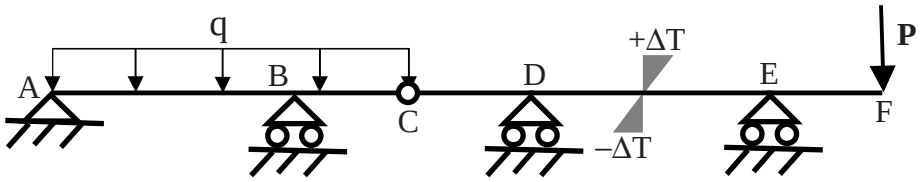
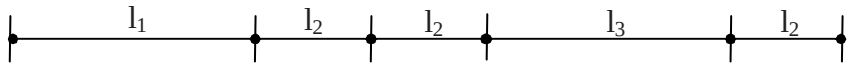
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA PARZIALE DI STATICA
FERRARA, 24/11/2009

A



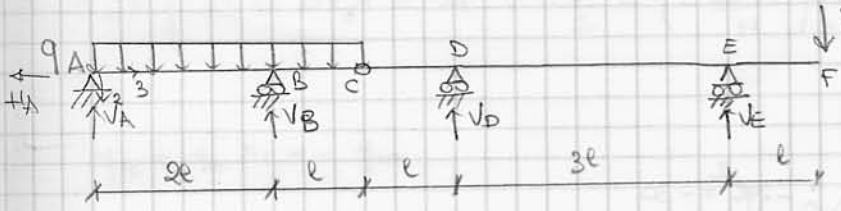


l_1 = 2 m, l_2= 1 m, l_3 = 3 m,
q = 15 kN/m, P = 30 kN,
E = 2.1 10^3 kN/cm^2, α= 10^{-5} °C^{-1}, ΔT = 20 °C

La travatura iperstatica di figura è realizzata con profilati IPE 200 (H= 200 mm, A = 28.4 cm^2, I_1 = 1943 cm^4).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo D.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel solo tratto DE e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, P che di ΔT.

A1)



$$\begin{cases} +H_A = 0 \\ V_A + V_B + V_D + V_E = 5q\ell \\ (C^{\curvearrowright})_{ABC} \quad V_A 3\ell + V_B \ell = \frac{9}{2} q\ell^2 \\ (C^{\curvearrowright})_{CDE} \quad V_D \ell + V_E 4\ell = 10 q\ell^2 \end{cases}$$

$q = 15$ kN/m

$\ell = 1$ m, $P = 30$ kN $= 2q\ell$

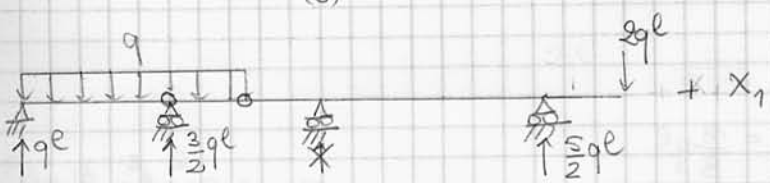
$$\begin{cases} H_A = 0 \\ V_B = \frac{9}{2} q\ell - 3V_A \\ V_D = 10q\ell - 4V_E \\ V_A + \frac{9}{2} q\ell - 3V_A + 10q\ell - 4V_E + V_E = 5q\ell \end{cases}$$

Trabatura una volta iperstatica.

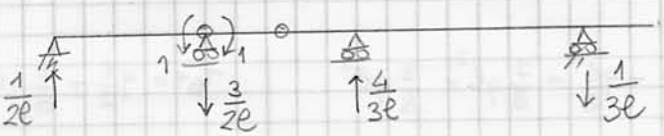
Incognita iperstatica: $X_1 = M_B$

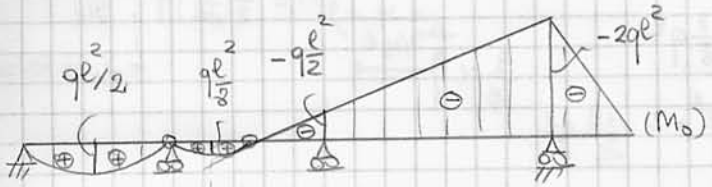
$$\begin{cases} H_A = 0 \\ V_B = \frac{9}{2} q\ell - 3V_A \\ V_D = 10q\ell - \frac{38}{3} q\ell + \frac{8}{3} V_A = -\frac{8}{3} q\ell + \frac{8}{3} V_A \\ V_E = \frac{19}{6} q\ell - 2\frac{V_A}{3} \end{cases}$$

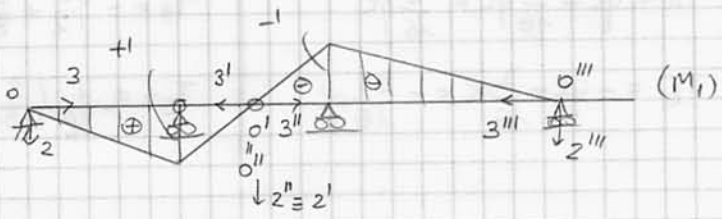
(0)

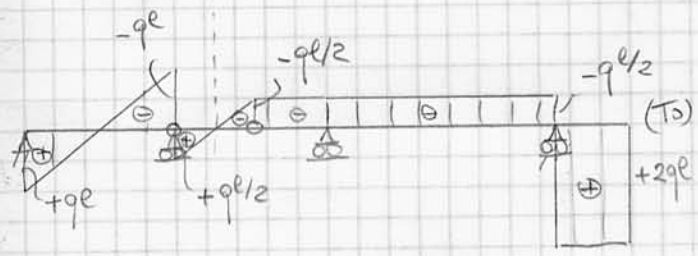


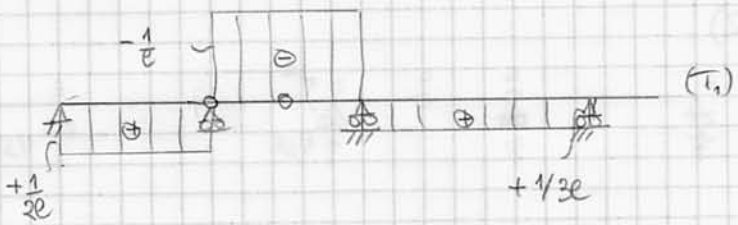
(1)











$$EI_1 M_{10} = \int_0^{2\ell} \left(\frac{x_3}{2\ell}\right)\left(q\ell x_3 - q\frac{x_3^2}{2}\right) dx_3 + \int_0^{\ell} \left(\frac{x_3'}{\ell}\right)\left(q\frac{\ell}{2} x_3' - q\frac{x_3'^2}{2}\right) dx_3' + \int_0^{\ell} \left(-q\frac{x_3''\ell}{2}\right)\left(-\frac{x_3''}{\ell}\right) dx_3'' +$$

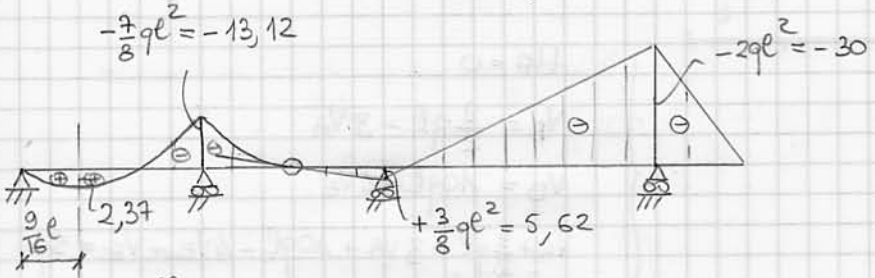
$$+ \int_0^{3\ell} \left(-\frac{x_3'''}{\ell}\right)\left(-2q\ell^2 + q\frac{\ell}{2} x_3'''\right) dx_3'''$$

$$= q\ell^3/3 + q\ell^3/24 + q\ell^3/6 + \frac{3}{2} q\ell^3 = 49/24\, q\ell^3$$

$$EI_1 M_{11} = \int_0^{2\ell} \left(\frac{x_3}{2\ell}\right)^2 dx_3 + 2\int_0^{\ell} \left(\frac{x_3'}{\ell}\right)^2 dx_3' + \int_0^{3\ell} \left(-\frac{x_3''}{\ell}\right)^2 dx_3'' = \frac{2\ell}{3} + \frac{2\ell}{3} + \ell = \frac{7}{3}\ell$$

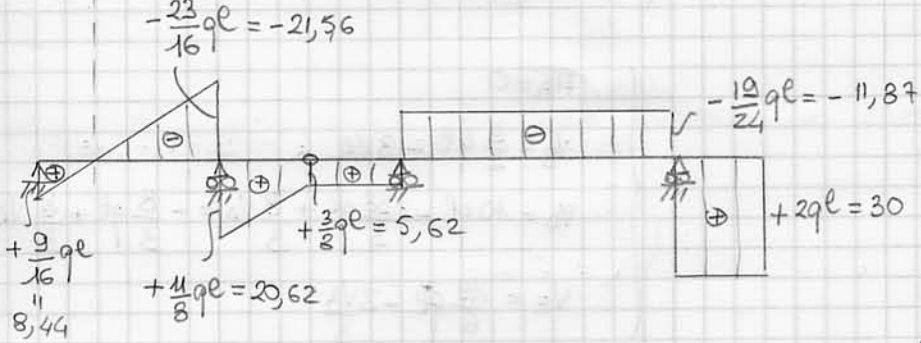
$$X_1 = -\frac{49}{24} q\ell^3 \cdot \frac{3}{7\ell} = -\frac{7}{8} q\ell^2 = -13{,}12 \text{ kNm}$$

Diagrammi delle c. s.:



(M) (kNm)

$q\ell^2 = 15$ kNm



(T) (kN)

$q\ell = 15$ kN

(N)=0

Calcoli

$$M_D = -q\frac{\ell^2}{2} + \frac{7}{8}q\ell^2 = \frac{3}{8}q\ell^2$$

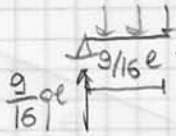
$$T_A = q\ell - \frac{7}{16}q\ell = \frac{9}{16}q\ell$$

$$T_B^- = -q\ell - \frac{7}{16}q\ell = -\frac{23}{16}q\ell$$

$$T_B^+ = q\frac{\ell}{2} + \frac{7}{8}q\ell = \frac{11}{8}q\ell$$

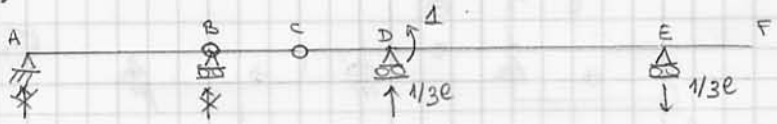
$$T_C = -q\frac{\ell}{2} + \frac{7}{8}q\ell = \frac{3}{8}q\ell$$

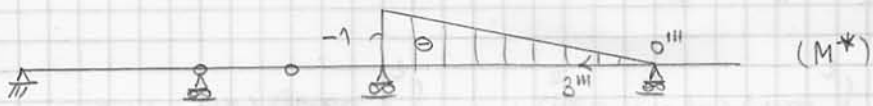
$$T_D^+ = -\frac{1}{3\ell}\left(\frac{3}{8} + 2\right)q\ell^2 = -\frac{19}{24}q\ell$$



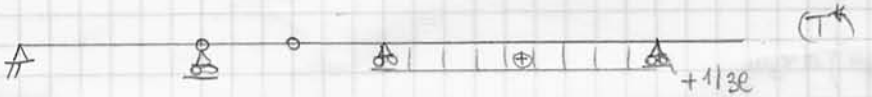
$$\overline{M} = \frac{1}{2}\left(\frac{9}{16}\right)^2 q\ell^2 = \frac{81}{512}q\ell^2 = 2{,}37 \text{ kNm}$$

A2)





(M^*)



(T^*)

$$1 \cdot \varphi_D = \frac{1}{EI_1}\int_0^{3\ell}\left(-\frac{x_3'''}{3\ell}\right)\left(-2q\ell^2 + \frac{19}{24}q\ell\, x_3'''\right)dx_3''' = \frac{5q\ell^3}{8EI_1} = \frac{5 \cdot 15 \cdot 10^{-2} \cdot 100^3}{8 \cdot 2{,}1 \cdot 10^3 \cdot 1943} = 1{,}31°$$

A3)

$$M_{1t} = \int_0^{3\ell} M_1 X_t \, d\alpha_{3''} = X_t \, (-1) \frac{3\ell}{2} = \left(-\frac{8\alpha\Delta T}{H}\right)\left(-\frac{3\ell}{2}\right) = +\frac{3\ell\alpha\Delta T}{H}$$

$$M_{1t} + M_{11} X_1 = - M_{10}$$

$$X_1 = -\frac{M_{10}}{M_{11}} - \frac{M_{1t}}{M_{11}} = -\frac{7}{8}q\ell^2 - \frac{3\ell\alpha\Delta T E I_1}{H}\frac{3}{7\ell} = -\frac{7}{8}q\ell^2 - \frac{9EI_1\alpha\Delta T}{7H}$$

$$= (-13{,}12 - 0{,}026) \text{ kN m}$$

$$= -13{,}14 \text{ kN m}$$

I diagrammi delle caratteristiche della sollecitazione non differiscono significativamente da quelli riportati al punto (1).