Moto circolare uniforme in coordinate cartesiane

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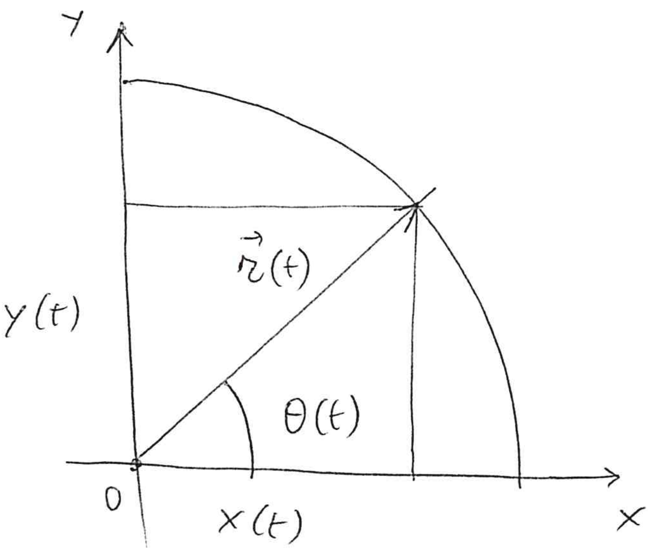


$$\theta(t) = \theta_0 + \omega t$$

$$\omega = \text{costante} = \frac{v}{R}$$

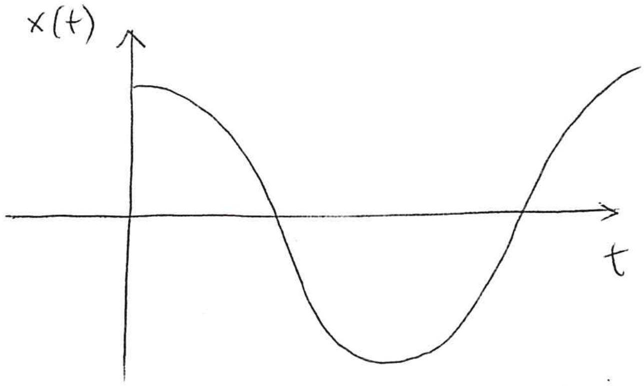
$$x^2(t) + y^2(t) = R^2 \quad \forall t$$

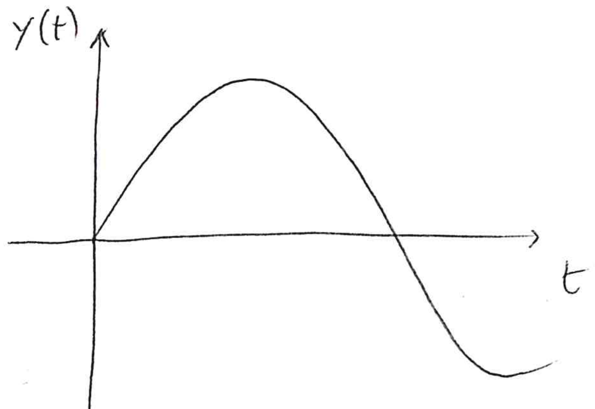
$$\operatorname{arctg} \frac{y(t)}{x(t)} = \theta(t)$$

$$\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j}$$

$$x(t) = R\cos\theta(t)$$
$$y(t) = R\sin\theta(t)$$

$$\vec{r}(t) = R\cos(\omega t + \theta_0)\,\vec{i} + R\sin(\omega t + \theta_0)\,\vec{j}$$





tali moti di proiezione sono detti armonici

$$|\vec{r}(t)| = \left[R^2\cos^2(\omega t + \theta_0) + R^2\sin^2(\omega t + \theta_0)\right]^{1/2} = R \quad \forall t$$

$$\vec{N} = -\cos\theta\,\vec{i} - \sin\theta\,\vec{j}$$

$$\boxed{\vec{r}(t) = -R\,\vec{N}}$$

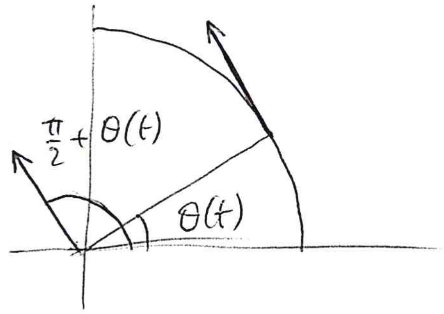
$\bullet\ \vec{v}(t) = \frac{d\vec{r}}{dt} = -R\omega \sin(\omega t + \theta_0)\,\vec{i} + R\omega\cos(\omega t + \theta_0)\,\vec{j}$

$\vec{r}(t)\cdot\vec{v}(t) = -R^2\omega \sin(\ )\cos(\ ) + R^2\omega \sin(\ )\cos(\ ) = 0$

$\Rightarrow\quad \vec{r}(t) \perp \vec{v}(t) \quad \forall t$

$\vec{T}(t) = -\sin\theta(t)\,\vec{i} + \cos\theta(t)\,\vec{j} = \cos\left(\theta(t) + \frac{\pi}{2}\right)\vec{i} + \sin\left(\theta(t) + \frac{\pi}{2}\right)\vec{j}$

$\vec{N}(t)\cdot\vec{T}(t) = 0$



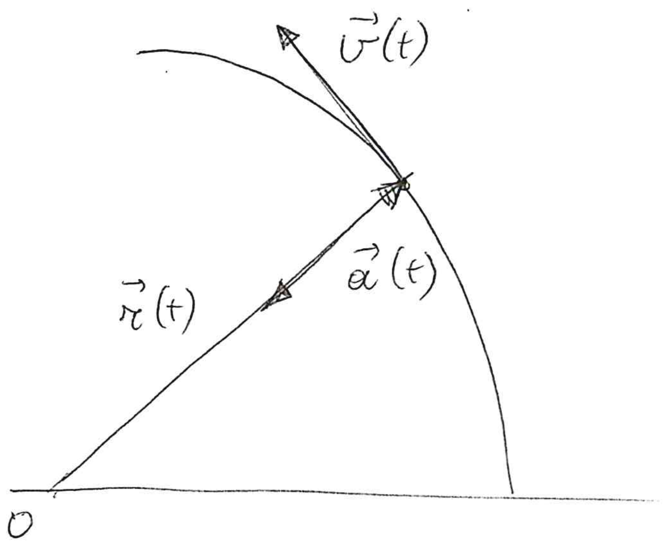
$\boxed{\vec{v}(t) = R\omega\,\vec{T}(t)}$

$|\vec{v}(t)| = R|\omega| =$ costante

$\bullet\ \vec{a}(t) = \frac{d\vec{v}}{dt} = -R\omega^2\cos(\omega t + \theta_0)\,\vec{i} - R\omega^2 \sin(\omega t + \theta_0)\,\vec{j} =$

$\boxed{\vec{a}(t) = -\omega^2\vec{r}(t)}$ $\qquad \vec{a}(t) = \omega^2 R\,\vec{N} =$ accel. centripeta

$|\vec{a}(t)| = R\omega^2 = |v\omega| = \frac{v^2}{R}$



Periodo, frequenza per il MCU

il MCU è un moto periodico, ossia $\exists\, T>0$

$$\vec r(t+T) = \vec r(t) \quad \forall t \qquad \boxed{T = \frac{2\pi}{\omega}}$$

il più piccolo T è detto periodo

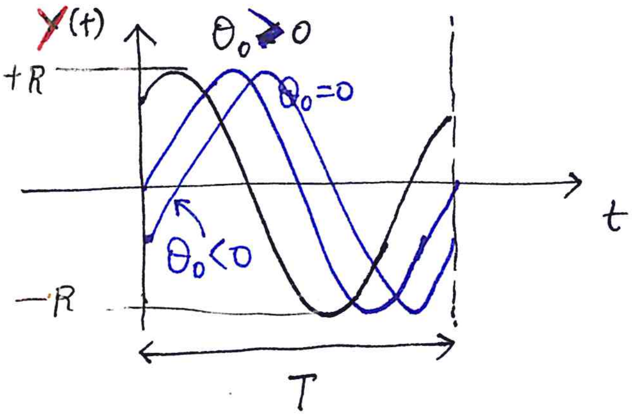
infatti

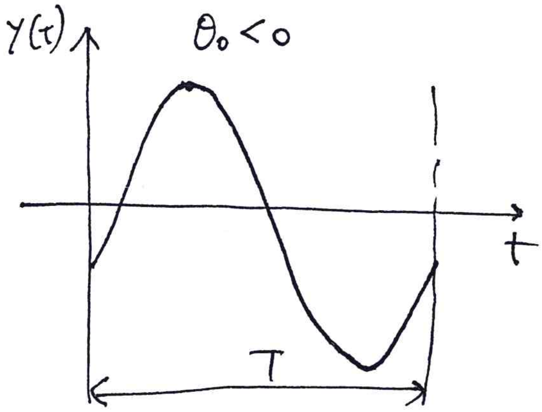
$$\vec r(t+T) = \cos(\omega t + \omega \underbrace{T}_{2\pi/\omega} + \theta_0)\,\vec i + \sin(\omega t + \omega \underbrace{T}_{2\pi/\omega} + \theta_0)\,\vec j =$$

$$= \cos(\omega t + \theta_0 + 2\pi)\,\vec i + \sin(\omega t + \theta_0 + 2\pi)\,\vec j =$$

$$= \cos(\omega t + \theta_0)\,\vec i + \sin(\omega t + \theta_0)\,\vec j = \vec r(t) \quad \forall t$$

Questo implica che anche $x(t)$, $y(t)$ sono periodiche



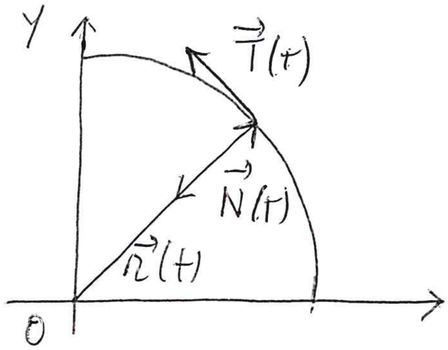


$$\text{Frequenza} = \frac{\text{numero giri}}{\text{int. tempo}} = \frac{\overbrace{\text{angolo percorso}}^{\Delta\theta}/2\pi}{\underbrace{\text{int. tempo}}_{\Delta t}} = \frac{\omega}{2\pi} = \nu$$

$$[\nu] = [L]^0 [T]^{-1} [M]^0 = [Hz]$$

$$\boxed{\nu = \frac{\omega}{2\pi} = \frac{1}{T}}$$

Moto circolare vario



$$\text{2-D}\begin{cases}\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j}\\ x(t) = R\cos\theta(t)\\ y(t) = R\sin\theta(t)\end{cases}$$

$$\text{1-D}\begin{cases} s(t) = R\,\tilde{\theta}(t)\\ v(t) = R\,\omega(t)\\ a(t) = R\,\alpha(t)\end{cases}$$

- $\vec{r}(t) = R\cos\theta(t)\,\vec{i} + R\sin\theta(t)\,\vec{j}$

  $\vec{N}(t) = -\cos\theta(t)\,\vec{i} - \sin\theta(t)\,\vec{j}$

  $$\boxed{\vec{r}(t) = -R\,\vec{N}(t)}$$

- $\vec{v}(t) = \dfrac{d\vec{r}}{dt} = -R\dfrac{d\theta}{dt}\sin\theta(t)\,\vec{i} + R\dfrac{d\theta}{dt}\cos\theta(t)\,\vec{j}$

  $\vec{r}(t)\cdot\vec{v}(t) = -R^2\dfrac{d\theta}{dt}\cos\theta(t)\sin\theta(t) + R^2\dfrac{d\theta}{dt}\cos\theta(t)\sin\theta(t) = 0\ \forall$

  $\vec{T}(t) = -\sin\theta(t)\,\vec{i} + \cos\theta(t)\,\vec{j}$

  $\vec{v}(t) = R\dfrac{d\theta}{dt}\vec{T}(t) = \underbrace{R\omega(t)}_{\text{vel. scal.}}\vec{T}(t) = \dfrac{ds}{dt}\vec{T}(t)$ $\qquad\boxed{\vec{v}(t) = \dfrac{ds}{dt}\vec{T}(t)}$

  $|\vec{v}(t)| = |R\omega\,\vec{T}(t)| = R|\omega| = |\text{vel. scal.}| = \left|\dfrac{ds}{dt}\right|$

- $\vec{a}(t) = \dfrac{d\vec{v}}{dt} = -R\dfrac{d^2\theta}{dt^2}\sin\theta(t)\,\vec{i} - R\left(\dfrac{d\theta}{dt}\right)^2\cos\theta(t)\,\vec{\lambda} +$

  $\qquad + R\dfrac{d^2\theta}{dt^2}\cos\theta(t)\,\vec{j} - R\left(\dfrac{d\theta}{dt}\right)^2\sin\theta(t)\,\vec{j} =$

  $$\boxed{\vec{a}(t) = R\frac{d^2\theta}{dt^2}\vec{T}(t) + R\left(\frac{d\theta}{dt}\right)^2\vec{N}(t)} =$$

  $$= R\,\alpha(t)\,\vec{T}(t) + R\,\omega^2_{(t)}\,\vec{N}(t) = \frac{d^2s}{dt^2}\vec{T}(t) + \frac{v^2}{R}\vec{N}(t)$$