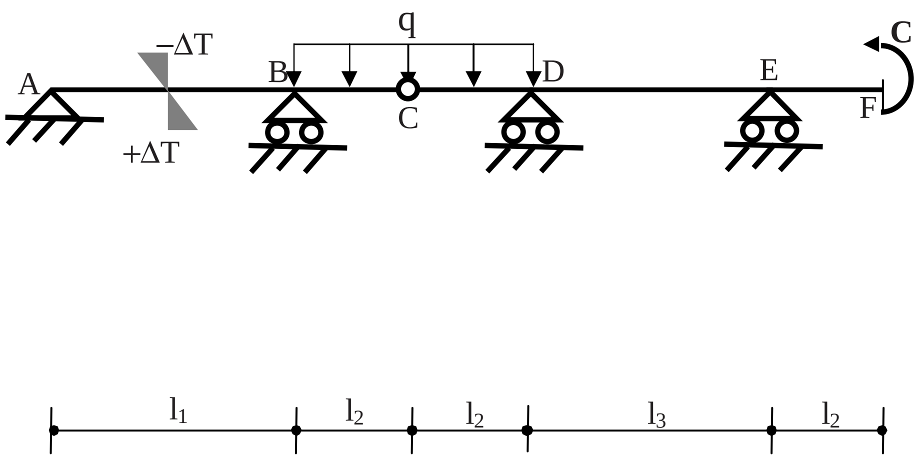
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA PARZIALE DI STATICA
FERRARA, 24/11/2009

B

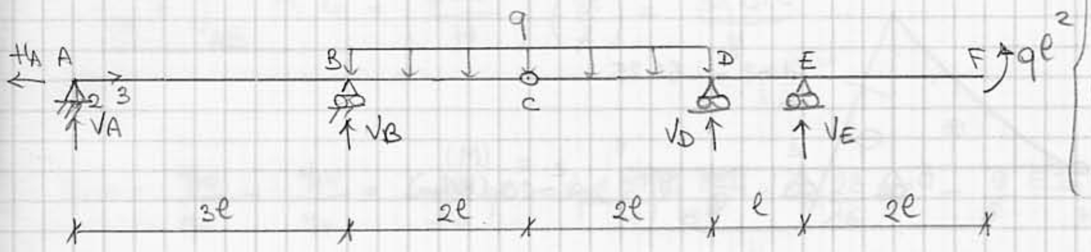


l_1 = 3 m, l_2= 2 m, l_3 = 1 m,
q = 20 kN/m, C = 20 kN m,
E = 2.1 10^3 kN/cm^2, α= 10^{-5} °C^{-1}, ΔT = 20 °C

La travatura iperstatica di figura è realizzata con profilati IPE 240 (H= 240 mm, A = 39.1 cm^2, I_1 = 3892 cm^4).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo B.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel solo tratto AB e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, C che di ΔT.

B1)



$$\begin{cases} (\rightarrow) & H_A = 0 \\ (C\circlearrowleft)_{CDEF} & V_D 2l + V_E 3l = 2ql^2 - ql^2 \\ (C\circlearrowleft)_{ABC} & V_A 5l + V_B 2l = 2ql^2 \\ (\uparrow) & V_A + V_B + V_D + V_E = 4ql \end{cases}$$

$q = 20\ kN/m$

$l = 1\ m, \quad C = 20\ kNm = 20ql^2$

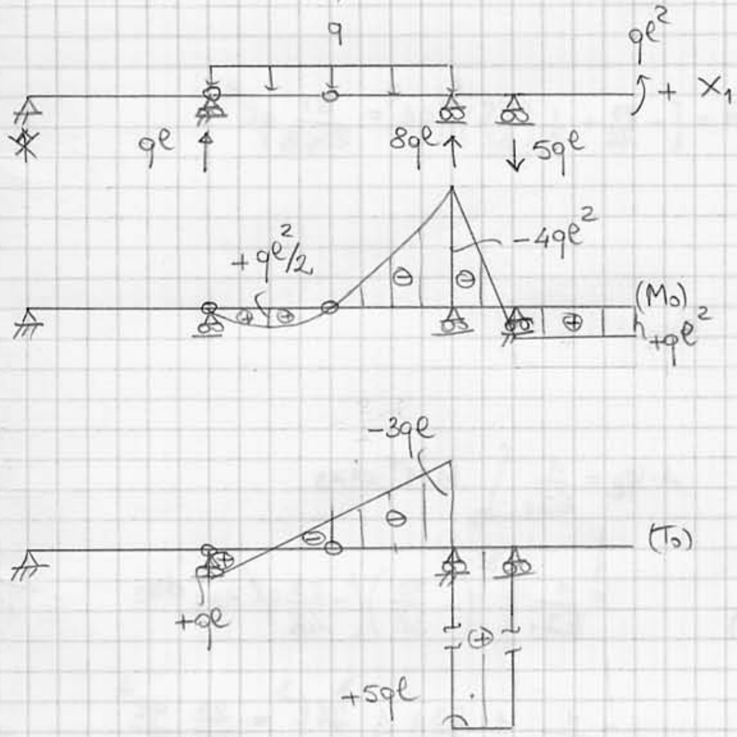
$$\begin{cases} H_A = 0 \\ V_D = q\frac{l}{2} - \frac{3}{2}V_E \\ V_B = ql - \frac{5}{2}V_A \\ V_A + ql - \frac{5}{2}V_A + q\frac{l}{2} - \frac{3}{2}V_E + V_E = 4ql \end{cases}$$

$$\begin{cases} H_A = 0 \\ V_D = q\frac{l}{2} - \frac{3}{2}V_E = 8ql + \frac{9}{2}V_A \\ V_B = ql - \frac{5}{2}V_A \\ V_E = -5ql - 3V_A \end{cases}$$

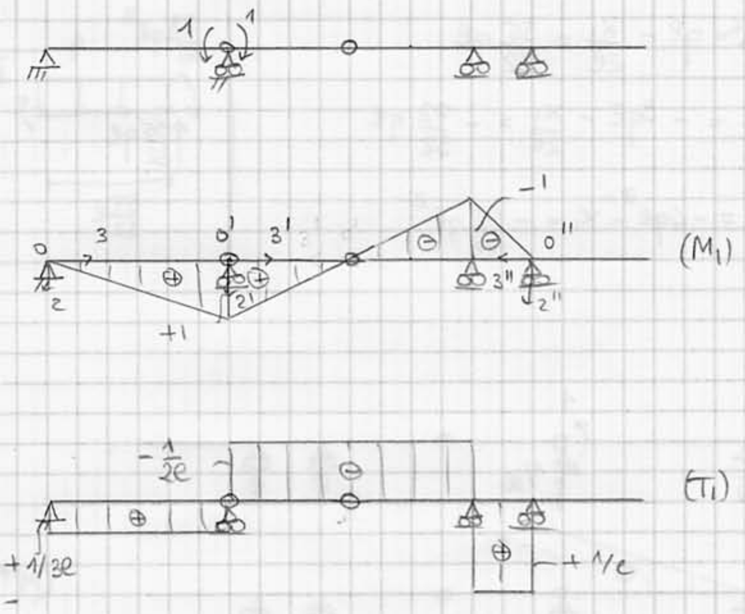
Struttura una volta iperstatica.

Incognita iperstatica: $X_1 = M_B$.

(0)



(1)

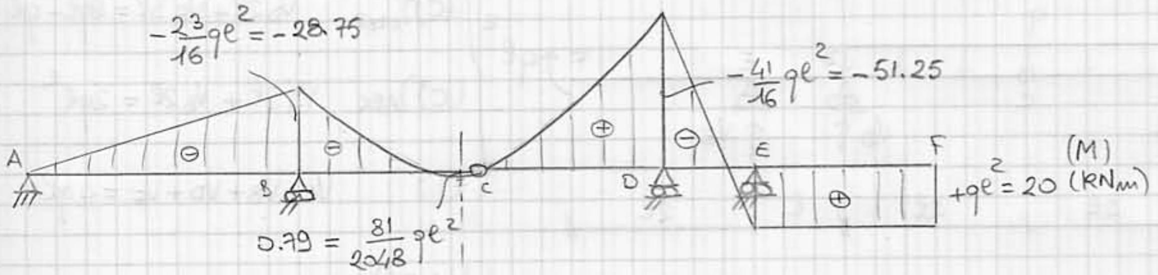


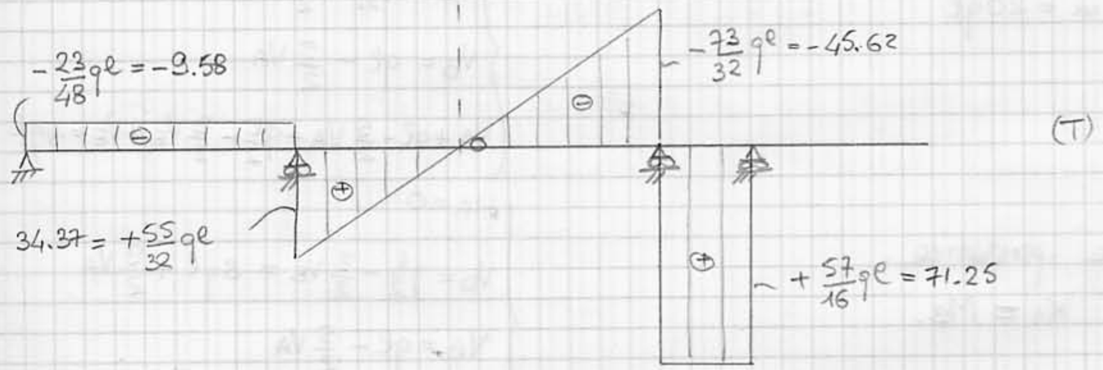
$$EI_1\eta_{10} = \int_0^{4l}\left(ql x_3' - q\frac{x_3'^2}{2}\right)\left(1 - \frac{x_3'}{2l}\right)dx_3' + \int_0^{l}\left(ql^2 - 5ql x_3''\right)\left(-\frac{x_3''}{l}\right)dx_3'' = \frac{8}{3}ql^3 + \frac{7}{6}ql^3 = \frac{23}{6}ql^3$$

$$EI_1\eta_{11} = \int_0^{3l}\left(\frac{x_3}{3l}\right)^2 dx_3 + 2\int_0^{2l}\left(1 - \frac{x_3'}{2l}\right)^2 dx_3' + \int_0^{l}\left(-\frac{x_3''}{l}\right)^2 dx_3'' = \frac{1}{3}3l + 2\frac{1}{3}2l + \frac{1}{3}l = \frac{8}{3}l$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{23}{\cancel{6}_2}ql^3\frac{\cancel{3}}{8l} = -\frac{23}{16}ql^2 = -28.75\ kNm$$

Diagrammi quotati:





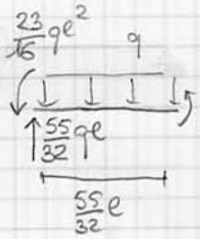
(N) = 0

Calcoli:

$$T_B^+ = q\ell - \frac{x_1}{2\ell} = \frac{55}{32}q\ell$$

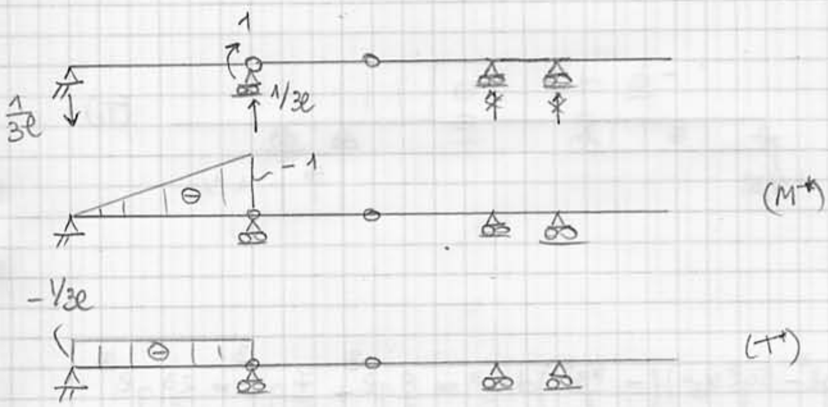
$$T_D^- = -3q\ell - \frac{x_1}{2\ell} = -\frac{73}{32}q\ell$$

$$M_D = -4q\ell^2 - x_1 = -\frac{41}{16}q\ell^2$$



$$\bar{M} = \left[-\frac{23}{16} + \frac{1}{2}\left(\frac{55}{32}\right)^2\right]q\ell^2 = \frac{81}{2048}q\ell^2$$

B2)



$$1 \cdot \varphi_B = \frac{1}{EI_1}\int_S M M^* \, dx_3$$

$$= \frac{1}{EI_1}\int_0^{3\ell}\left(-\frac{x_3}{3\ell}\right)\left(-\frac{23}{48}q\ell x_3\right)dx_3$$

$$= \frac{1}{EI_1}\,\frac{1}{\cancel{3}}\,\frac{23}{\underset{16}{\cancel{48}}}q\,\frac{1}{\cancel{3}}\,\overset{3}{\cancel{27}}\ell^3 = \frac{23}{16}\frac{q\ell^3}{EI_1}$$

$$= 2{,}01^\circ$$

B3)

$$\eta_{1t} = \int_{AB} M_1 \chi_t = \left(\frac{\cancel{7}\alpha \Delta T}{H}\right)\left(\frac{3l}{\cancel{7}}\right) = \frac{3\alpha \Delta T l}{H}$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1T}}{\eta_{11}} = -\frac{23}{16} q l^2 - \frac{3\alpha \Delta T \cancel{l}}{H} \frac{3EI_1}{8\cancel{l}} = -\frac{23}{16} q l^2 - \frac{9}{8}\frac{EI_1 \alpha \Delta T}{H}$$

$$= -28.75 - 0{,}76 \quad \text{kNm}$$

$$= -29.51 \ \text{kNm}$$

I diagrammi delle c.s. complessivi sia di q, che di ΔT non differiscono in modo significativo da quelli calcolati al punto (1).