Funzione di una variabile

$$X = x_m \pm \delta x = x_m\left(1 \pm \frac{\delta x}{|x_m|}\right) \qquad \frac{\delta x}{|x_m|} \ll 1$$

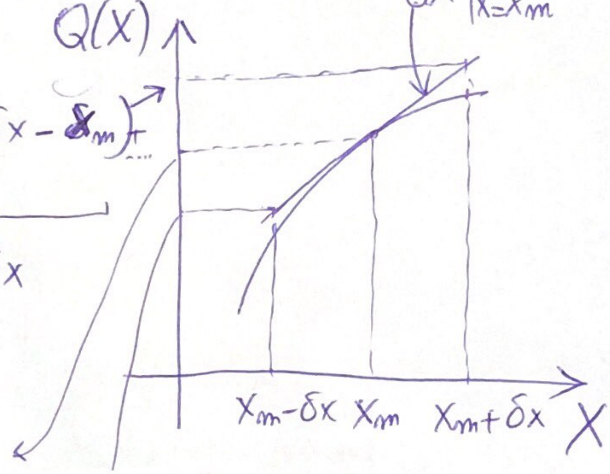
$$Q = Q(X)$$

$$Q = q_m \pm \delta q \qquad q_m = ? \quad \delta q = ?$$

retta tangente
$$Q(x_m) + \left.\frac{dQ}{dX}\right|_{X=x_m}(X - x_m)$$

1) Q funzione crescente

$$Q(x_m+\delta x) = \underbrace{Q(x_m)}_{q_m} + \underbrace{\left.\frac{dQ}{dX}\right|_{X=x_m}}_{0 < \left.\frac{dQ}{dX}\right|_{X=x_m}} \underbrace{(x_m+\delta x - x_m)}_{\delta x} + \dots$$

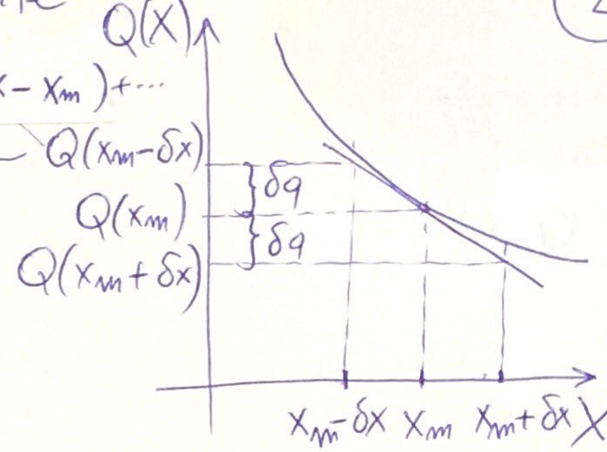


$$Q(x_m) = q_m$$

$$Q(x_m-\delta x) = \underbrace{Q(x_m)}_{q_m} + \underbrace{\left.\frac{dQ}{dX}\right|_{X=x_m}}_{\left.\frac{dQ}{dX}\right|_{X=x_m} > 0} \underbrace{(x_m - \delta x - x_m)}_{-\delta x}$$

$$\Rightarrow \quad \delta Q = \underbrace{|Q(x+x_m) - Q(x_m)|}_{>0} = \left|\left.\frac{dQ}{dX}\right|_{X=x_m}\right| \delta x$$

2) Q funzione decrescente



$$Q(x_m-\delta x) = \underbrace{Q(x_m)}_{q_m} + \underbrace{\left.\frac{dQ}{dX}\right|_{X=x_m}}_{<0} + \underbrace{(x_m-\delta x-x_m)}_{-\delta x} + \cdots$$

$$Q(X) = q_m - \left.\frac{dQ}{dX}\right|_{X=x_m} \delta x$$

$$Q(x_m+\delta x) = \underbrace{Q(x_m)}_{q_m} + \underbrace{\left.\frac{dQ}{dX}\right|_{X=x_m}}_{<0} \underbrace{(x_m+\delta x-x_m)}_{\delta x} + \cdots$$

$$\delta q = \Big|\underbrace{Q(x_m+\delta x) - Q(x_m)}_{<0}\Big| = \left|\left.\frac{dQ}{dX}\right|_{X=x_m} \delta x\right| =$$

$$= \left|\left.\frac{dQ}{dX}\right|_{X=x_m}\right| \delta x$$

In generale:

$$\boxed{\delta q = \left|\left.\frac{dQ}{dX}\right|_{X=x_m}\right| \delta x}$$

Es 1

$$Q = \lg X \quad \Rightarrow \quad \delta q = \frac{1}{|x_m|}\delta x$$

Es 2

$$Q = XY \qquad \lg Q = \lg X + \lg Y$$

$$\boxed{\frac{\delta q}{|q_m|} = \frac{\delta x}{|x_m|} + \frac{\delta y}{|y_m|}}$$

è la regola del prodotto!

(3)

Funzione di due variabile

$X = x_m \pm \delta x$

$Y = y_m \pm \delta y$

$Q = Q(X, Y)$

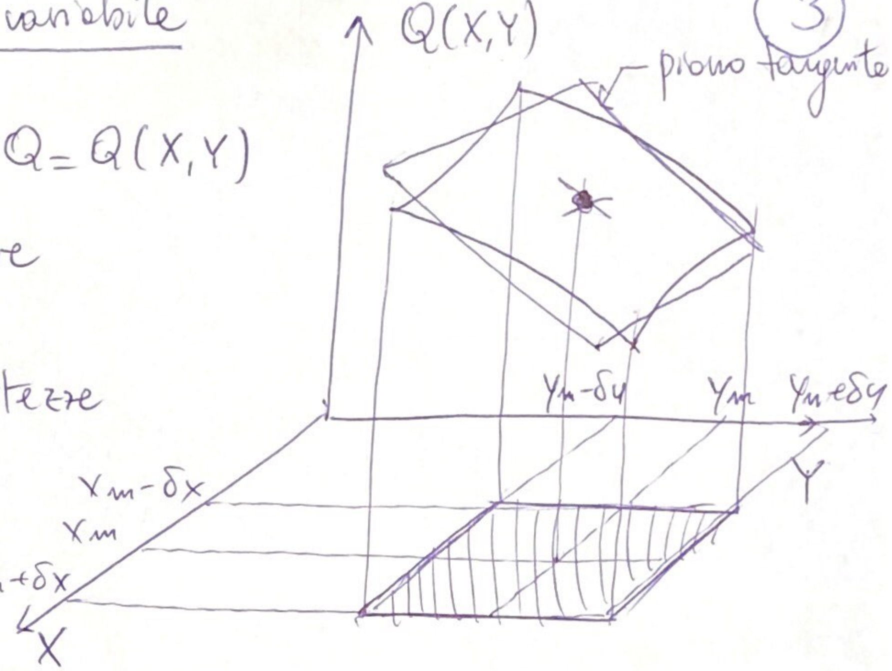
si intende valutare

$Q = q_m \pm \delta q$

per piccole incertezze

$\delta x \ll |x_m|$

$\delta y \ll |y_m|$



Si ricava che

$q_m = q(x_m, y_m)$

Eseguo uno sviluppo di Taylor al I ordine

$$Q(X,Y) = Q(x_m, y_m) + \left.\frac{\partial Q}{\partial X}\right|_{\substack{X=x_m \\ Y=y_m}} (X - x_m) + \left.\frac{\partial Q}{\partial Y}\right|_{\substack{X=x_m \\ Y=y_m}} (Y - y_m)$$

$$\delta q = \left| Q(x_m \pm \delta x, y_m \pm \delta y) - Q(x_m, y_m) \right| =$$

$$= \left| \pm \left.\frac{\partial Q}{\partial X}\right|_{\substack{X=x_m \\ Y=y_m}} \underbrace{(X - x_m)}_{\delta x} \pm \left.\frac{\partial Q}{\partial Y}\right|_{\substack{X=x_m \\ Y=y_m}} \underbrace{(Y - y_m)}_{\delta y} \right| \leq$$

stima prudente

$$\boxed{\delta q \leq \left| \left.\frac{\partial Q}{\partial X}\right|_{\substack{X=x_m \\ Y=y_m}} \right| \delta x + \left| \left.\frac{\partial Q}{\partial Y}\right|_{\substack{X=x_m \\ Y=y_m}} \right| \delta y}$$

Es 1

$Q = X + Y$

$\delta q \approx \delta x + \delta y$

Generalizzazione

$$\boxed{\delta q = \sum_{j=1}^{n} \left| \left.\frac{\partial Q}{\partial X_j}\right|_m \right| \delta x_j}$$

Es 2

$Q = XY$

$\delta q = |y_m| \delta x + |x_m| \delta y \Rightarrow \frac{\delta q}{|q_m|} = \frac{\delta x}{|x_m|} + \frac{\delta y}{|y_m|}$

Oss

Nel caso di variabili indipendenti la somma lineare è sostituita dalla "somma in quadratura":

$$\delta q = \left[\sum_{J=1}^{n} \left(\frac{\partial Q}{\partial X_J}\bigg|_m \right)^2 (\delta x_J)^2 \right]^{1/2}$$

Es

$Q = XY$ $\quad$ X e Y indipendenti $\qquad$ $q_m = x_m y_m$

$$\delta q = \left[(y_m)^2 (\delta x)^2 + (x_m)^2 (\delta y)^2 \right]^{1/2}$$

$$\Rightarrow \frac{\delta q}{q_m} = \left[\left(\frac{\delta x}{x_m} \right)^2 + \left(\frac{\delta y}{y_m} \right)^2 \right]^{1/2}$$