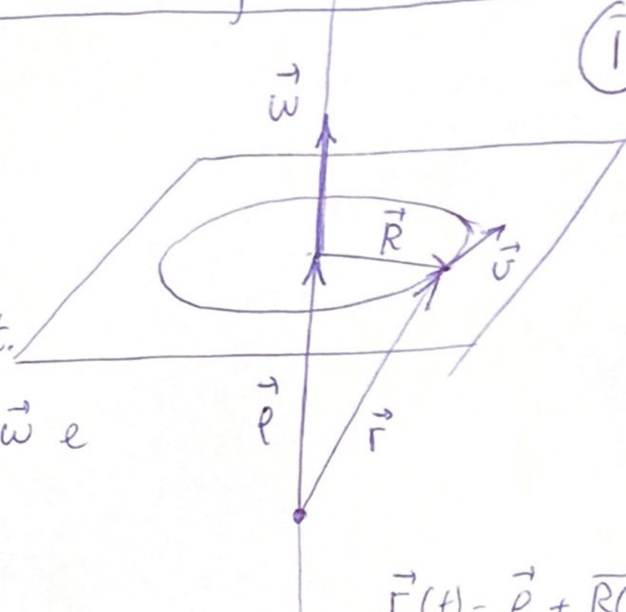
Equazione di un vettore di modulo costante in rotazione uniforme attorno ad un asse fisso di rotazione

Dal MCU si sa:

$$\vec{v}(t) = \vec{\omega} \times \vec{R}(t)$$

$$\vec{\omega} = \text{costante}$$

$$|\vec{v}(t)| = \text{cost.} \quad |\vec{R}(t)| = \text{cost.}$$



$$\vec{r}(t) = \vec{\rho} + \vec{R}(t)$$

Sia $\vec{\rho}$ vettore parallelo a $\vec{\omega}$ e costante

$$\frac{d\vec{\rho}}{dt} = 0 \qquad \vec{\rho} \times \vec{\omega} = 0$$

Dalla prima equazione

$$\frac{d\vec{R}}{dt} = \vec{\omega} \times \vec{R}(t)$$

Per cui

$$\frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} + \underbrace{\frac{d\vec{\rho}}{dt}}_{0} = \vec{\omega} \times \vec{R}(t) + \underbrace{\vec{\omega} \times \vec{\rho}}_{0} = \vec{\omega} \times \underbrace{\left(\vec{R}(t) + \vec{\rho}\right)}_{\vec{r}(t)}$$

$$\boxed{\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}(t)}$$

Tale espressione è utile per semplificare le formule della cinematica dei moti relativi.

I versori della base di S_R sono tali che:

$$\boxed{\frac{d\vec{E}_J}{dt} = \vec{\omega} \times \vec{E}_J(t)} \qquad J = 1, 2, 3$$

Moto di S_R in rotazione uniforme rispetto a S_R (2)

$$\vec{r}_A(t) = \vec{r}_R(t) + \vec{r}_\Omega(t) \qquad \frac{d\vec{E}_J}{dt} = \vec{\omega} \times \vec{E}_J \quad J = 1, 2, 3$$

- $$\vec{v}_A(t) = \vec{v}_R(t) + \vec{v}_{tr}(t)$$

$$\vec{v}_{tr}(t) = \vec{v}_\Omega(t) + \sum_{J=1}^{3} X_J \frac{d\vec{E}_J}{dt} = \vec{v}_\Omega(t) + \underbrace{\sum_{J=1}^{3} X_J \vec{\omega} \times \vec{E}_J}_{\vec{\omega} \times \underbrace{\sum_{J=1}^{3} X_J \vec{E}_J}_{\vec{r}_R(t)}}$$

$$\boxed{\vec{v}_{tr}(t) = \vec{v}_\Omega(t) + \vec{\omega} \times \vec{r}_R(t)}$$

- $$\vec{a}_A(t) = \vec{a}_R(t) + \vec{a}_{tr}(t) + \vec{a}_{co}(t)$$

$$\vec{a}_{co}(t) = 2 \sum_{J=1}^{3} \frac{dX_J}{dt} \frac{d\vec{E}_J}{dt} = 2 \sum_{J=1}^{3} \frac{dX_J}{dt} \vec{\omega} \times \vec{E}_J = 2 \vec{\omega} \times \vec{v}_R(t)$$

$$\boxed{\vec{a}_{co}(t) = 2 \vec{\omega} \times \vec{v}_R(t)}$$

$$\vec{a}_{tr}(t) = \vec{a}_\Omega(t) + \sum_{J=1}^{3} X_J \frac{d^2\vec{E}_J}{dt^2} = \vec{a}_\Omega(t) + \sum_{J=1}^{3} X_J \frac{d}{dt}\left[\vec{\omega} \times \vec{E}_J\right]$$

$$= \vec{a}_\Omega(t) + \vec{\omega} \times \sum_{J=1}^{3} X_J \frac{d\vec{E}_J}{dt} + \vec{a}_\Omega(t) + \vec{\omega} \times \left(\sum_{J=1}^{3} X_J \vec{\omega} \times \vec{E}_J\right) +$$

$$= \vec{a}_\Omega(t) + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_R\right)$$

$$\boxed{\vec{a}_{tr}(t) = \vec{a}_\Omega(t) + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_R(t)\right)}$$

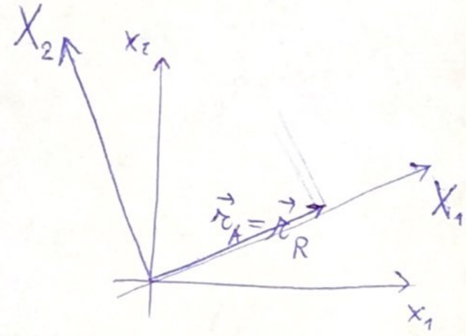
Le formule di $\vec{v}_{tr}(t)$, $\vec{a}_{tr}(t)$ e $\vec{a}_{co}(t)$ risultano semplificate

Es (Studio del MCU) 3

sistema S_R rotante uniformemente ($\vec{\omega} = \text{cost}$) attorno a S_R

$\vec{r}_\Omega = 0$

(a) caso $\vec{v}_R = 0$



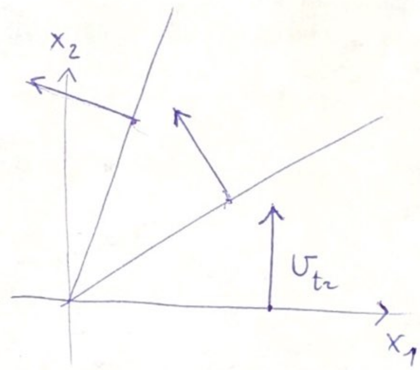
$$\vec{r}_A = \vec{r}_\Omega + \vec{r}_R = \vec{r}_R$$

$$\vec{v}_A = \vec{v}_R + \vec{v}_{tr} = \vec{v}_{tr}$$

$$\vec{v}_{tr} = \underbrace{\vec{v}_\Omega}_{0} + \sum_{J=1}^{3} X_J \frac{d\vec{E}_J}{dt} = \sum_{J=1}^{3} X_J (\vec{\omega} \times \vec{E}_J) = \vec{\omega} \times \sum_{J=1}^{3} X_J E_J =$$

$$= \vec{\omega} \times \vec{r}_R$$

$$\boxed{\vec{v}_{tr} = \vec{\omega} \times \vec{r}_R}$$



$$\vec{a}_A = \underbrace{\vec{a}_R}_{0} + \vec{a}_{tr} + \vec{a}_{Co}$$

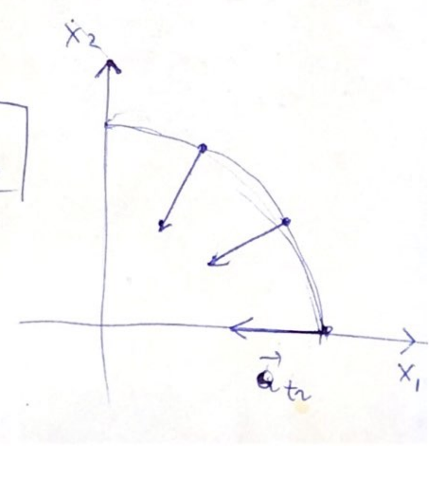
$$\vec{a}_{Co} = 2\vec{\omega} \times \vec{v}_R = 0$$

$$\vec{a}_{tr} = \underbrace{\vec{a}_\Omega}_{0} + \sum_{J=1}^{3} X_J \frac{d^2\vec{E}_J}{dt^2} =$$

$$= \sum_{J=1}^{3} X_J \frac{d}{dt}(\vec{\omega} \times E_J) = \sum_{J=1}^{3} X_J \vec{\omega} \times \frac{d\vec{E}_J}{dt} = \sum_{J=1}^{3} X_J \vec{\omega} \times (\vec{\omega} \times \vec{E}_J) =$$

$$= \vec{\omega} \times \Big(\vec{\omega} \times \underbrace{\sum_{J=1}^{3} X_J E_J}_{\vec{r}_R}\Big) = \vec{\omega} \times \vec{v}_{tr}$$

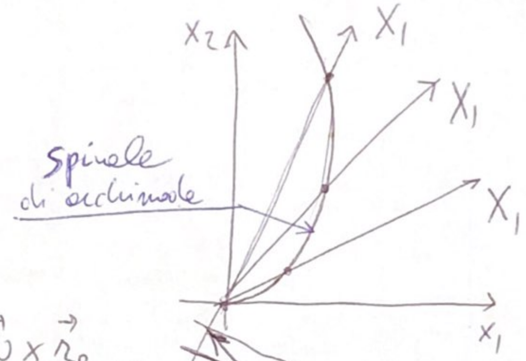
$$\boxed{\vec{a}_{tr} = \vec{\omega} \times \vec{v}_{tr}} \qquad \boxed{\vec{a}_A = \vec{\omega} \times \vec{v}_{tr}}$$



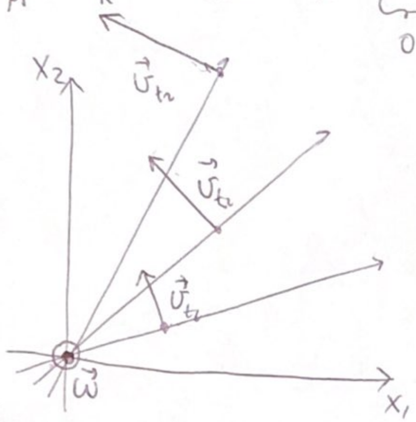
(b) caso $\vec{v}_R \neq 0$ lungo X_1 con MRU $\vec{v}_R = \vec{v} = cost = v\vec{E}_1$

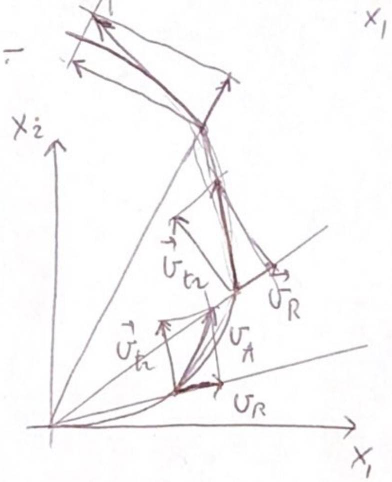
$$\vec{r}_A = \vec{r}_R = \boxed{v t \, \vec{E}_1(t)}$$

$$= v t \left(\cos \omega t \, \vec{e}_1 + \sin \omega t \, \vec{e}_2\right)$$

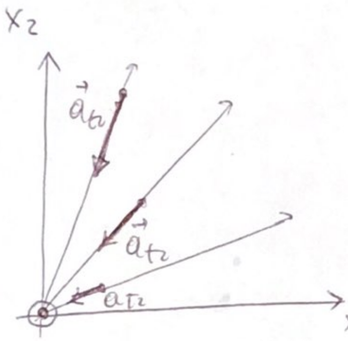


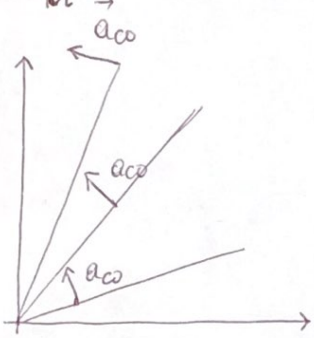
$$\vec{v}_A = \vec{v}_R + \vec{v}_{tr} = \vec{v}_R + \underbrace{\vec{v}_\Omega}_{0} + \underbrace{\vec{\omega} \times \vec{r}_R}_{v_{tr}} =$$

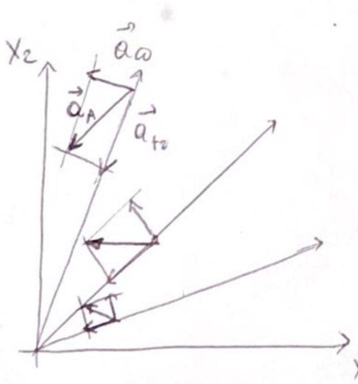




$$\vec{a}_A = \underbrace{\vec{a}_R}_{0} + \underbrace{\vec{a}_{tr}}_{\vec{a}_\Omega + \vec{\omega} \times (\vec{\omega} \times \vec{r}_R) = \vec{\omega} \times \vec{v}_{tr}} + \overbrace{\vec{a}_{co}}^{2\vec{\omega} \times \vec{v}_R}$$







Oss.

L'esempio trova applicazione nella progettazione delle pompe centrifughe, in particolare nel dimensionamento delle palette