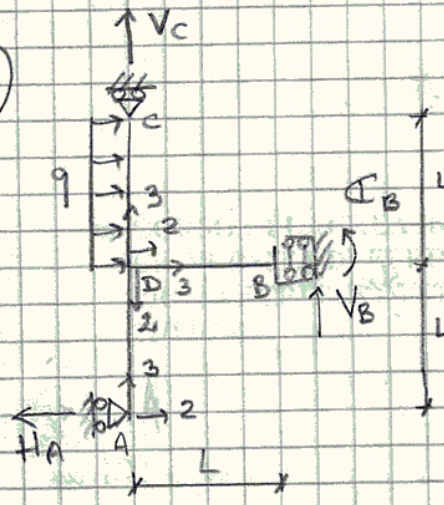
$$X_1 = -\frac{M_{10}}{M_{11}} + \frac{M_1}{M_{11}} = -\frac{3}{8}qL + \frac{3EI_1\delta_c}{4L^3}$$

$$= \left(-18.75 + \frac{3\cdot 210\cdot 10^9\cdot 2772\cdot 10^{-8}\cdot 10^{-2}\cdot 10^{-3}}{4\cdot 8}\right)\text{kN}$$

$$= (-18.75 + 5{,}45)\ \text{kN} = -13.3\ \text{kN}$$



B)



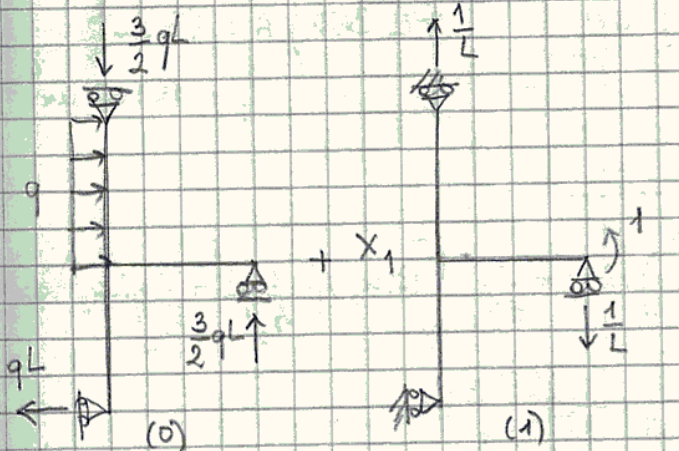
$(\rightarrow)\quad H_A = qL$

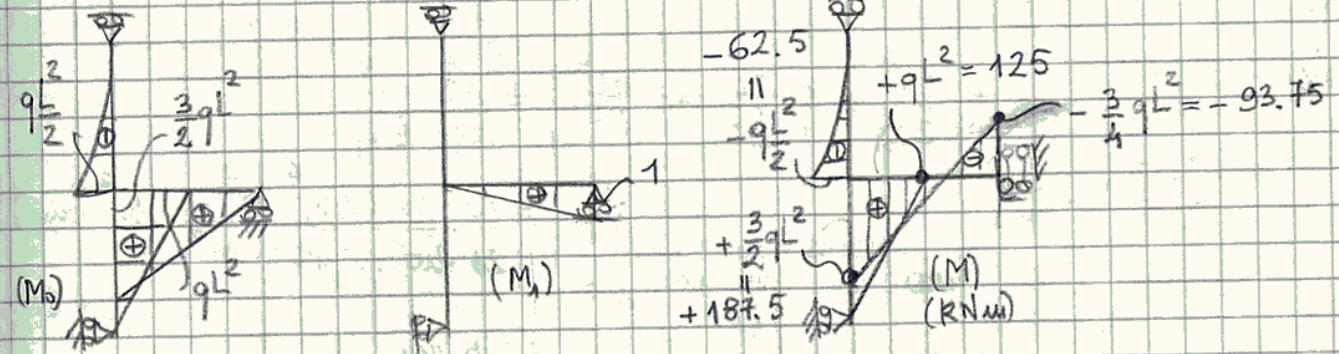
$(\uparrow)\quad V_B + V_C = 0$

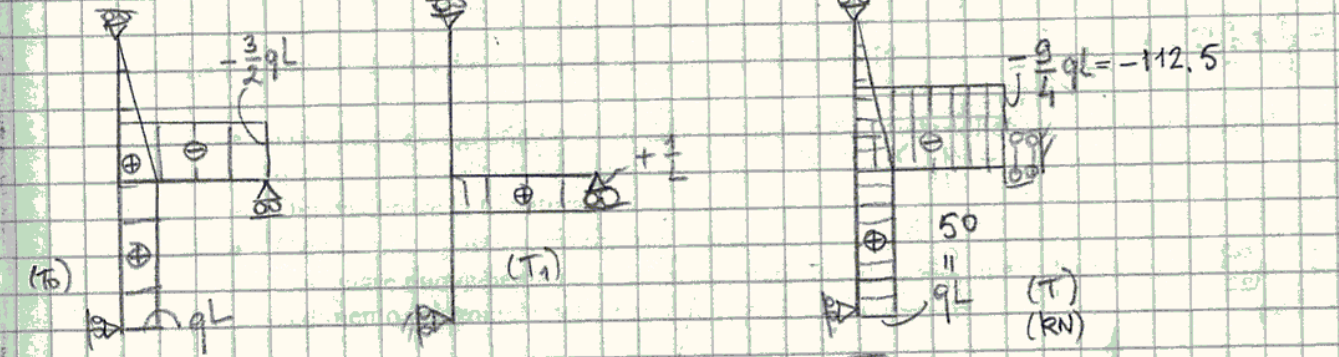
$(A\circlearrowleft)\quad C_B + V_B L - \frac{3}{2} qL^2 = 0$

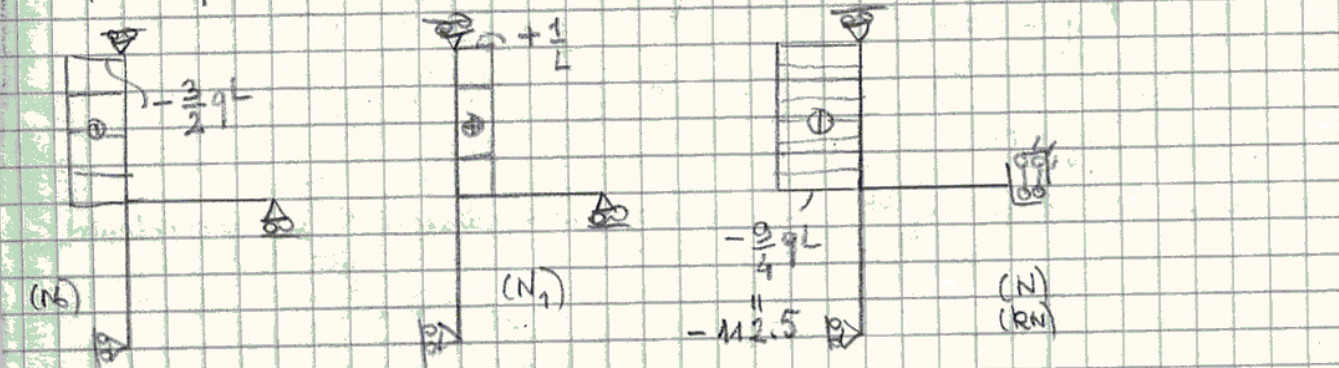
La struttura è una volta iperstatica.

Incognita iperstatica $X_1 = C_B$.









$$EI_1 \eta_{10} = \frac{1}{2} L \frac{2}{2} qL^2 \cdot 1 = \frac{qL^3}{4}$$

$$EI_1 \eta_{11} = \frac{1}{3} L \cdot 1 \cdot 1 = \frac{L}{3}$$

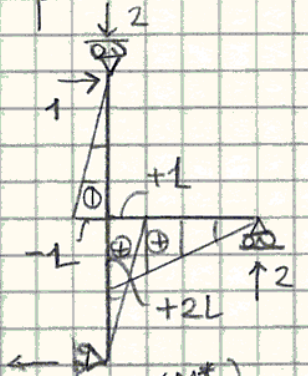
$$X_1 = -\frac{qL^3}{4} \cdot \frac{3}{L} = -\frac{3}{4} qL^2 = -\frac{3}{4} \cdot 125 \text{ kNm} = -93{,}75 \text{ kNm}$$

Dimensionamento:

$$W_1 \geq \frac{3/2\, qL^2}{6AMM} = \frac{187.5 \cdot 10^6}{240} \text{ cm}^3 = 781 \text{ cm}^3$$

IPE 360 $\begin{cases} W_1 = 904 \text{ cm}^3 \\ A = 73 \text{ cm}^2 \\ I_1 = 16270 \text{ cm}^4 \end{cases}$

Spostamento orizzontale:



$$1 \cdot v_c = \frac{1}{EI_1} \int_0^L (-x_3')\left(-q\frac{x_3'^2}{2}\right) dx_3' + \frac{1}{EI_1} \int_0^L (x_3')\left(-\frac{3}{2}qL^2 + \frac{9}{2}qLx_3'\right) dx_3'$$

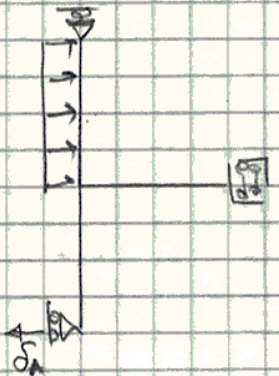
$$+ \frac{1}{EI_1} \frac{1}{3} qL^2 \cdot L \cdot L$$

$$= + \frac{qL^4}{8EI_1} + \frac{3}{4}\frac{qL^4}{EI_1} + \frac{qL^4}{3EI_1} = \frac{29}{24}\frac{qL^4}{EI_1}$$

$$= \frac{29 \cdot 20 \cdot (2{,}5)^4 \cdot 10^3}{24 \cdot 2 \cdot 10^4 \cdot 16270} 10^2 \text{ cm}$$

$$= 2{,}8 \text{ cm}$$

Cedimento $\delta_A = 1.5$ cm



$$\eta_{10} + \eta_{11} X_1 = \eta_1$$

$$\eta_{10} = \frac{qL^3}{4EI_1}$$

$$\eta_{11} = \frac{L}{3EI_1}$$

$$\eta_1 = 0$$

Il cedimento non ha alcun effetto sui diagrammi disegnati in precedenza.