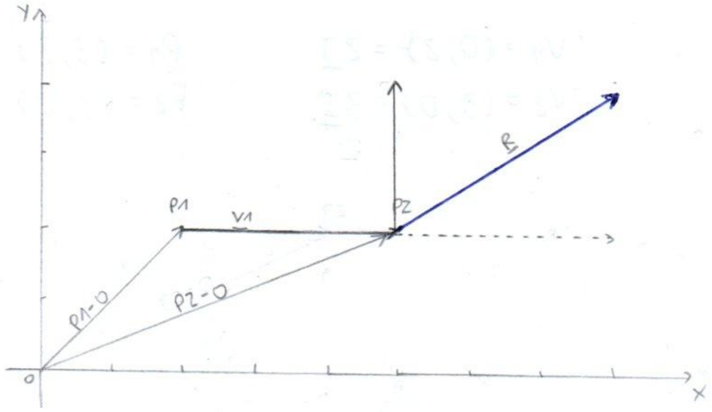
3



$\underline{V}_1 = 3\underline{I}$ $\quad P_1 = (2;2)$

$\underline{V}_2 = 2\underline{J}$ $\quad P_2 = (5;2)$

$? = \underline{M}(O)$

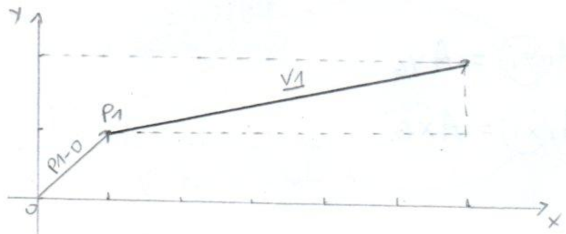
$$\underline{M}(O) = \begin{array}{c} \\ P_1 - O \\ V_1 \end{array} \begin{bmatrix} \underline{I} & \underline{J} & \underline{K} \\ 2 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \underline{I} & \underline{J} & \underline{K} \\ 5 & 2 & 0 \\ 0 & 2 & 0 \end{bmatrix} = -6\underline{K} + 10\underline{K} = 4\underline{K}$$

in P2 il momento risultante é 0, quindi la retta d'azione della risultante é anche l'asse centrale

quindi $M'(O) = M(O)$, in cui $M'(O)$ é il momento del risultante applicato all'asse centrale

$$M'(O) = \begin{array}{c} \\ P_2 - O \\ \underline{R} \end{array} \begin{bmatrix} \underline{I} & \underline{J} & \underline{K} \\ 5 & 2 & 0 \\ 3 & 2 & 0 \end{bmatrix} = \underline{K}\,(10-6) = 4\underline{K}$$

4

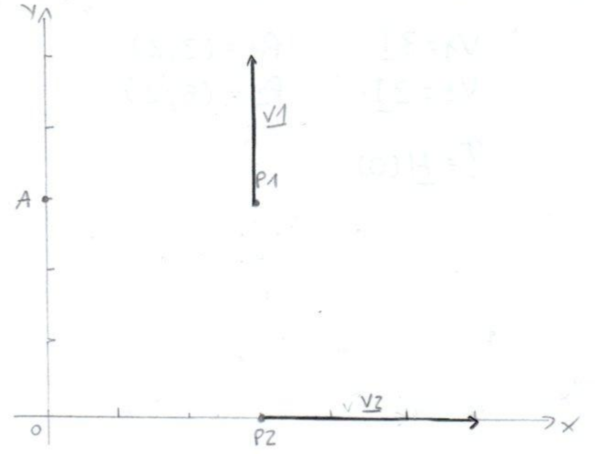


$P_1 = (1;1) \quad V_1 = (5;1)$

$\underline{M}(O) = (P - O) \times V_1$

$$\underline{M}(O) = \begin{bmatrix} \underline{I} & \underline{J} & \underline{K} \\ 1 & 1 & 0 \\ 5 & 1 & 0 \end{bmatrix} = (1-5)\,\underline{K} = -4\underline{K}$$

(5)



$V_1 = (0;2) = 2\underline{I}$ $\quad P_1 = (3;3)$

$V_2 = (3;0) = 3\underline{I}$ $\quad P_2 = (3;0)$

$\underline{M}(P_2) = \underline{0}$ perché P_2 sta sul vettore 2 e sul prolungamento di P_1

$\underline{M}(P_1) = 9\underline{K}$ (modulo di V_2 · distanza da P_1)

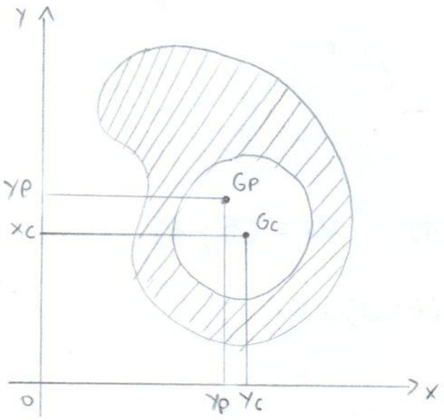
$\underline{M}(0) = 6\underline{K}$

$\underline{M}(A) = 6\underline{K} + 9\underline{K} = 15\underline{K}$ $\quad (6\underline{K} = V_1 = \underbrace{2}_{\text{modulo}} \cdot \underbrace{3}_{\text{distanza}} \quad ; \quad 9\underline{K} = V_2 = \underbrace{3}_{\text{modulo}} \cdot \underbrace{3}_{\text{distanza}})$

TEOREMA DI HOYGHENS

Data una figura con cavità, determina momento statico

(1)



Gp = baricentro figura piena

Gc = baricentro figura cava

$S_x = \sum_i A_i y_i = A y_G$ — y_i: distanza del baricentro dall'asse x

$S_y = \sum_i A_i x_i = A x_G$ — x_i: distanza del baricentro dall'asse y

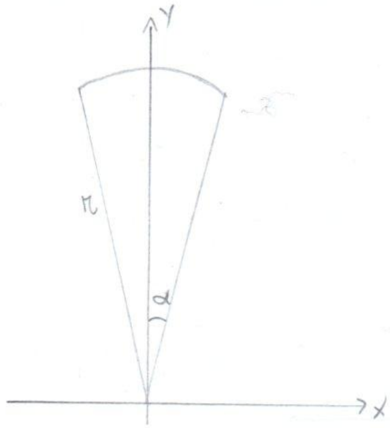
$S_x = A_p y_p - A_c y_c$ — A_c: area figura cava; A_p: area figura piena

$S_y = A_p x_p - A_c x_c$

$I_x = I_{xp} - I_{xc}$ — I_x: momento d'inerzia rispetto all'asse x

$I_y = I_{yp} - I_{yc}$ — I_y: momento d'inerzia rispetto all'asse y

2

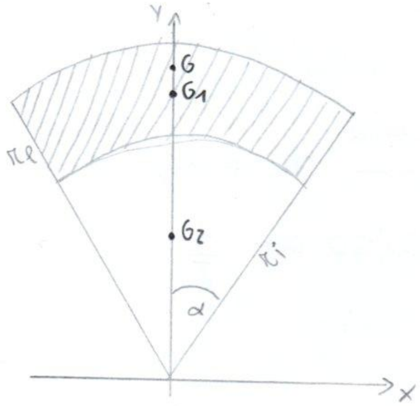


$$Y_G = \frac{2r \cdot \text{sen}\,\alpha}{3\alpha_{rad}}$$

$$\left[\begin{array}{l} \alpha = 45^\circ \\ Y_G = \dfrac{2r\sqrt{2}\cdot 4}{3\cdot 2\pi} = \dfrac{4\sqrt{2}\,r}{3\pi} \end{array}\right]$$

$$\left[\begin{array}{l} \alpha = 90^\circ \\ Y_G = \dfrac{2r\cdot 2}{3\pi} = \dfrac{4r}{3\pi} \end{array}\right]$$

3



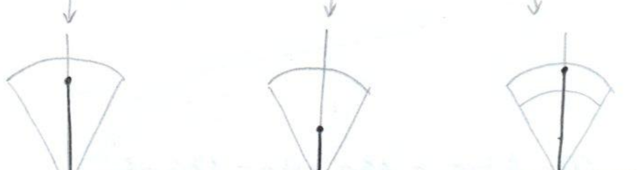
G = baricentro settore di corona circolare
G_1 = baricentro figura totale
G_2 = baricentro figura cava

r_e = raggio esterno
r_i = raggio interno

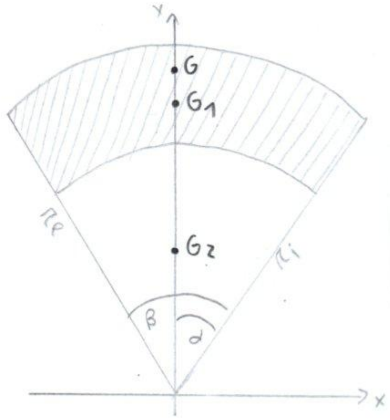
$$Y_{G_1} = \frac{2r_e \cdot \text{sen}\,\alpha}{3\alpha_{rad}}$$

$$Y_{G_2} = \frac{2r_i \cdot \text{sen}\,\alpha}{3\alpha_{rad}}$$

$$S_x = A_1 \cdot Y_{G_1} - A_2 \cdot Y_{G_2} = A \cdot Y_G$$



4



$\alpha = 45°$

$\beta = 90° = 2\alpha$

$r_e = 6\ cm$

$r_i = 3\ cm$

1 = settore totale

2 = settore cavo

$$A_1 = \frac{A\,cerchio \cdot \beta°}{360°} = \frac{\pi r_e^2 \cdot 90°}{360°} = 28,26\ cm^2$$

$$A_2 = \frac{\pi r_i^2 \cdot 90°}{360°} = 7,065\ cm^2$$

$$A\ settore\ pieno = A_1 - A_2 = 21,195\ cm^2$$

$$Y_{G1} = \frac{2}{3} \cdot r_e \cdot sen\alpha \cdot \frac{4}{\pi} = 4,33\ cm$$

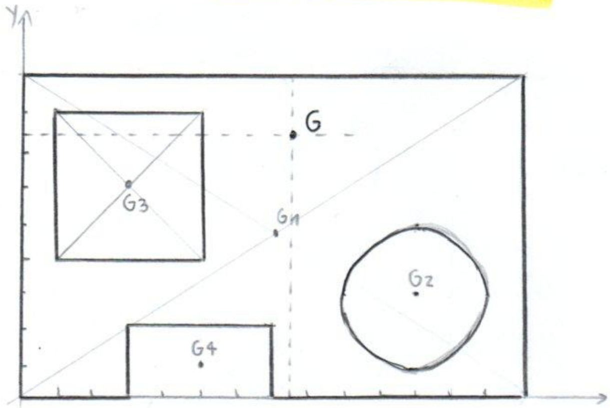
sarebbe $\dfrac{\frac{2}{3} \cdot r_e \cdot sen\alpha}{\frac{\pi}{4}}$ che è equivalente a moltiplicare tutto per $\frac{4}{\pi}$

$$Y_{G2} = \frac{2}{3} \cdot r_i \cdot sen\alpha \cdot \frac{4}{\pi} = 2,165\ cm$$

$$Y_G = \frac{\underbrace{A_1 \cdot Y_{G1}}_{S_{x1}} - \underbrace{A_2 \cdot Y_{G2}}_{S_{x2}}}{A\ settore\ pieno} = \frac{28,26 \cdot 4,33 - 7,065 \cdot 2,165}{21,195} = 5,05\ cm$$

baricentro settore di corona circolare

RICERCA DEI BARICENTRI



1 = $A\,tot. = 14a \cdot 9a = 126a^2$

2 = $A\,cerchio = \pi r^2 = \pi \cdot (2a)^2 = \pi 4a^2$

3 = $A\,quadrato = 4a \cdot 4a = 16a^2$

4 = $A\,rettangolo = 4a \cdot 2a = 8a^2$

$X_{G1} = 7a$	$Y_{G1} = 4,5a$
$X_{G2} = 11a$	$Y_{G2} = 3a$
$X_{G3} = 3a$	$Y_{G3} = 6a$
$X_{G4} = 5a$	$Y_{G4} = 1a$

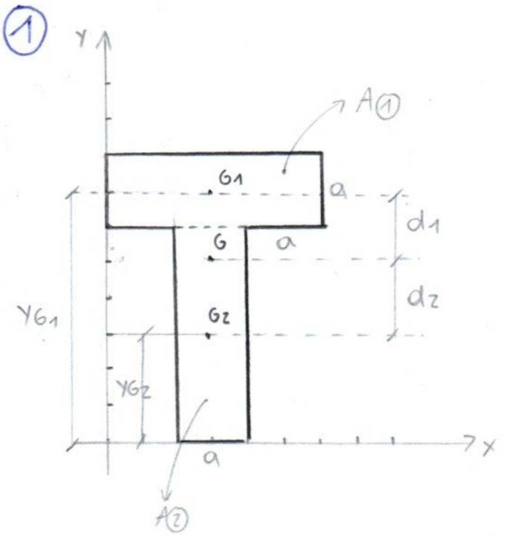
baricentro area con cavità

$$X_G = \frac{A_1 \cdot Y_{G1} - A_2 \cdot Y_{G2} - A_3 \cdot Y_{G3} - A_4 \cdot Y_{G4}}{A_1 - A_2 - A_3 - A_4} = 7,3a$$

$$Y_G = \frac{A_1 \cdot X_{G1} - A_2 \cdot X_{G2} - A_3 \cdot X_{G3} - A_4 \cdot X_{G4}}{A_1 - A_2 - A_3 - A_4} = 7,33a$$

MOMENTI D'INERZIA

1



ascissa baricentro $x_G = \frac{3}{2}a$

$$Y_G = \sum_i \frac{A_i y_i}{A_{tot}} = \frac{3a^2 \cdot \frac{7}{2}a + 3a^2 \cdot \frac{3}{2}a}{6a^2} = \frac{5}{2}a = 2{,}5a$$

$A① = 3a^2$
$A② = 3a^2$
$A_{tot.} = 6a^2$

momento d'inerzia $I_{x_0} = I_{x_0}^{①} + I_{x_0}^{②}$

$I_{x_0}^{①} = I_{x_1} + A_1 \cdot d_1^2$ asse baricentrico

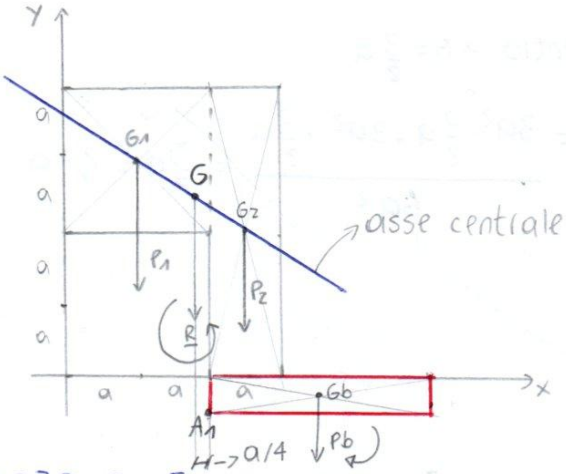
$$= \frac{3a \cdot a^3}{12} + 3a^2 \cdot a^2 = \frac{3a^4}{12} + 3a^4 = \frac{39}{12}a^4 = 3{,}25a^4$$

$I_{x_0}^{②} = I_{x_2} + A_2 \cdot d_2^2$

$$= \frac{a \cdot 27a^3}{12} + 3a^2 \cdot a^2 = \frac{27}{12}a^4 + 3a^4 = \frac{63}{12}a^4 = 5{,}25a^4$$

$$I_{x_0} = \frac{39}{12}a^4 + \frac{63}{12}a^4 = \frac{102}{12}a^4 = 8{,}5a^4$$

RIBALTAMENTO



$A_1 = 2a \cdot 2a = 4a^2$
$A_2 = a \cdot 4a = 4a^2$

$y_G = 2a + \frac{a}{2} = \frac{5}{2}a = 2{,}5a$

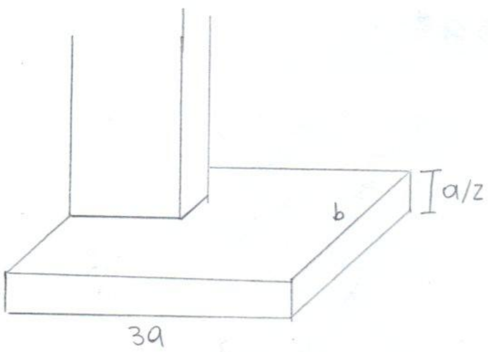
$x_G = a + \frac{3}{4}a = \frac{7}{4}a = 1{,}75a$

coordinate baricentro intera figura ⟶ la figura non si ribalta perché il baricentro si trova all'interno della figura

$P_1 = \underbrace{\underbrace{A_1}_{\text{area}} \cdot \underbrace{s}_{\text{spessore}}}_{\text{volume}} \cdot \gamma$ → peso specifico materiale

$P_2 = A_2 \cdot s \cdot \gamma$

aggiungo una base d'appoggio:



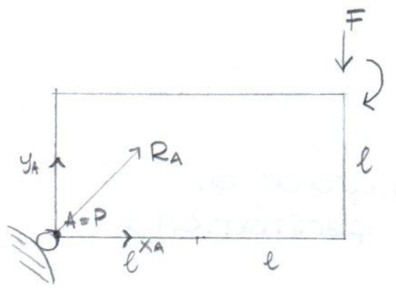
$P_{base} = 3a \cdot b \cdot \frac{a}{2} \cdot \gamma_b$

$M_r = R \cdot a/4$

$M_s = P_b \cdot \frac{3}{2}a$

ESERCIZI CON VINCOLI

①



$g.\ell = 3$
$m = 2$

$g.\ell > m$

$g.\ell - m = 1 \rightarrow$ gradi di labilità

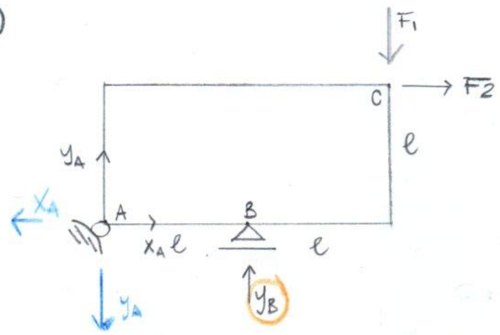
$\underline{R} = \underline{0}$ $\begin{cases} R_x = \sum_i x_i = 0 \rightarrow x_A = 0 \\ R_y = \sum_i y_i = 0 \rightarrow y_A - F = 0 \Rightarrow y_A = F \end{cases}$

$M_{(P)} = 0$

$M = F \underbrace{2\ell}_{\text{braccio}}$

$M_{(P)} = -Fb = -F2\ell = 0 \rightarrow$ il problema è IMPOSSIBILE

②



$g.\ell = 3$
$m = 3$

$\rightarrow g.\ell. = m$ _ STATICAMENTE DETERMINATO

$\underline{R} = \underline{0}$ $\begin{cases} R_x = \sum_i x_i = 0 \rightarrow x_A + F_2 = 0 \Rightarrow x_A = -F_2 \\ R_y = \sum_i y_i = 0 \rightarrow y_A - F_1 + y_B = 0 \rightarrow y_A = F_1 - y_B \end{cases}$

$y_A = F_1 - 2F_1 - F_2$

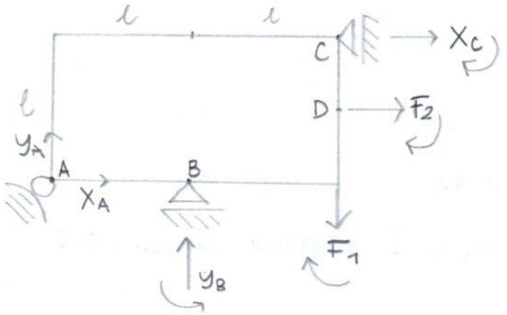
$y_A = -F_1 - F_2$

$M_{(A)} = 0 \quad - M = y_B \cdot \ell - F_1 \cdot 2\ell - F_2 \cdot \ell$

Ricavo y_B

$y_B = 2F_1 + F_2$

3



$m = 4$

$g.l. = 3$

$m > g.l.$

$m - g.l. = 1 \rightarrow$ grado di iperstaticità

$\downarrow$

STATICAMENTE INDETERMINATO

$R_x = \sum_i x_i = 0 \qquad X_A + X_C + F_2 = 0$

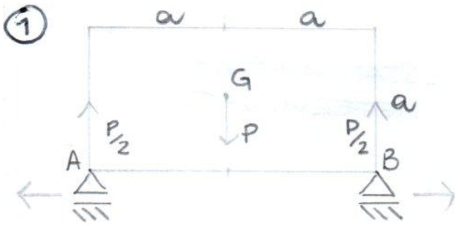
$R_y = \sum_i y_i = 0 \qquad y_A + y_B - F_1 = 0$

$M_{(P)} = M_{(A)} = -X_C \cdot l + y_B \cdot l - F_1 2l - F_2 \frac{l}{2}$

$X_A = K \in \mathbb{R}$

CASI AMBIGUI

1



$m = 2$

$g.l. = 3$

$m < g.l.$

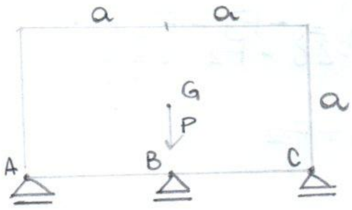
$\sum_i x_i = 0$

$\sum_i y_i = 0 \qquad y_A + y_B - P = 0$

$\sum_i M_{i(A)} = 0 \qquad -P \cdot a + y_B \cdot 2a = 0 \rightarrow y_B = \frac{P}{2}$

$y_A + \frac{P}{2} - P = 0 \rightarrow y_A = \frac{P}{2}$

2



$m = 3$

$\sum_i x_i = 0$

$\sum_i y_i = 0 \qquad y_A + y_B + y_C - P = 0$

$\sum_i M_{(A)} = 0 \qquad y_B \cdot a + y_C \cdot 2a - Pa = 0$

2 equazioni e 3 incognite

n° eq. < n° inc.

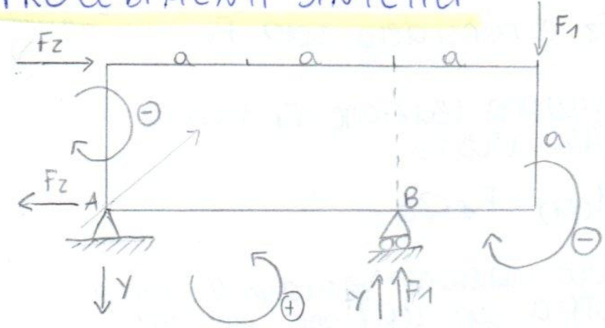
IL PROBLEMA RISULTA STATICAMENTE INDETERMINATO

pongo $y_B = K \in \mathbb{R}$

ricavo così y_A e y_C in funzione del valore K:

$y_A(K), y_C(K)$.

PROCEDIMENTI SINTETICI



(9)

①

m (n° incognite) = 3 — 1 da carrello, 2 da cerniera

g.l. = 3 (n° equazioni = gradi di libertà)

$\sum_i x_i = 0$
$\sum_i y_i = 0$
$\sum_i M_{(A)} = 0$

RISOLVO SENZA UTILIZZARE LE EQUAZIONI

① metto in evidenza le reazioni vincolari e le scompongo negli assi

- F_1 carrello = F_1 dall'alto → equilibrio verticale verificato

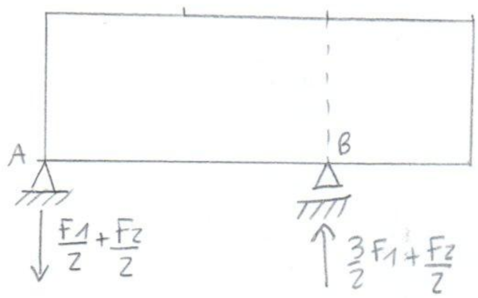
generano momento orario
$M_{(F_1)} = F_1 \cdot a$

- con reazioni vincolari di cerniera e carrello viene generato momento antiorario contrapposto a quello generato da F_1
- pongo $M_{(F_1)} = M_{(reagente)} = Y \cdot 2a \longrightarrow Y = \frac{F_1 \cdot a}{2a} = \frac{F_1}{2}$
- quindi carrello sviluppa reazione $F_1 + \frac{F_1}{2} = \frac{3}{2} F_1$

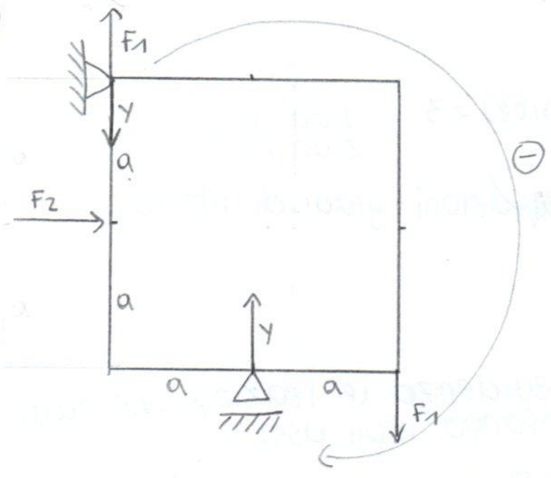
• Finora ho tralasciato F_2, ora tralascio F_1:

$M_{(F_2)} = F_2 \cdot \cancel{a} = Y \cdot 2\cancel{a} \longrightarrow 2Y = F_2 \longrightarrow Y = \frac{F_2}{2}$

- sommo gli effetti dopo averli considerati separatamente:



2



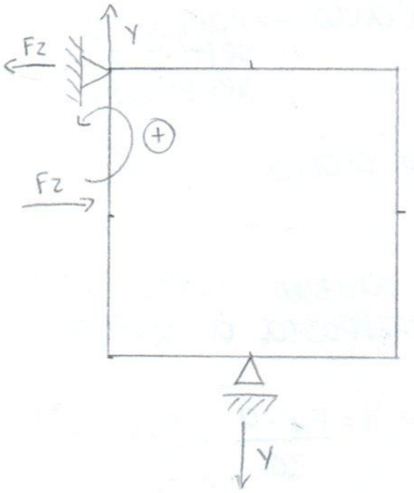
1 trascuro F_2 e considero solo F_1

cerniera = sviluppa reazione F_1 rivolta verso l'alto

$$M_{(F_1)} = F_1 \cdot 2a$$

carrello = solo reazione verticale e forma coppia con reazione verticale della cerniera (verso il basso)

$$M_{(F_1)} = F_1 \cdot 2\cancel{a} = Y \cdot \cancel{a} \longrightarrow Y = 2F_1$$



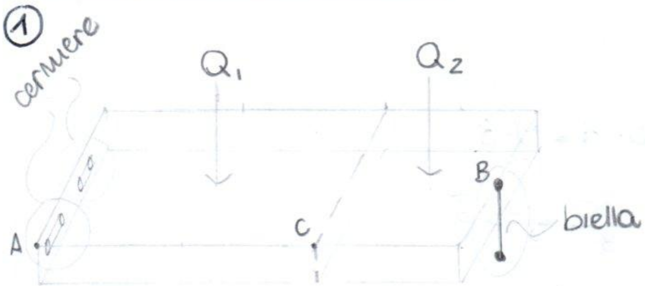
2 trascuro F_1 e considero solo F_2

$$M_{(F_2)} = F_2 \cdot \cancel{a} = Y \cdot \cancel{a} \longrightarrow Y = F_2$$

- reazioni complessive nei punti vincolati:
  - reaz. Y in cerniera = $F_1 + F_2 - 2F_1$
  - reaz. Y in carrello = $2F_1 - F_2$

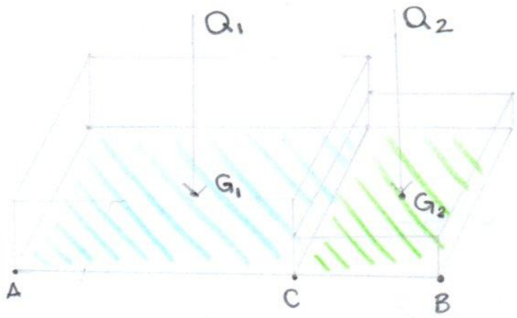
ESERCIZIO VINCOLI

①



$q_1 = 100 \ kg/m^2$

$q_2 = 50 \ kg/m^2$



$Q_1 = q_1 \cdot A_1 = 100 \cdot 2 \cdot 0{,}5 = 100 \ kg$

$Q_2 = q_2 \cdot A_2 = 50 \cdot 1 \cdot 0{,}5 = 25 \ kg$

Per trovare il carico lineare:

$q_1^* = \frac{Q_1}{L_{AC}} = \frac{100}{2} = 50 \ kg/m^2$ $\qquad q_2^* = \frac{Q_2}{L_{CB}} = \frac{25}{1} = 25 \ kg/m^2$

$\sum_i x_i = 0$

$\sum_i y_i = 0 \quad y_A + y_B - Q_1 - Q_2 = 0$

$\sum_i M_{i(A)} = 0 \quad y_B \cdot 3 - Q_1 \cdot 1 - Q_2 \cdot 2{,}5 = 0$

ricavo y_B

$y_B \cdot 3 = Q_1 + Q_2 \cdot 2{,}5$

$y_B = \frac{Q_1 + 2{,}5 Q_2}{3} = \frac{100 + 2{,}5 \cdot 25}{3} = 54{,}1$

ricavo y_A

$y_A = Q_1 + Q_2 - y_B = 70{,}8$

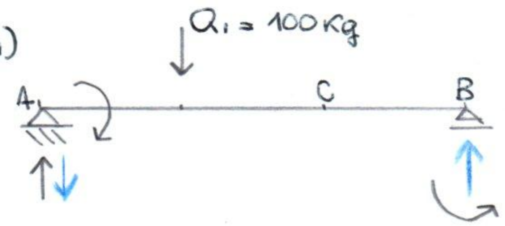
↓

Risolvo anche con il procedimento sintetico

↓

PROCEDIMENTO SINTETICO:

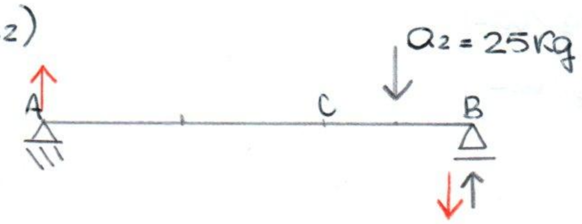
Q_1)



$$M(Q_1) = 100 \cdot 1 = y \cdot 3$$

$$\rightarrow y = \frac{100}{3} = 33,\overline{3}$$

Q_2)



$$M(Q_2) = 25 \cdot 0,5 = 12,5 = y \cdot 3$$

$$\rightarrow y = \frac{12,5}{3}$$

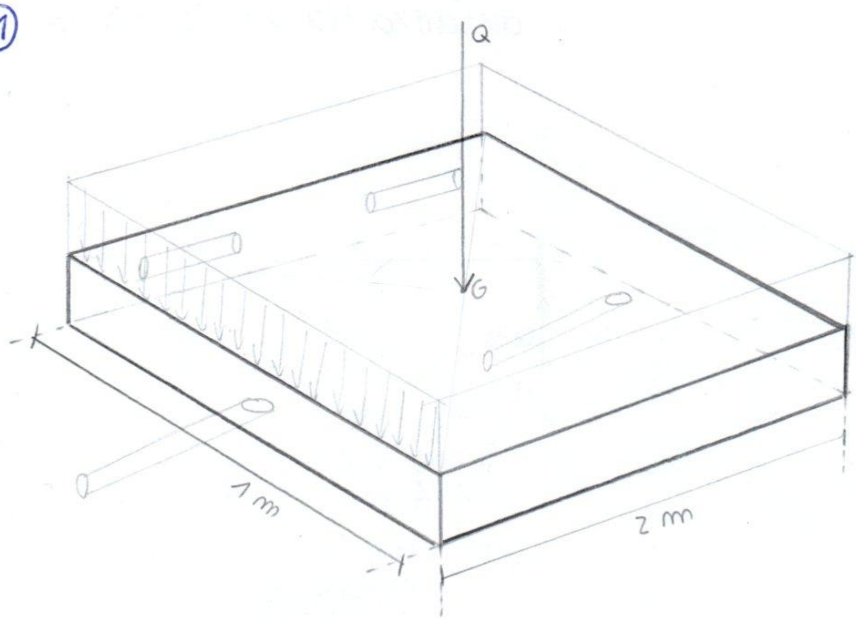
• Sommo le reazioni vincolari ottenute con Q_1 e Q_2:

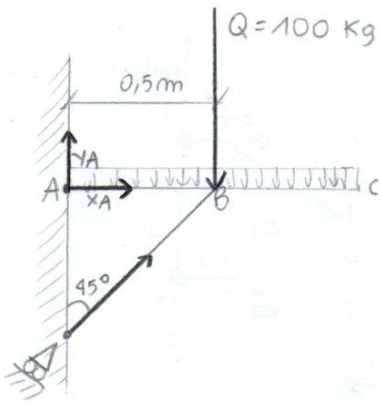
REAZIONE VERTICALE CERNIERA : $Q_1 - y + y = 100 - 33,\overline{3} + \frac{12,5}{3} = 70,8$

REAZIONE VERTICALE CARRELLO : $Q_2 - y + y = 25 - \frac{12,5}{3} + 33,3 = 54,1$

ESERCIZI VINCOLI

(1)



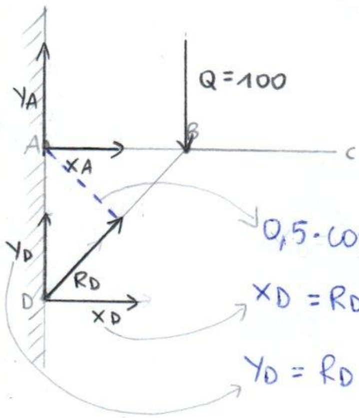


q = densità di carico = 5 Kg/cm²

Q = q · area = 50 · 2 = 100 Kg

A = 2 m²

q* = carico per metro lineare = Q/lunghezza = $\frac{100}{1}$ = 100 Kg/m



$0{,}5 \cdot \cos(45) = 0{,}5 \cdot \frac{\sqrt{2}}{2}$

$X_D = R_D \cdot \cos(45) = R_D \cdot \frac{\sqrt{2}}{2}$

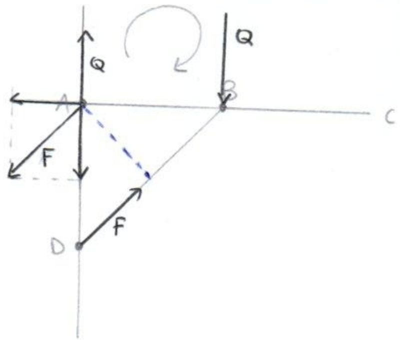
$Y_D = R_D \cdot \mathrm{sen}(45) = R_D \cdot \frac{\sqrt{2}}{2}$

$\sum_i x_i = 0 \longrightarrow X_A + R_D \cdot \frac{\sqrt{2}}{2} = 0$

$\sum_i y_i = 0 \longrightarrow Y_A + R_D \cdot \frac{\sqrt{2}}{2} - 100 = 0$ Q

$\sum_i M_{i(P)} = 0 \longrightarrow R_D \cdot 0{,}5 \cdot \frac{\sqrt{2}}{2} - 100 \cdot 0{,}5 = 0$

PROCEDIMENTO SINTETICO



$M_{(Q)} = Q \cdot 0{,}5 = 100 \cdot 0{,}5 = 50$

$M_{(Q)} = M_{(F)} = Q \cdot 0{,}5 = F \cdot 0{,}5 \cdot \frac{\sqrt{2}}{2}$

$F = \frac{2 \cdot Q \cdot \cancel{0{,}5}}{\cancel{0{,}5} \cdot \sqrt{2}} = \frac{200}{\sqrt{2}}$

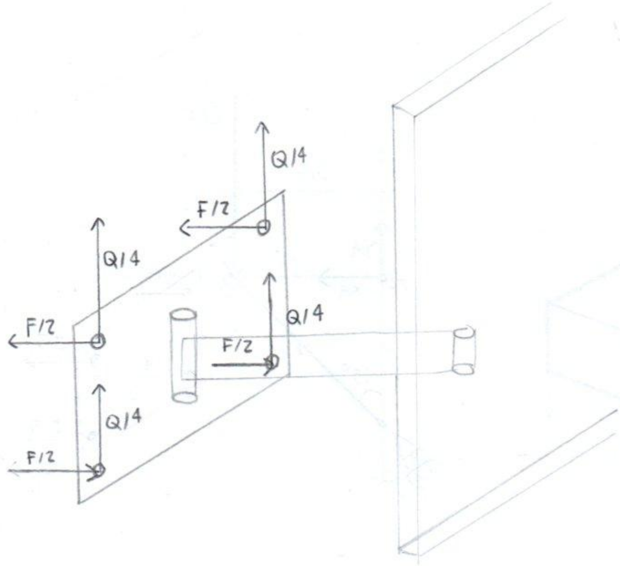
reazione biella = $F = \frac{200}{\sqrt{2}}$

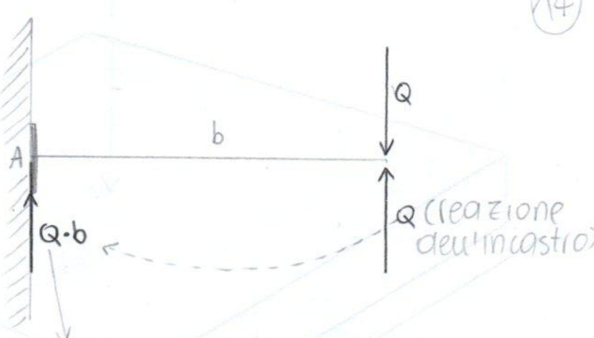
reazione cerniera verticale = $Q - F \cdot \frac{\sqrt{2}}{2}$ — componente verticale di F

reazione cerniera orizzontale = $-F \cdot \frac{\sqrt{2}}{2}$ — componente orizzontale F

2

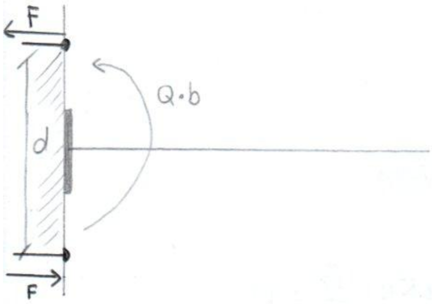
Q = 10 Kg
lunghezza staffa (b) = 30 cm
distanza tra viti (d) = 5 cm





Sposto Q e lo applico ad A e diventa Q · braccio

- Q · b sviluppa momento antiorario



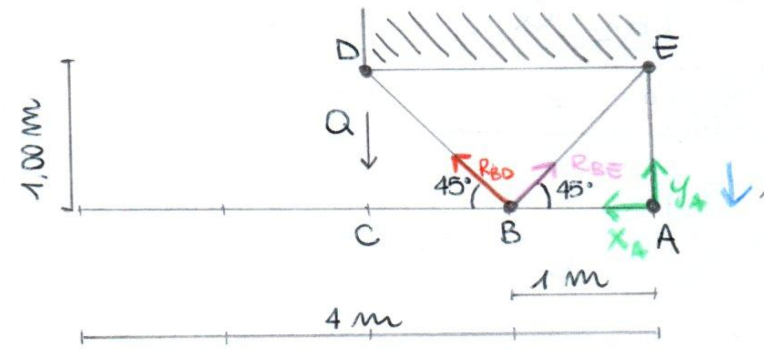
$$Q \cdot b = F \cdot d$$

$Q \cdot b$ → momento antiorario

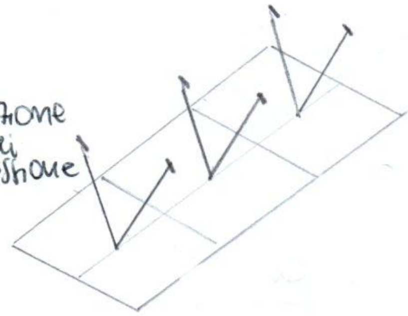
$F \cdot d$ → momento orario

$$F = \frac{Q \cdot b}{d}$$

③



reazione di compressione



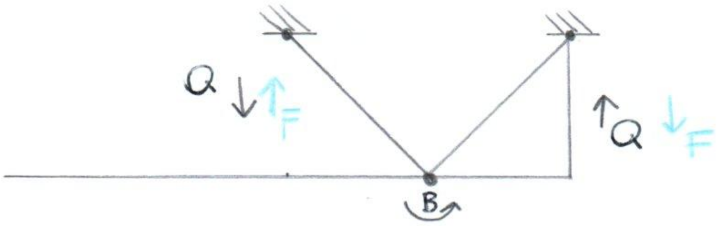
$$\sum_i x_i = 0 \Rightarrow R_{BE} \cdot \frac{\sqrt{2}}{2} - R_{BD} \frac{\sqrt{2}}{2} = 0$$

$$R_{BE} = R_{BD}$$

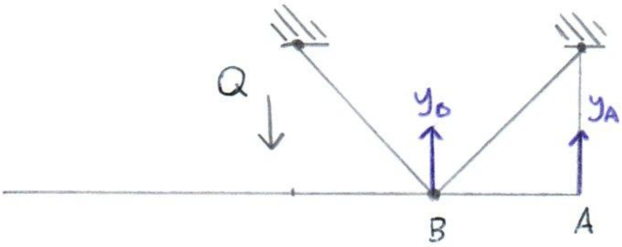
$$\sum_i y_i = 0 \Rightarrow y_A + R_{BE} \frac{\sqrt{2}}{2} + R_{BD} \frac{\sqrt{2}}{2} - Q = 0 \rightarrow Q + R_{BE} \frac{\sqrt{2}}{2} + R_{BD} \frac{\sqrt{2}}{2} - Q = 0$$

$$\sum_i M_i = 0 \Rightarrow y_A \cdot 1 + Q \cdot 1 = 0 \Rightarrow y_A = Q$$

PROCEDIMENTO SINTETICO:

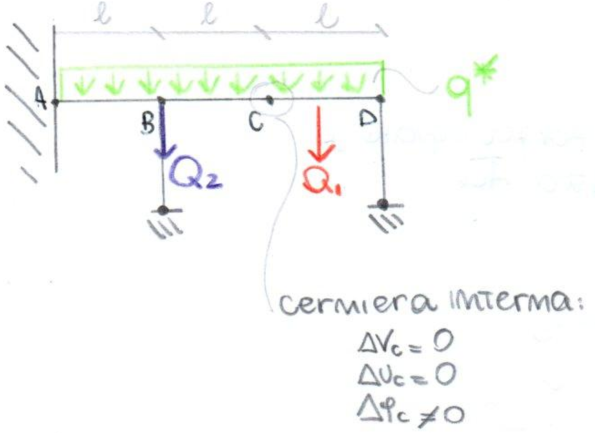


$$Q \cdot 2 = F \cdot 1$$
$$F = 2Q$$



$$\sum_i x_i = 0$$
$$\sum_i y_i = 0 \Rightarrow y_A + y_B - Q = 0$$
$$y_B = 2Q$$
$$M_{(B)} - Q \cdot 1 + y_A \cdot 1 = 0$$
$$y_A = -Q$$

4



$q \cdot l = 6$

$m = 6$

$Q_1 = q^* \cdot l$

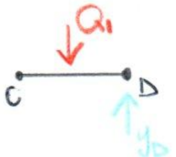
$Q_2 = q^* \cdot 2l$

$\sum_i x_i = 0 \rightarrow$ non ci sono carichi orizzontali

$\sum_i y_i = 0 \rightarrow y_A + y_B + y_D - Q_1 - Q_2 = 0$

$\sum_i M_{i(A)} = 0 \rightarrow y_B \cdot l + y_D \cdot 3l - Q_2 l - Q_1 \frac{5}{2} l = 0$

n° eq < n° incognite $\rightarrow$ il problema sarebbe STATICAMENTE INDETERMINATO ma devo considerare che la cerniera interna non può trasmettere momento per cui in C il momento risultante deve essere posto uguale a Ø



$M_{CD(C)} = 0$

$y_D \cdot l - Q \cdot l/2 = 0$

$y_D = \frac{Q_1}{2}$

↓

EQ. AUSILIARIA: utile a risolvere il problema

- ricavo y_B:

$y_B \cdot \cancel{l} + \frac{Q_1}{2} \cdot 3\cancel{l} - Q_2 \cancel{l} - Q_1 \cdot \frac{5}{2} \cancel{l} = 0$

$y_B = Q_1 - Q_2$

- ricavo y_A:

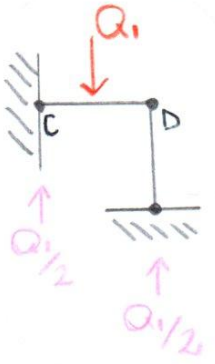
$y_A \cancel{+ Q_1} - Q_2 + \frac{Q_1}{2} \cancel{- Q_1} - Q_2 = 0$

$y_A = 2Q_2 - \frac{Q_1}{2}$

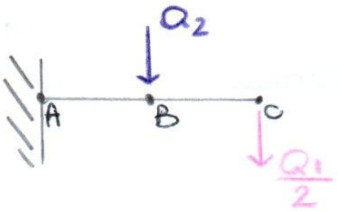
↓

PROCEDIMENTO SINTETICO

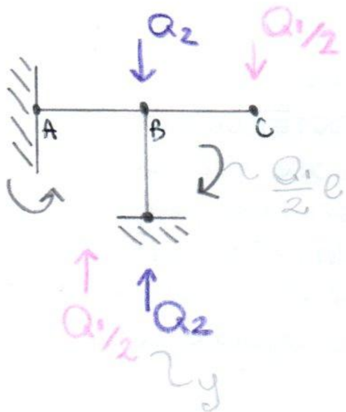
• considero solo $\overline{CD}$: tratto la cerniera sul p.to C come una cerniera esterna.



trasmette una forza uguale e contraria al tratto $\overline{AC}$



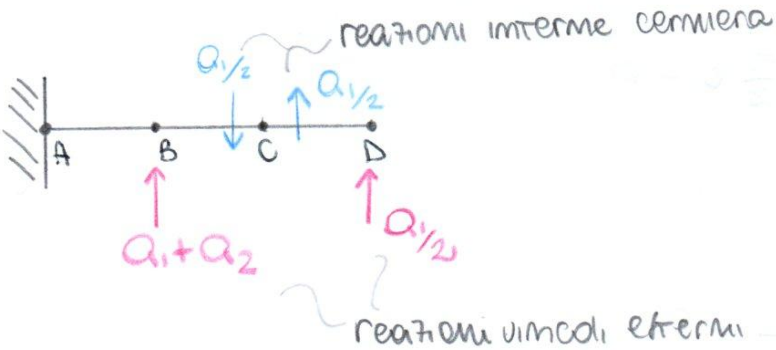
• considero solo $\overline{AC}$:



$$\frac{Q_1}{2} \cdot \cancel{\ell} = y \cdot \cancel{\ell}$$

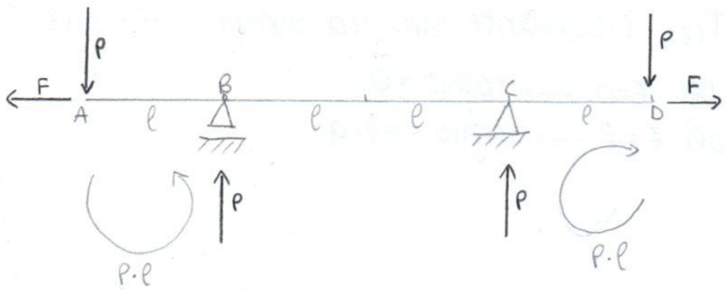
$$y = Q_1/2$$

• conclusione :



CARATTERISTICHE DELLA SOLLECITAZIONE

1



$P = 100$ Kg
$l = 1$ m
$F = 100$ Kg

N = costante lungo tutta l'asta e pari ad F

1. sezione molto prossima ad A = TAGLIO NEGATIVO ⊖
2. sezione tra A e B = TAGLIO NEGATIVO
3. sezione tra B e C = TAGLIO NULLO
4. sezione tra C e D = TAGLIO POSITIVO

$N(z)\ \overline{AD} = F$
$T(z)\ \overline{AB} = -P$
$T(z)\ \overline{BC} = 0$
$T(z)\ \overline{CD} = P$

ANALISI DEL MOMENTO

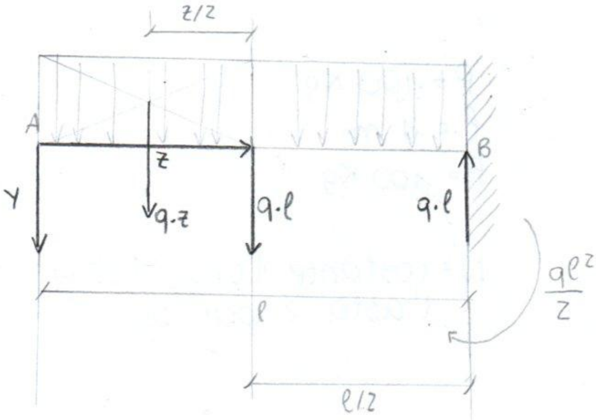
$M_{(A)} = P\cdot l + P\cdot 3l - P(4l) = 0$

$M(z)\ \overline{AB} = -P\cdot z$

$M(z)\ \overline{BC} = -P\cdot l$

$M(z)\ \overline{CD} = -P\cdot l + P\cdot(z-3l)$ → da punto C a taglio

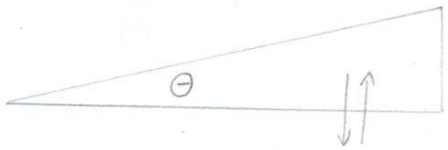
②



$T_{(z)}$ = risultante sinistra sempre $qz = -qz$

rivolto verso il basso

- per $z=0 \longrightarrow$ taglio $=0$
- per $z=l \longrightarrow$ taglio $= -l\cdot q$

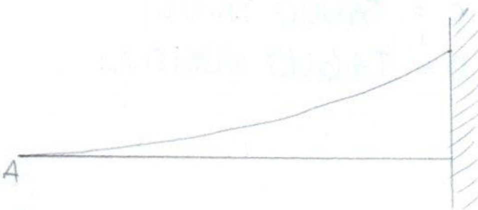


• con carico distribuito = taglio lineare

$$M_{(z)} = (-qz)\cdot\left(\frac{z}{2}\right) = -\frac{qz^2}{2}$$

risultante del carico

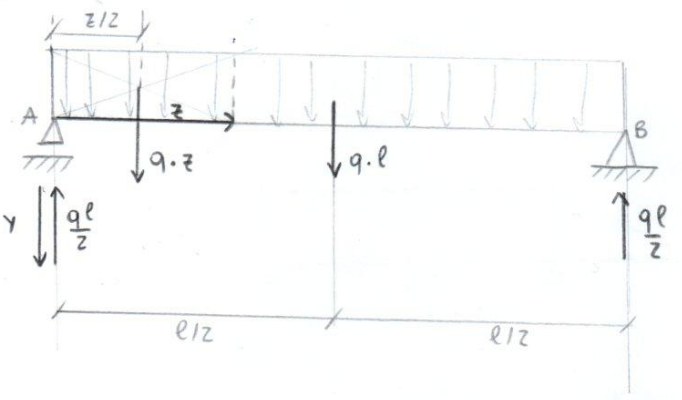
braccio



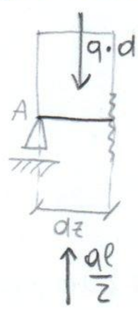
cavità = dalla parte del carico

$$M_{(z=l)} = -\frac{ql^2}{2}$$

③



① sezione infinitamente prossima ad A:



$$T_{(A)} = \frac{ql}{2} - \cancel{q\cdot dz}$$

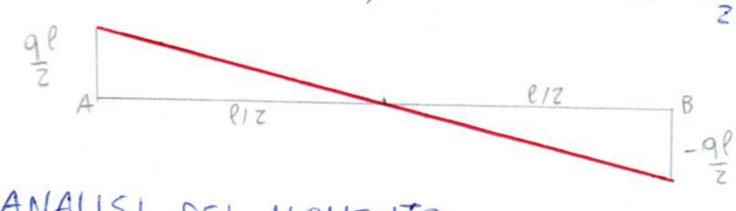
essendo infinitesimo non lo considero

rivolto in alto = positivo

verso il basso

② $T_{(z)} = \frac{ql}{2} = q\cdot z \longrightarrow$ per $z=0$, $T_{(A)} = \frac{ql}{2}$

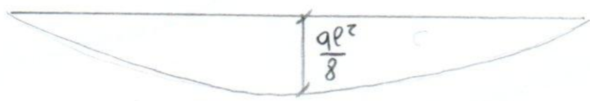
③ sezione in B, $z=l \longrightarrow T_{(B)} = \frac{ql}{2} - ql = -\frac{ql}{2}$



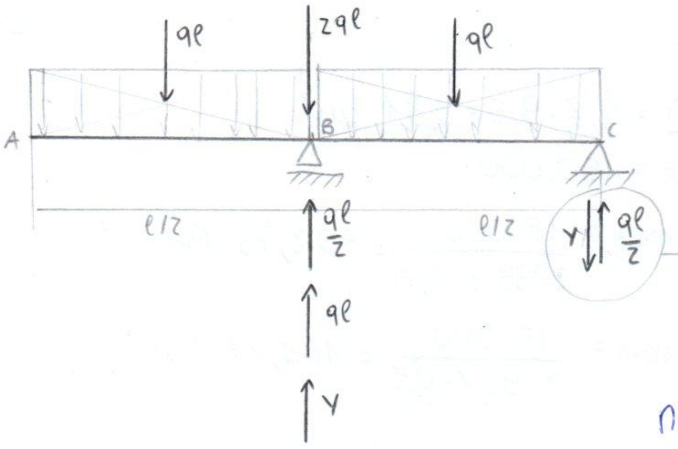
ANALISI DEL MOMENTO

$M_{(z)} = \frac{ql}{2}\cdot z - qz\cdot\frac{z}{2} \longrightarrow$ positivo

M di mezzeria $\left(z=\frac{l}{2}\right) = \frac{ql}{2}\cdot\frac{l}{2} - \frac{ql}{2}\cdot\frac{l}{4} = \frac{ql^2}{8}$



4



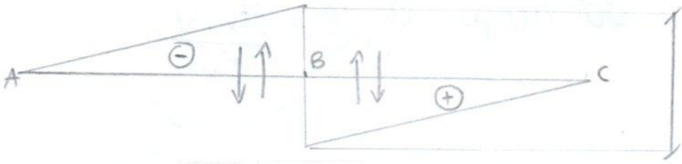
$$\frac{q\ell^2}{2} = Y \cdot \ell \longrightarrow Y = \frac{q\ell}{2}$$

→ essendo $Y = \frac{q\ell}{2}$, la cerniera non reagisce perché $\frac{q\ell}{2} - \frac{q\ell}{2} = 0$

nel carrello = reazione totale = $2q\ell$

$\left(\frac{q\ell}{2} + \frac{q\ell}{2} + q\ell\right)$

TAGLIO



$T_{(z)}\ \overline{AB} = -q \cdot z$

$T_{(z)}\ \overline{BC} = -qz + 2q\ell$

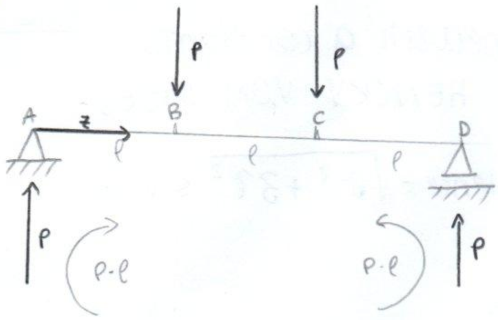
$T_{(C)} = 0 \quad (z = \ell)$

MOMENTO

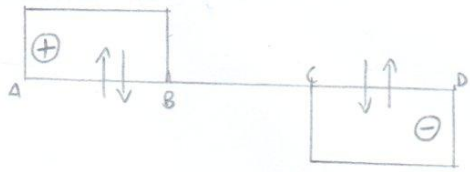
$M_{(z)}\ \overline{AB} = -\frac{qz^2}{2}$

$M_{(z)}\ \overline{BC} = -\frac{qz^2}{2} + 2q\ell(z-\ell)$

5



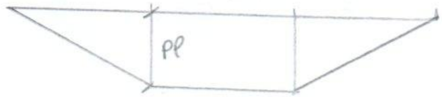
TAGLIO



MOMENTO

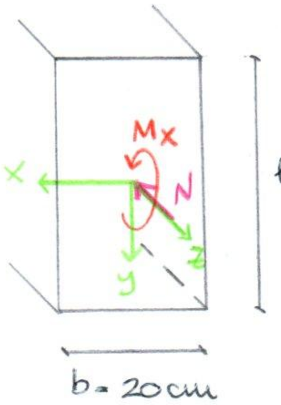
$M_{(z)}\ \overline{BC} = P \cdot \ell - P(z-\ell)$

momento della coppia



SFORZO NORMALE CENTRATO

①



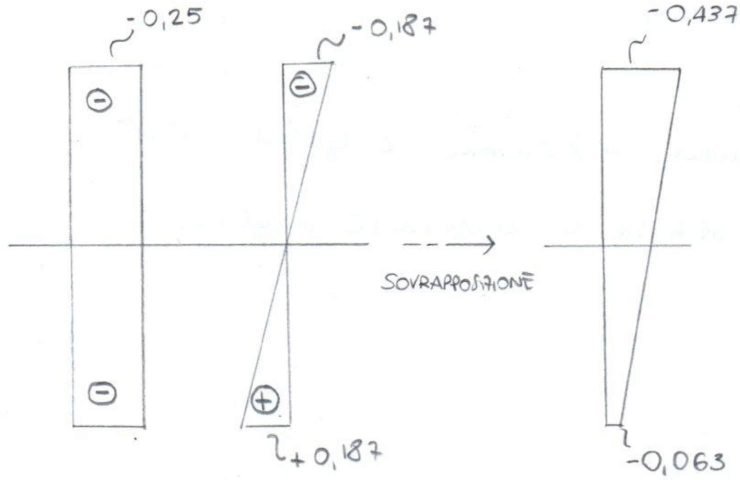
$N = 200 \text{ kg}$

$M_x = 1000 \text{ kg} \cdot \text{cm}$

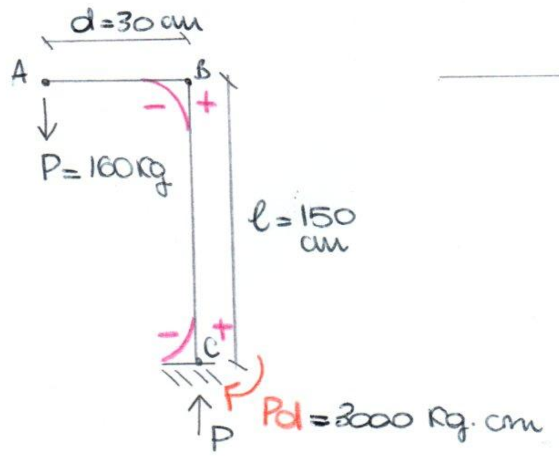
DISTRIBUZIONE UNIFORME ← $\sigma = \dfrac{N}{A} = \dfrac{200}{800} = 0{,}25 \text{ kg/cm}^2$

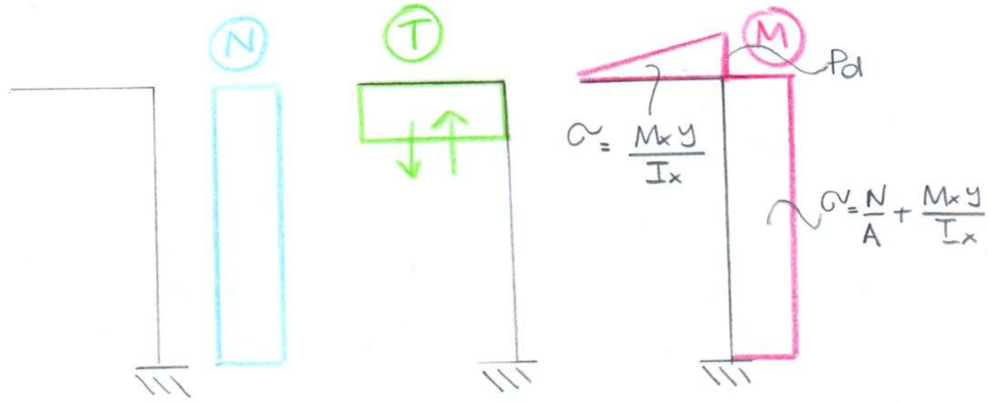
MOMENTO DI INERZIA ← $I_x = \dfrac{bh^3}{12} = 106'666 \text{ cm}^4$

FORMULA DI NAVIER ← $\sigma_{MAX} = \dfrac{M_x \cdot y}{I_x} = \dfrac{1000 \cdot 20}{106'666} = 0{,}187 \text{ kg/cm}^2$

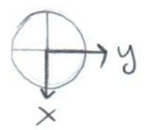


②





sezione



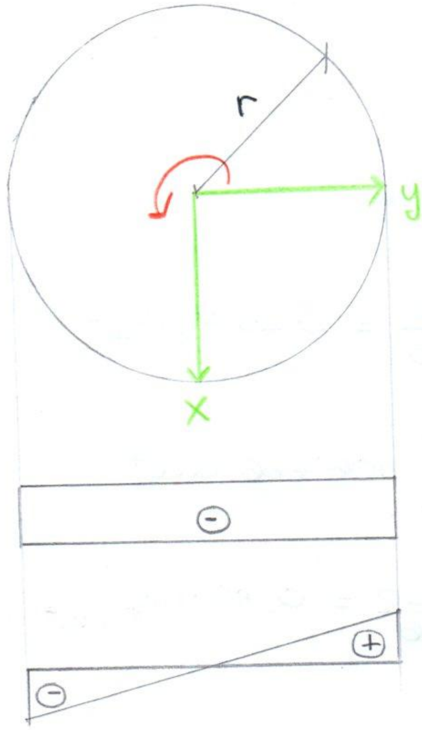
⊖ perché è di compressione

⊖ perché è nell'asse negativo

$$\sigma = \frac{N}{A} + \frac{M_x y}{I_x} = \frac{-100}{28{,}26} + \frac{3000\,(-3)}{63{,}5} \cong -146 \text{ kg/cm}^2$$

considero la sezione circolare per ricavare il momento di inerzia

◦ considero la sezione dell'oggetto:



$r = 3\,cm$

$A = \pi r^2 = 28{,}26\,cm^2$

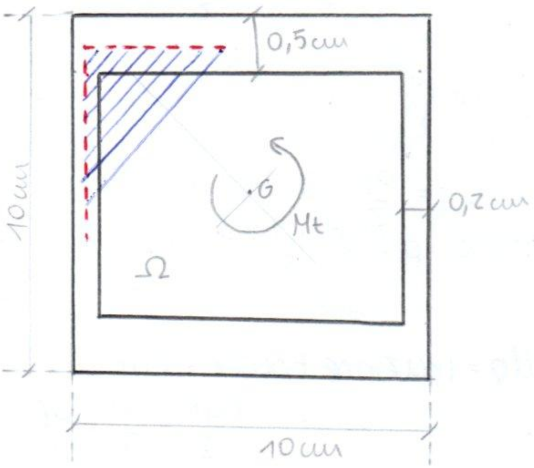
$I_x = \dfrac{\pi R^4}{4} = 63{,}5\,cm^4$

→ compressione

→ tensione negativa dovuta al momento flettente

Se σ_{amm} fosse 70 Kg/cm² avrei $\sigma > \sigma_{amm}$ e quindi, non essendo possibile dovrei aumentare la sezione o diminuire il carico.

SEZIONI CAVE – BREDT



$\Omega = 9,8 \cdot 9,5 = 93,1 \ cm^2$

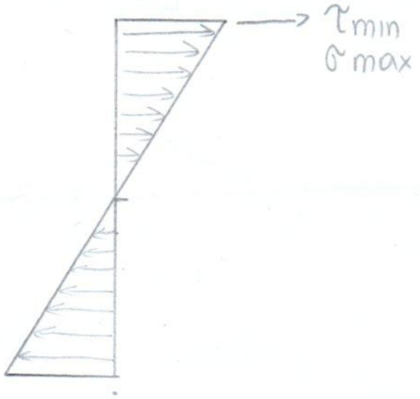
$Mt = 150.000$

$\tau_{max} = \dfrac{15.000}{2 \cdot 93,1 \cdot 0,2} = 402,79 \ Kg/cm^2$

$\tau_{min} = \dfrac{15.000}{2 \cdot 93,1 \cdot 0,5} = 161,11 \ Kg/cm^2$

supponiamo momento flettente che dà luogo a $\sigma = \dfrac{M_x \cdot y}{I_x}$ → tensione normale

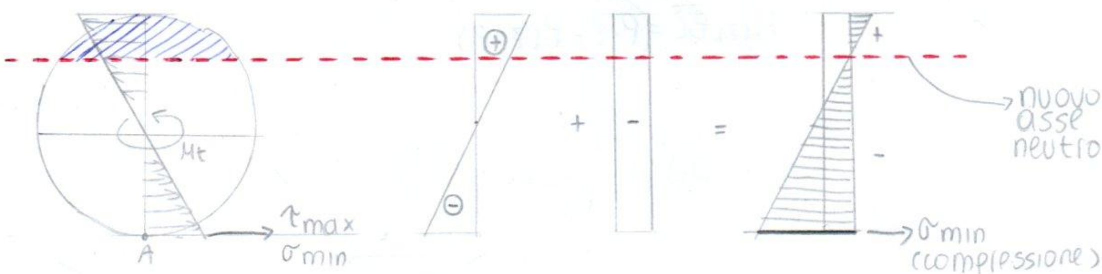
$\sigma_{max} = 1250 \ Kg/cm^2$



da una parte ho sigma e tau, dall'altra ho la sigma ammissibile ⟹ per metterli a confronto = HENCKY VON MISES

$\sigma_{equivalente} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma_{amm}$

$\sigma_{eq.} = \sqrt{1250^2 + 3 \cdot 161,11^2} = 1280 \ Kg/cm^2 < \sigma_{amm}$ → per Fe 360 = σ_{amm} : 1600 Kg/cm²



$I_G = \dfrac{\pi R^4}{2}$

$I_G = I_x + I_y$

$\tau = \dfrac{Mt \cdot r}{I_G}$

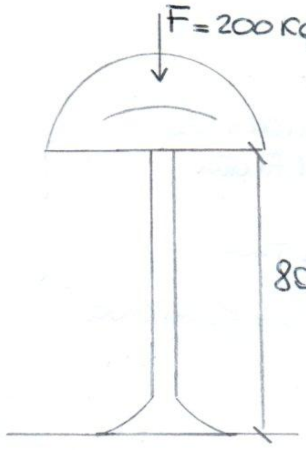
$\sigma_{tot} = \dfrac{M_x \cdot Y}{I_x} + \dfrac{N}{A}$

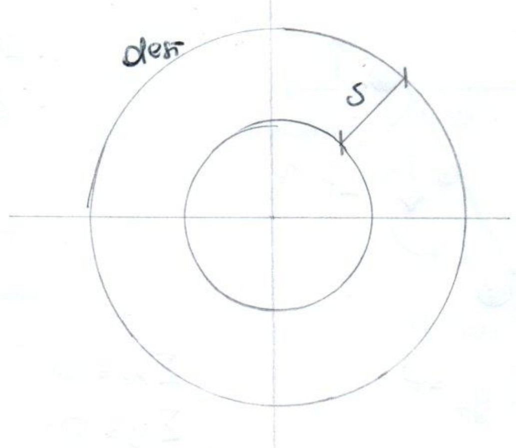
Y → R

N → ⊖

I_x → $\dfrac{\pi R^4}{2}$

ESERCIZIO SGABELLO





$d_{est} = 33{,}7$ mm

$s = 2{,}6$ mm

$A = 2{,}56$ cm²

Fe 360

$\sigma_{am} = 1600$ kg/cm²

$$\lambda = \frac{l_0}{\rho_{min}} \qquad \rho_{min} = \sqrt{\frac{I}{A}} = 1{,}10 \text{ cm}$$

Schema che posso seguire: asta incastrata a terra

$$l_0 = 2l$$

$$\lambda = \frac{2l}{1{,}10} = \frac{160}{1{,}10} = 145{,}45 \quad > 100 \rightarrow \text{l'asta è snella}$$

SFORZO NORMALE CRITICO $\rightarrow$ $N_{cr} = \dfrac{\pi^2 E I_{min}}{l_0} = \dfrac{\pi^2 E \; 3{,}09}{160} \ggg 200$

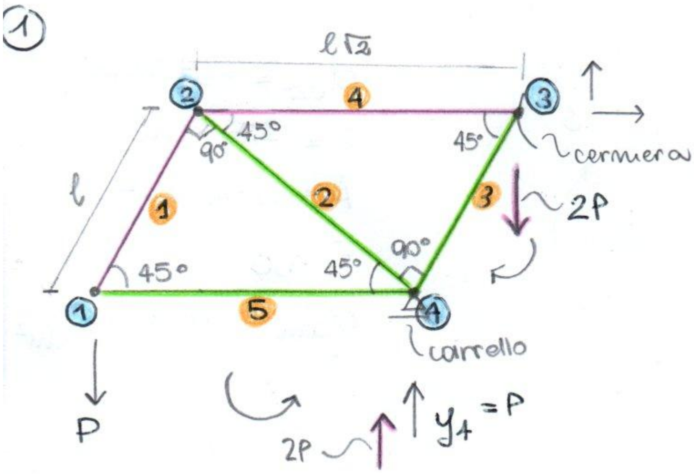
calcoliamo ω corrispondente alla sezione:

$\lambda = 146$
$\omega = 2{,}84$ } dati ricavati dalle tabelle

$$\sigma = \frac{\omega \cdot \overset{\text{carico reale}}{N}}{A} = \frac{2{,}84 \cdot 200}{2{,}54} = 223 \text{ kg/cm}^2 \lll \sigma_{am}$$

STRUTTURE RETICOLARI

1



$2n = a + 3$

$8 = 5 + 3$ - condizione verificata

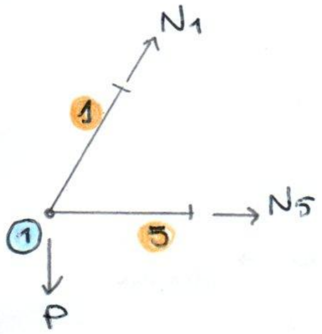
trazione = TIRANTE

compressione = PUNTONE

$\Sigma x = 0$

$\Sigma y = 0$

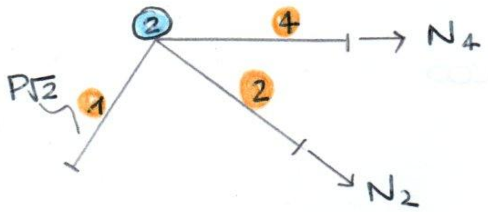
$\Sigma M_{(3)} = 0 \qquad P\cancel{l\sqrt{2}} = y\,\frac{\cancel{l\sqrt{2}}}{2} \qquad y = 2P$



$\Sigma x = 0 \qquad \begin{cases} N_1 \frac{\sqrt{2}}{2} + N_5 = 0 \\ N_1 \frac{\sqrt{2}}{2} - P = 0 \end{cases}$

$\Sigma y = 0$

$\begin{cases} N_1 = P\sqrt{2} \\ P\sqrt{2} \cdot \frac{\sqrt{2}}{2} + N_5 = 0 \rightarrow N_5 = -P \end{cases}$

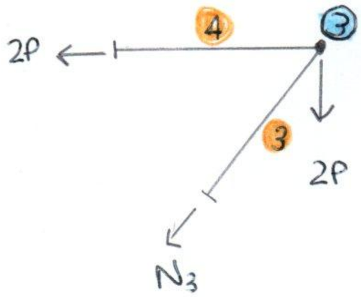


$\Sigma x = 0 \qquad \begin{cases} N_2 \frac{\sqrt{2}}{2} + N_4 - P\cancel{\sqrt{2}}\,\cancel{\frac{\sqrt{2}}{2}} = 0 \\ N_2\left(-\frac{\sqrt{2}}{2}\right) - P\cancel{\sqrt{2}}\,\cancel{\frac{\sqrt{2}}{2}} = 0 \end{cases}$

$\Sigma y = 0$

$N_2 = -\sqrt{2}P$

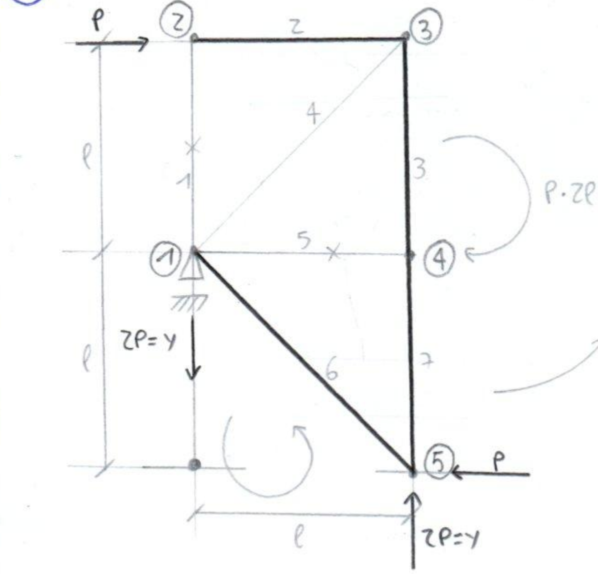
$N_4 = 2P$



$\Sigma x = 0 \qquad -N_3 \frac{\sqrt{2}}{2} - 2P = 0$

$N_3 = -\frac{4}{\sqrt{2}} P$

2



n = nodi
a = aste

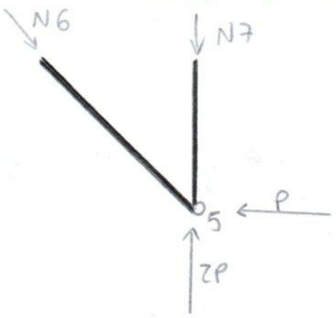
$2n = a+3 \longrightarrow 10 = 7+3$ ✓

- non ci sono maglie quadrate
- non ci sono sovrapposizioni

ISOSTATICO

$2P \cdot \ell = Y \cdot \ell \longrightarrow Y = 2P$

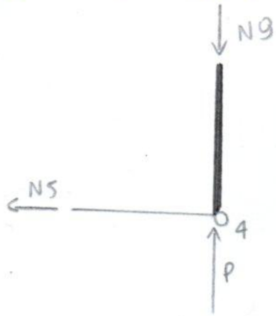
1 isolo nodo 5 in cui concorrono aste 6 e 7



$\Sigma x = 0 \longrightarrow -N6 \frac{\sqrt{2}}{2} - P = 0 \longrightarrow N6 = -\frac{2}{\sqrt{2}} P = -P\sqrt{2}$

$\Sigma y = 0 \longrightarrow N7 + N6 \frac{\sqrt{2}}{2} + 2P = 0 \longrightarrow N7 - \frac{P\sqrt{2}\sqrt{2}}{2} + 2P = 0 \longrightarrow N7 = -P$

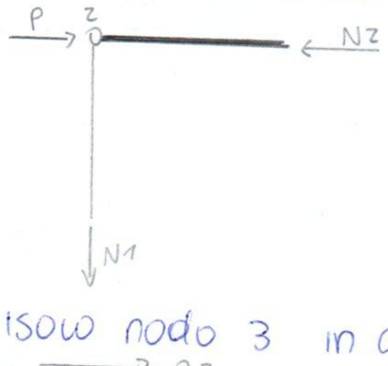
2 isolo nodo 4 in cui concorrono aste 3 e 5



$\Sigma x = 0 \longrightarrow N5 = 0$

$\Sigma y = 0 \longrightarrow N3 + P = 0 \longrightarrow N3 = -P$

3 isolo nodo 2 in cui concorrono aste 2 e 1



$\Sigma x = 0 \longrightarrow N2 + P = 0 \longrightarrow N2 = -P$

$\Sigma y = 0 \longrightarrow N1 = 0$

4 isolo nodo 3 in cui concorre l'asta 4

$\Sigma x = 0 \longrightarrow -N4 \cdot \frac{\sqrt{2}}{2} + P = 0 \longrightarrow N4 = \frac{2}{\sqrt{2}} P = P\sqrt{2}$