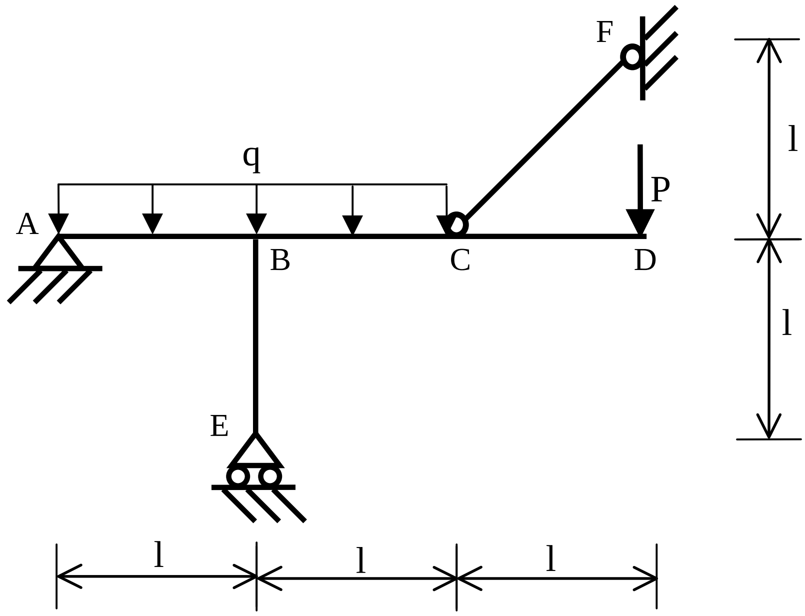
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 3/12/2009

A

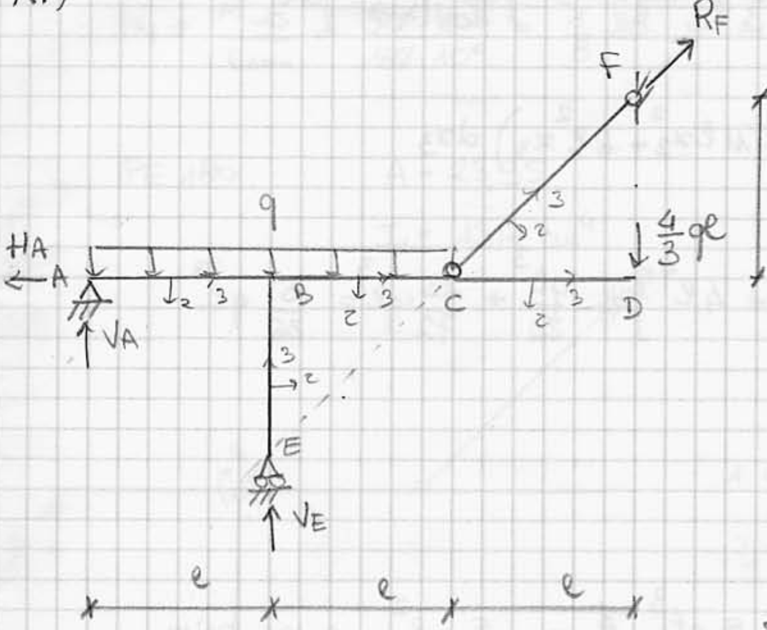


l = 1 m, q = 15 kN/m, P = 20 kN
$E = 2.1\ 10^{3}$ kN/cm^{2}, σ_{AMM}= 160 MPa

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è possibile trascurare le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo B.
4. Risolvere nuovamente la travatura considerando anche un abbassamento verticale del vincolo in B pari a 1 cm e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q e P che del cedimento.

A1)



(→) $H_A = R_F \sqrt{2}/2$

(↑) $V_A + V_E + R_F \frac{\sqrt{2}}{2} = \frac{7}{3} q\ell$

(E↺) $V_A \cancel{\ell} = -\frac{4}{3} q\ell \, 2\cancel{\ell} + H_A \cancel{\ell}$

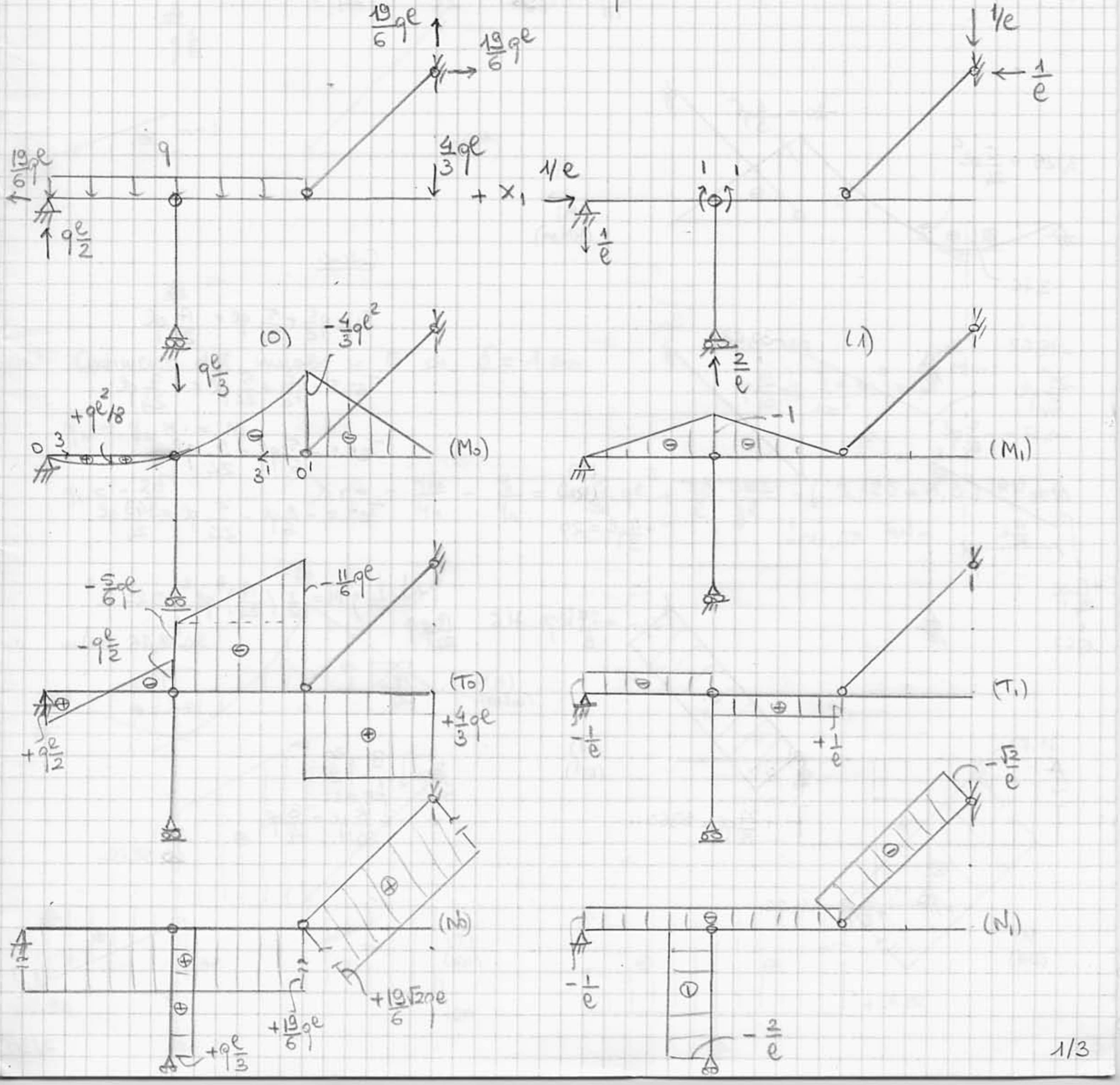
Travatura una volta iperstatica.

Incognita iperstatica: $X_1 = M_B^-$

Equilibrio del nodo triplo B:

$X_1 = M_B^-$ B M_B^+ N.B. $M_{B_E} = 0$!

Dunque: $M_B^+ = M_B^- = X_1$



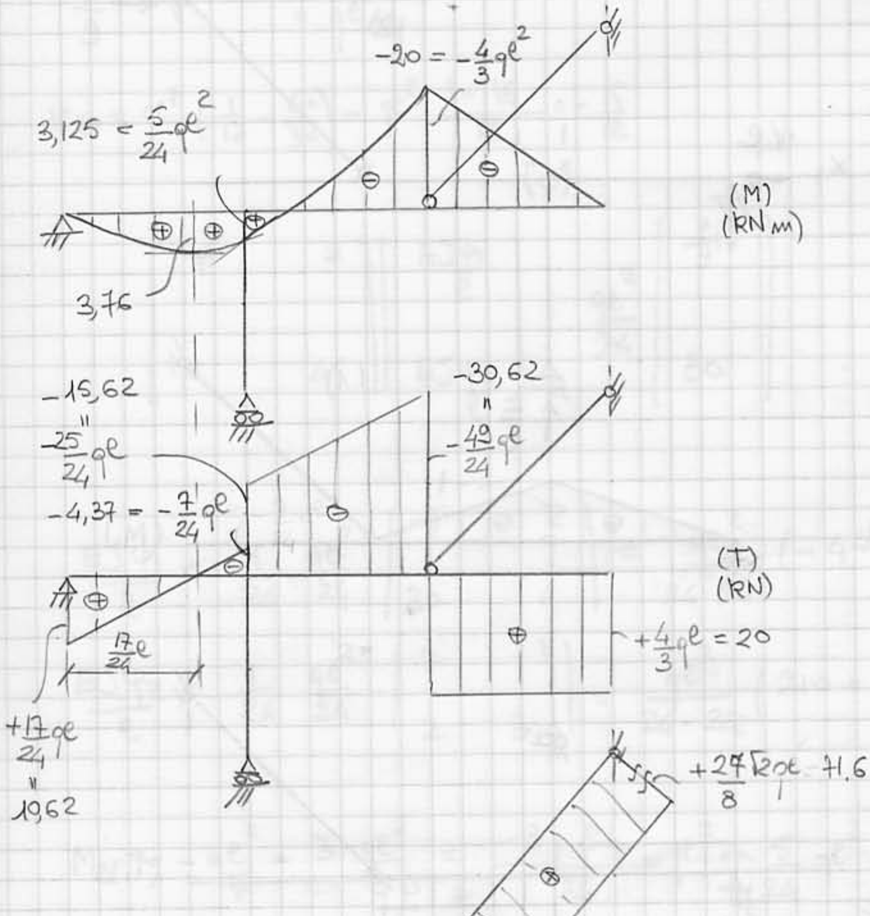
$$EI_1\eta_{10} = \int_0^\ell \left(q\frac{\ell}{2}x_3 - q\frac{x_3^2}{2}\right)\left(-\frac{x_3}{\ell}\right)dx_3 + \int_0^\ell \left(-\frac{4}{3}q\ell^2 + \frac{11}{6}q\ell x_3 - q\frac{x_3^2}{2}\right)\left(-\frac{x_3}{\ell}\right)dx_3$$

$$= \frac{q}{2\ell}\int_0^\ell \left(x_3^3 - \ell x_3^2\right)dx_3 + \frac{q}{6\ell}\int_0^\ell \left(3x_3^3 - 11\ell x_3^2 + 8\ell^2 x_3\right)dx_3$$

$$= \frac{q}{2\ell}\left[\frac{\ell^4}{4} - \frac{\ell^4}{3}\right] + \frac{q}{6\ell}\left[\frac{3}{4}\ell^4 - \frac{11}{3}\ell^4 + 4\ell^4\right] = -\frac{q\ell^3}{24} + \frac{13}{72}q\ell^3 = \frac{5}{36}q\ell^3$$

$$EI_1\eta_{11} = 2\int_0^\ell \left(-\frac{x_3}{\ell}\right)^2 dx_3 = \frac{2}{\ell^2}\frac{\ell^3}{3} = \frac{2}{3}\ell$$

$$\eta_{10} + \eta_{11}X_1 = 0 \iff X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{5}{36}q\ell^3\frac{3}{2\ell} = -\frac{5}{24}q\ell^2 = -3{,}125 \text{ kNm}$$



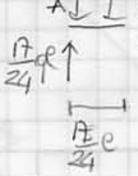
Calcoli

$T_A = q\frac{\ell}{2} + \frac{5}{24}q\ell = \frac{17}{24}q\ell$

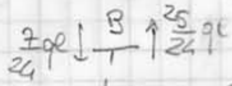
$T_{B^-} = -q\frac{\ell}{2} + \frac{5}{24}q\ell = -\frac{7}{24}q\ell$

$T_{B^+} = -\frac{5}{6}q\ell - \frac{5}{24}q\ell =$

$T_{C^-} = -\frac{11}{6}q\ell - \frac{5}{24}q\ell = \frac{49}{24}q\ell$



$\bar{M} = \frac{1}{2}\left(\frac{17}{24}\right)^2 q\ell^2 = 0{,}25\,q\ell^2 = 3{,}76$ kNm



$\frac{18}{24}q\ell = \frac{3}{4}q\ell$

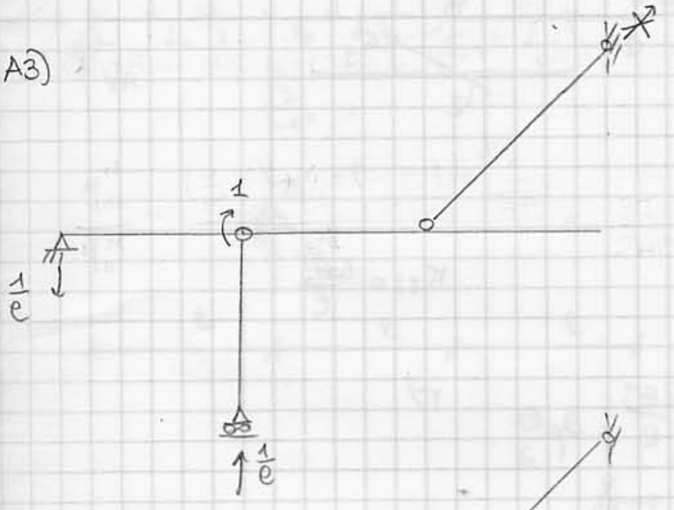
A2)

$$W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{2\cancel{0}\cdot 10^{\cancel{3}}}{16\cancel{0}\cdot 10^{\cancel{6}}} = \frac{1}{8}10^{-3}\ m^3 = \frac{1}{8}10^{-3}\cdot 10^{6}\ cm^3 = 125\ cm^3$$

IPE 180 $\quad A = 23,95$

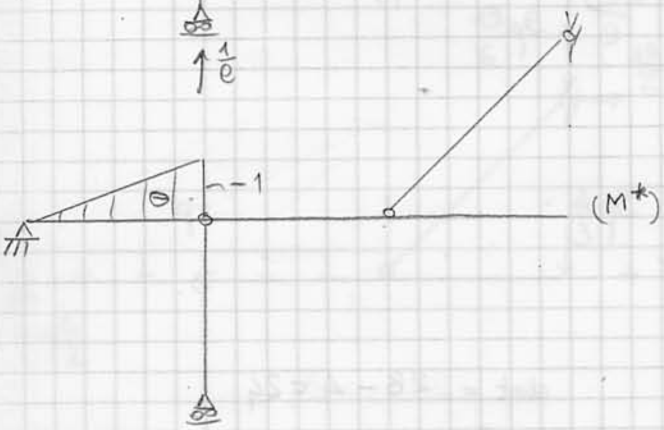
$I_1 = 1317\ cm^4$

A3)



$$1\cdot\varphi_B = \frac{1}{EI_1}\int_0^\ell \left(\frac{17}{24}q\ell x_3 - q\frac{x_3^2}{2}\right)\left(-\frac{x_3}{\ell}\right)dx_3$$

$$= -\frac{q\ell^3}{9EI_1} = -0,34°$$



A4) Cedimento del vincolo in E di $\delta = 1$ cm.

1317

$\left(\frac{3\cdot 2,1\cdot 10^3}{100^2}\right)$ kN/cm²

10^{-2} m

$$\eta_{10} + \eta_{11}X_1 = \eta_1$$

$$\eta_1 = -\frac{2\delta}{\ell}$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_1}{\eta_{11}} = -\frac{5}{24}q\ell^2 - \frac{\cancel{2}\delta}{\ell}\frac{3EI_1}{\cancel{2}\ell} = (-3,125-8,3)\ \text{kN m}$$

$$\stackrel{!}{=} -11,42\ \text{kNm}$$