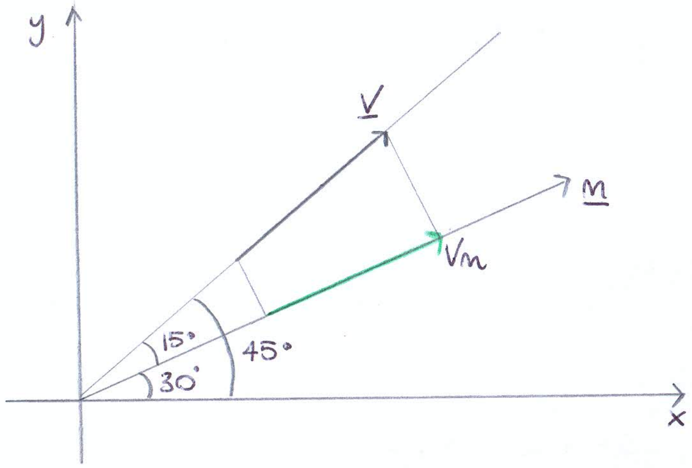
COMPONENTE DI UN VETTORE SULLA RETTA ORIENTATA

Dato il vettore e la retta orientata determina la proiezione ortogonale del vettore sulla retta.

1



$\underline{V} = (2{,}5 \,;\, 2{,}5)$

$\underline{M} = (\cos 30° \,;\, \cos 60°)$

$\cos 30° \to \frac{\sqrt{3}}{2}$ $\qquad \cos 60° \to \frac{1}{2}$

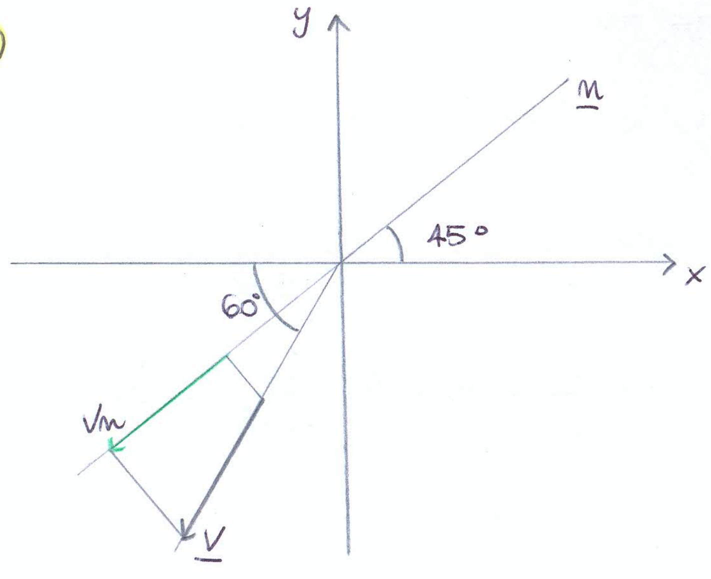
PROCEDIMENTO 1

$V_m = V \cos 15° = \left(\sqrt{2{,}5^2 + 2{,}5^2}\right) \cdot \cos 15° = 3{,}415$

PROCEDIMENTO 2 (prodotto scalare)

$\underline{V} \cdot \underline{M} = V_x \cdot M_x + V_y \cdot M_y = 2{,}5 \cdot \frac{\sqrt{3}}{2} + 2{,}5 \cdot \frac{1}{2} = 3{,}415$

2



$\underline{V} = (-4 \cos 60° \,;\, -4 \cos 30°)$

$\cos 60° \to \frac{1}{2}$ $\qquad \cos 30° \to \frac{\sqrt{3}}{2}$

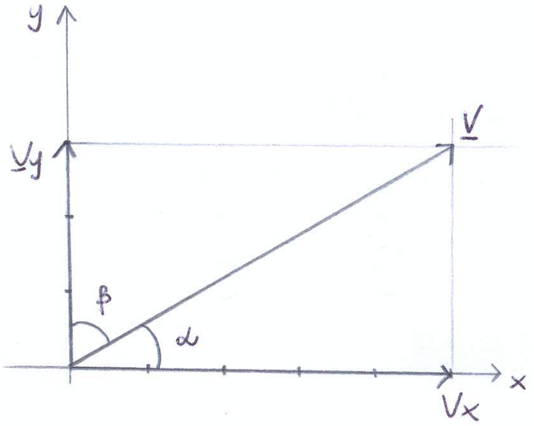
$\underline{M} = (\cos 45° \,;\, \cos 45°)$

$\cos 45° \to \frac{\sqrt{2}}{2}$ $\qquad \cos 45° \to \frac{\sqrt{2}}{2}$

$V_m = \underline{V} \cdot \underline{M} = V_x \cdot M_x + V_y \cdot M_y = \left(-2 \cdot \frac{\sqrt{2}}{2}\right) + \left(-\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{2}}{2}\right) = -2{,}2307$

Ricavare gli angoli α e β dato il vettore $\underline{V}$.

(1)



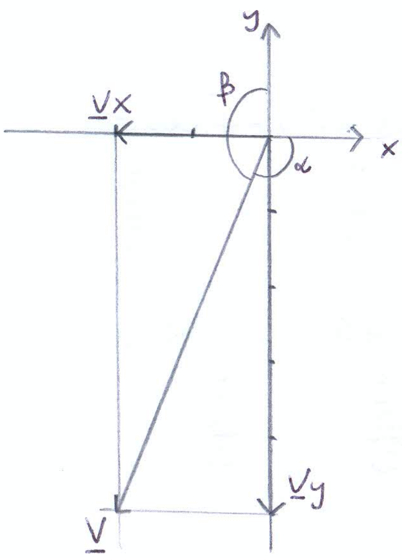
$$\underline{V} = (5,3)$$

$$V = |\underline{V}| = \sqrt{Vx^2 + Vy^2} = \sqrt{25+9} = \sqrt{34} = 5{,}83$$

$$\cos\alpha = \frac{Vx}{V} = \frac{5}{5{,}83} = 0{,}8576 \rightarrow \alpha = \arccos 0{,}8576 = 30{,}95°$$

$$\cos\beta = \frac{Vy}{V} = \frac{3}{5{,}83} = 0{,}5145 \rightarrow \beta = \arccos 0{,}5145 = 59.05°$$

(2)



$$\underline{V} = (-2,-5)$$

$$V = |\underline{V}| = \sqrt{Vx^2 + Vy^2} = \sqrt{4+25} = \sqrt{29} = 5{,}38$$

$$\alpha = \arccos \frac{Vx}{V} = \arccos \frac{-2}{5{,}38} = 111°$$

$$\beta = \arccos \frac{Vy}{V} = \arccos \frac{-5}{5{,}38} = 158{,}3°$$

Dati due vettori ricava la risultante.

$\underline{U} = (-2;3;4)$

$\underline{V} = (-5;-2;3)$

$\underline{R} = \underline{U} + \underline{V} = (-7,1,7) = -7\underline{i} + 1\underline{j} + 7\underline{k}$

$|\underline{R}| = \sqrt{(-7)^2 + (1)^2 + (7)^2} = 9,9498$

Dati due vettori e la risultante determina gli angoli che la risultante forma con gli assi coordinati.

$\underline{U} = (-3;2)$

$\underline{V} = (1;-3)$

$\underline{R} = (-2;-1)$

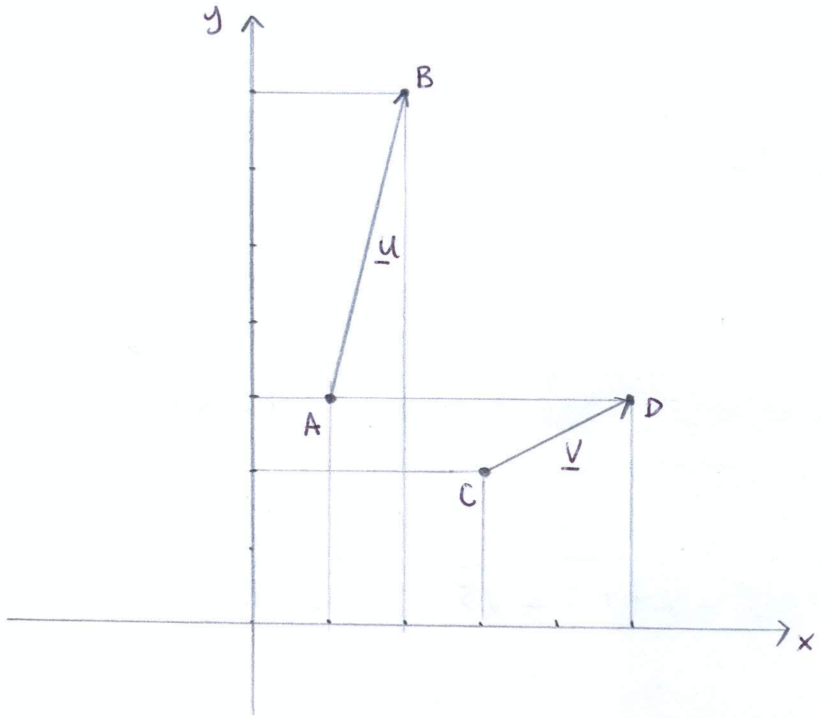
$|\underline{R}| = \sqrt{Rx^2 + Ry^2} = \sqrt{4+1} = \sqrt{5}$

$Rx = |\underline{R}| \cos\alpha \rightarrow \cos\alpha = \dfrac{Rx}{|\underline{R}|} = \dfrac{-2}{\sqrt{5}} \longrightarrow \alpha = \arccos(-2/\sqrt{5}) = 153,4°$

$Ry = |\underline{R}| \cos\beta \rightarrow \cos\beta = \dfrac{Ry}{|\underline{R}|} = \dfrac{-1}{\sqrt{5}} \longrightarrow \beta = \arccos(-1/\sqrt{5}) = 116,5°$

PRODOTTO VETTORIALE TRA DUE VETTORI

Dati due vettori ricava il prodotto vettoriale.



$\underline{U} = (B - A)$

$\underline{V} = (D - C)$

$A = (1;3)$

$B = (2;6)$

$C = (3;2)$

$D = (5;3)$

Le coordinate dei vettori:

$$\left.\begin{aligned} U_x &= X_B - X_A = 2 - 1 = 1 \\ U_y &= Y_B - Y_A = 6 - 3 = 3 \end{aligned}\right\} \quad \underline{U} = (1;3)$$

$$\left.\begin{aligned} V_x &= X_D - X_C = 5 - 3 = 2 \\ V_y &= Y_D - Y_C = 3 - 2 = 1 \end{aligned}\right\} \quad \underline{V} = (2;1)$$

Prodotto vettoriale:

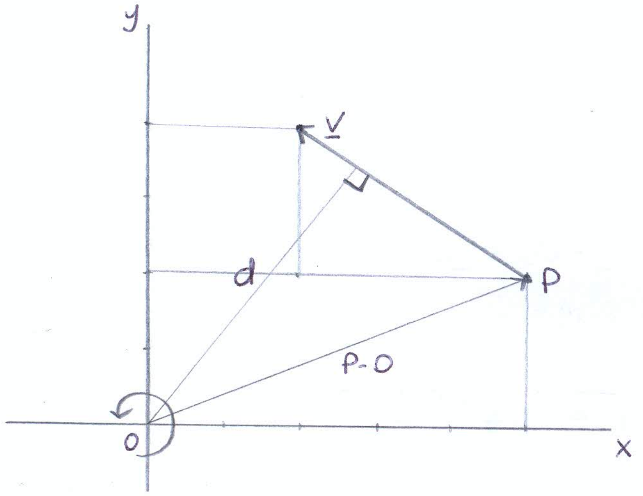
$\underline{U} \times \underline{V} = \underline{W}$

$$\begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix} = \underline{i}\,\underbrace{(3 \cdot 0 - 1 \cdot 0)}_{=0} - \underline{j}\,\underbrace{(1 \cdot 0 - 2 \cdot 0)}_{=0} + \underline{k}\,(1 \cdot 1 - 2 \cdot 3) = -5\,\underline{k}$$

"5" è il modulo del vettore risultante $\underline{W}$.

VETTORI APPLICATI

Dato un vettore e il suo punto di applicazione trova il momento rispetto al punto O.



$\underline{V} = (-3;2) = -3i + 2j$

$P = (5;2)$

$\underline{M}(O) = (P-O) \times \underline{V}$

$|\underline{M}|(O) = |(P-O)| \cdot |\underline{V}| \cdot \operatorname{sen}\alpha = |\underline{V}| \cdot d$

d

componenti (P-O):

$(P-O) = [(x_P - x_O), (y_P - y_O)]$

$= [(5-0), (2-0)]$

$= (5;2) = 5\underline{i} + 2\underline{j}$

$$\begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 2 & 0 \\ -3 & 2 & 0 \end{bmatrix} = \underline{k}\,(5 \cdot 2 - (-3 \cdot 2)) = \underline{k}\,(10+6) = 16\underline{k}$$

Dati due vettori e il loro punto di applicazione trova il momento rispetto al punto O.

$\underline{V}_1 = (-3\underline{i} + 2\underline{j})$ $\qquad$ $\underline{V}_2 = (2\underline{i} - 4\underline{j})$

$P_1 = (2; -4)$ $\qquad$ $P_2 = (-3; 5)$

$O = (1,2)$

$$M(O) = \sum_{i=1}^{2} (P_i - O) \times \underline{V}_i = (P_1 - O) \times \underline{V}_1 + (P_2 - O) \times \underline{V}_2$$

$$(P_1 - O) = [(x_{P_1} - x_o); (y_{P_1} - y_o)] = (1; -6) = 1\underline{i} - 6\underline{j}$$

$$(P_2 - O) = [(x_{P_2} - x_o); (y_{P_2} - y_o)] = (-4; 3) = -4\underline{i} + 3\underline{j}$$

$$\underline{M}(O) = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -6 & 0 \\ -3 & 2 & 0 \end{bmatrix} + \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ -4 & 3 & 0 \\ 2 & -4 & 0 \end{bmatrix}$$

$$= \underline{k}[(1 \cdot 2) - (-6 \cdot (-3))] + \underline{k}[(-4 \cdot (-4)) - (3 \cdot 2)]$$

$$= \underline{k}(2 - 18) + \underline{k}(16 - 6) = -16k + 10k = -6k$$

MOMENTO RISPETTO AD UN NUOVO POLO O' AVENDO M(O)

$V_1 = (2; -4)$

$V_2 = (1; 5)$

$V_3 = (-6; 2)$

$O = (3; -1) \qquad \underline{M}(O) = -15\underline{k}$

$O' = (-2; 5) \qquad M(O') = ?$

$\underline{R} = [(2+1-6); (-4+5+2)] = (-3; 3)$

$\underline{M}(O) + (O - O') \times \underline{R}$ => Formula di trasposizione dei momenti

$\underline{R} \downarrow \sum_i \underline{V}_i$

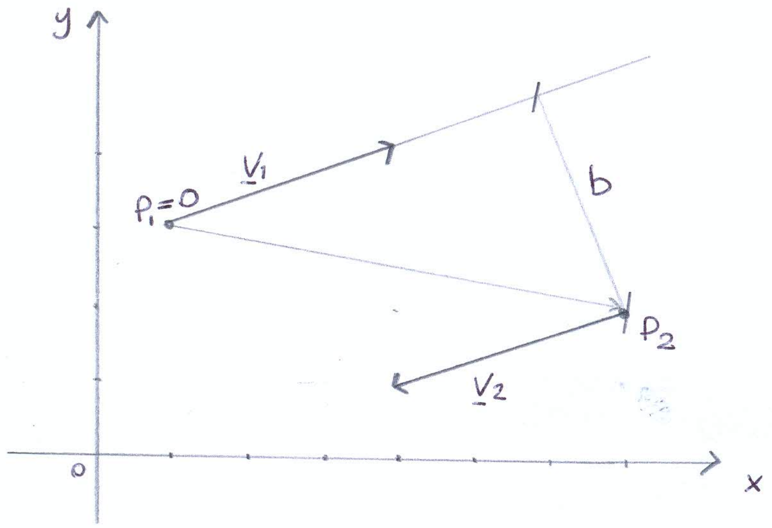
$(O-O') = [(x_O - x_{O'}); (y_O - y_{O'})] = (5; -6)$

$$(O-O') \times \underline{R} = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & -6 & 0 \\ -3 & 3 & 0 \end{bmatrix} = \underline{k}(15-18) = -3\underline{k}$$

$M(O') = M(O) - 3k = -15k - 3k = -18k$

COPPIA DI VETTORI

Dati due vettori calcolare il momento della coppia.



$V_1 = (3;1)$

$V_2 = (-3;-1)$

$P_1 = (1;3)$

$P_2 = (7;2)$

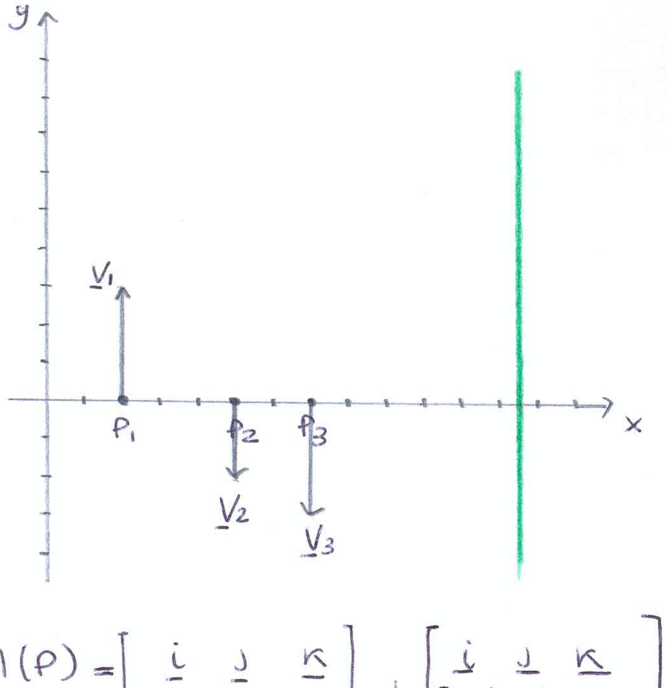
$O = P_1$

$(P_2 - P_1) = (6;-1)$

$$\underline{M}(O=P_1) = (P_2 - P_1) \times V_2 = \begin{bmatrix} \underline{i} & \underline{J} & \underline{K} \\ 6 & -1 & 0 \\ -3 & -1 & 0 \end{bmatrix} = (-6+3)\underline{K} = -3\underline{K}$$

ASSE CENTRALE DI UN SISTEMA PIANO DI VETTORI APPLICATI

① Dati tre vettori e i loro punti di applicazione trova l'asse centrale



$V_1 = 3J = (0;3)$ $P_1 = (2;0)$

$V_2 = -2J = (0;-2)$ $P_2 = (5;0)$

$V_3 = -3J = (0;3)$ $P_3 = (7;0)$

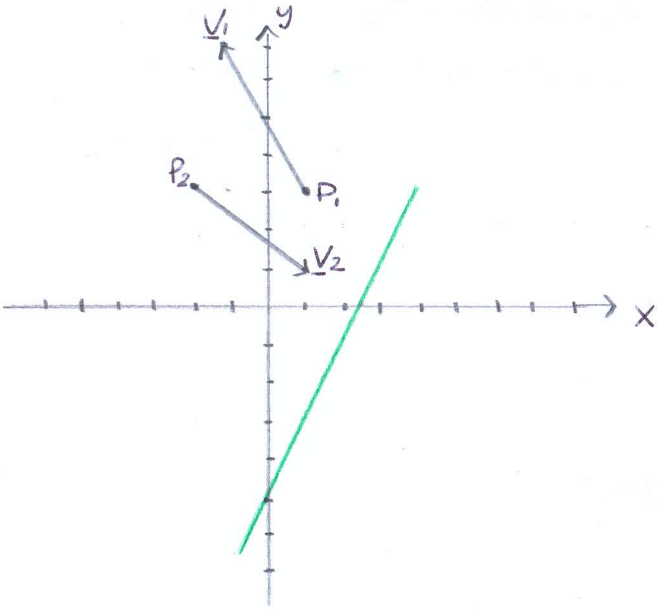
$P(x;y) \rightarrow \underline{M}(P) = 0$

$$\underline{M}(P) = \begin{bmatrix} \underline{i} & \underline{J} & \underline{K} \\ 2-x & -y & 0 \\ 0 & 3 & 0 \end{bmatrix} + \begin{bmatrix} \underline{i} & \underline{J} & \underline{K} \\ 5-x & -y & 0 \\ 0 & -2 & 0 \end{bmatrix} + \begin{bmatrix} \underline{i} & \underline{J} & \underline{K} \\ 7-x & -y & 0 \\ 0 & -3 & 0 \end{bmatrix} = \underline{K}(6-3x) + \underline{K}(-10+2x) + \underline{K}(-21+3x) = \underline{K}(-25+2$$

impongo momento risultante = 0

$\underline{K}(-25+2x) = 0$ $2x - 25 = 0$ $x = \frac{25}{2} = 12,5 \rightarrow$ equazione della retta dell'asse centrale

② Dati due vettori e i loro punti di applicazione trova l'asse centrale



$V_1 = (-2; 4)$ $\quad P_1 = (1; 3)$

$V_2 = (3; -2)$ $\quad P_2 = (-2; 3)$

$P(x; y) \rightarrow \underline{M}(P) = 0$

$$\underline{M}(P) = \sum_{i=1}^{2} (P_i - P) \times \underline{V_i} = (P_1 - P) \times V_1 + (P_2 - P) \times V_2$$

$$(P_1 - P) = [(x_{P_1} - x_P); (y_{P_1} - y_P)] = (1-x; 3-y)$$

$$(P_2 - P) = [(x_{P_2} - x_P); (y_{P_2} - y_P)] = (-2-x; 3-y)$$

$$M(P) = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1-x & 3-y & 0 \\ -2 & 4 & 0 \end{bmatrix} + \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2-x & 3-y & 0 \\ 3 & -2 & 0 \end{bmatrix} = \underline{k}[(1-x)4 - (-2)(3-y)] + \\ + \underline{k}[(-2-x)(-2) - 3(3-y)] =$$

$$= \underline{k}(4 - 4x + 6 - 2y) + \underline{k}(4 + 2x - 9 + 3y) = \underline{k}(5 - 2x + y)$$

$$\cancel{\underline{k}}(5 - 2x + y) = 0$$

$y - 2x + 5 = 0$ → equazione dell'asse centrale

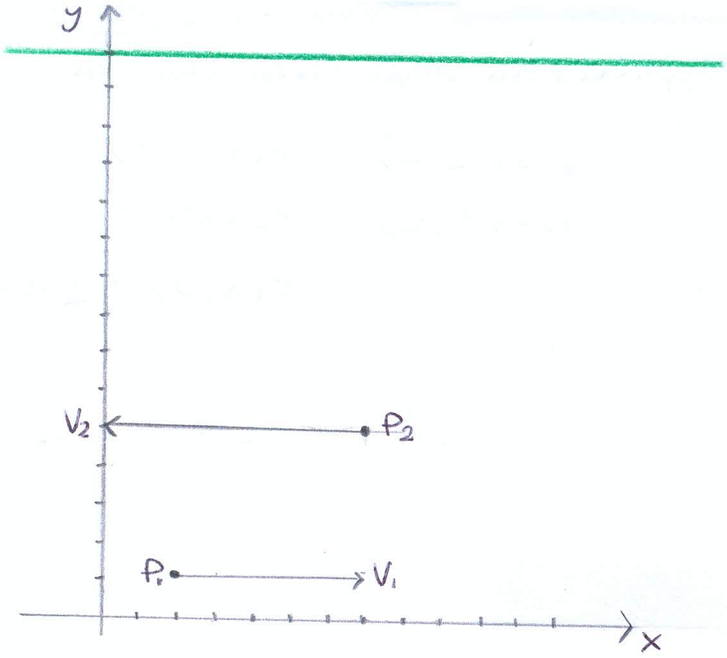
per ricavare y impongo $x = 0$ ⇒ $y - 2(0) + 5 = 0$

$y = -5$

per ricavare x impongo $y = 0$ ⇒ $(0) - 2x + 5 = 0$

$x = \frac{5}{2}$

③



$V_1 = (5;0) = 5\underline{i}$ $\quad P_1 = (2;1)$

$V_2 = (-7;0) = -7\underline{i}$ $\quad P_2 = (7;5)$

$P(x;y) \rightarrow \underline{M}(P) =$

$$\underline{M}(P) = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2-x & 1-y & 0 \\ 5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7-x & 5-y & 0 \\ -7 & 0 & 0 \end{bmatrix} = \underline{k}\,(-5+5y) + \underline{k}\,(35-7y) =$$

$$= \underline{k}\,(30-2y)$$

$\cancel{\underline{k}}(30-2y) = 0$

$30 - 2y = 0$ → equazione dell'asse centrale

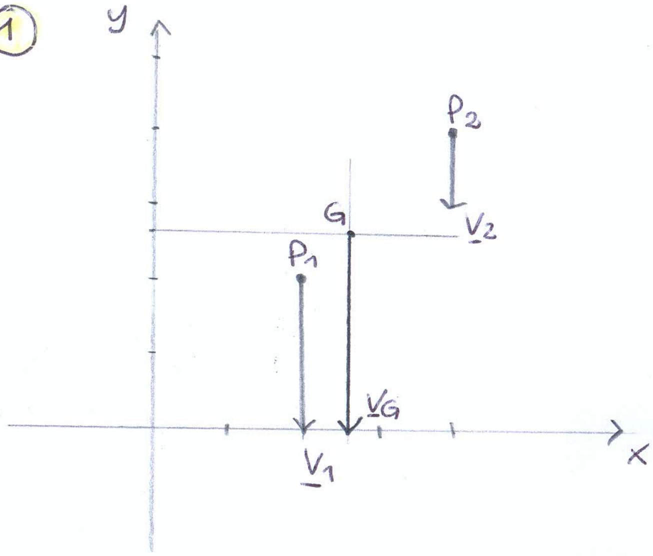
$y = 15$

BARICENTRO DI UN SISTEMA DI VETTORI

Dati due vettori e i loro punti di applicazione, calcola il loro baricentro.

① $V_1 = (0;-2)$ $P_1 = (2;2)$

$V_2 = (0;-1)$ $P_2 = (4;4)$



G = punto di applicazione del baricentro

$$X_G = \sum_{i=1}^{2} \frac{V_i x_i}{\sum_i V_i} = \frac{(-2)2 + (-1)4}{-3} = \frac{-8}{-3} = \frac{8}{3} = 2{,}\bar{6}$$

$$y_G = \sum_{i=1}^{2} \frac{V_i y_i}{\sum_i V_i} = \frac{(-2)2 + (-1)4}{-3} = \frac{-8}{-3} = \frac{8}{3} = 2{,}\bar{6}$$

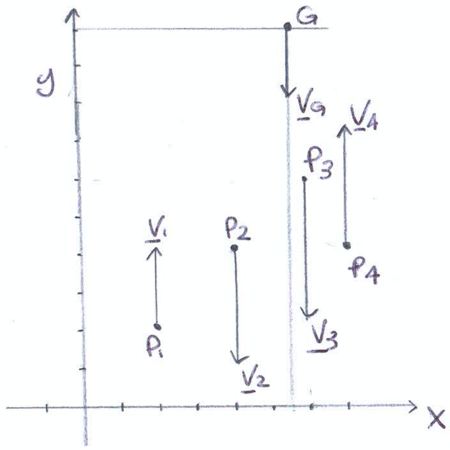
$\underline{R} = (0;-3) \rightarrow |\underline{R}| = -3 = |\underline{V}_G|$

② $V_1 = (0;2)$ $P_1 = (2;2)$

$V_2 = (0;-3)$ $P_2 = (4;4)$

$V_3 = (0;-4)$ $P_3 = (6;6)$

$V_4 = (0;3)$ $P_4 = (7;4)$



$$X_G = \sum_{i=1}^{4} \frac{V_i x_i}{\sum_i V_i} = \frac{(2\cdot 2) + (-3)4 + 6(-4) + 21}{2-3-4+3} = \frac{-11}{-2} = 5{,}5$$

$$y_G = \sum_{i=1}^{4} \frac{V_i y_i}{\sum_i V_i} = \frac{4 - 12 - 24 + 12}{-2} = \frac{-20}{-2} = 10$$

$\underline{R} = (0;-2) \rightarrow |\underline{R}| = -2 = |\underline{V}_G|$