ALGEBRA BOOLE

$A, B, C \ldots \quad 0 / 1$

FALSO VERO

AND

OR

NOT

$A \cdot B = 1 \iff A = 1$ E $B = 1$

$A + B = 1$ se almeno A o $B = 1$

$\overline{A}$ inverte lo stato della variabile.

A	B	AND $A \cdot B$	OR $A + B$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

2^2 combinazioni

2^{NVAR}

A	$\overline{A}$
0	1
1	0

TEOREMA DI DE MORGAN

1) $\overline{A+B} = \overline{A} \cdot \overline{B}$

2) $\overline{A \cdot B} = \overline{A} + \overline{B}$

$F = A \oplus (B \cdot C) \oplus D$

SIGNIFICATO

1) $\overline{\overline{A+B}} = \overline{\overline{A} \cdot \overline{B}}$

$\boxed{A+B = \overline{\overline{A} \cdot \overline{B}}}$

OR come combinazione $\boxed{\text{AND e NOT}}$

2) $\overline{\overline{A \cdot B}} = \overline{\overline{A} + \overline{B}}$

$\Rightarrow \boxed{A \cdot B = \overline{\overline{A} + \overline{B}}}$

AND combinazione $\boxed{\text{OR e NOT}}$

$\boxed{\text{AND e NOT}}$ oppure $\boxed{\text{OR e NOT}}$

$\boxed{\text{PORTE UNIVERSALI}}$

T. DE MORGAN

$\overline{A+B} = \overline{A} \cdot \overline{B}$

$\overline{A \cdot B} = \overline{A} + \overline{B}$

$\overline{A+B+C+\cdots+F} = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \cdots \cdot \overline{F}$

$\overline{A \cdot B \cdot C \cdots F} = \overline{A} + \overline{B} + \overline{C} + \cdots + \overline{F}$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

$\Rightarrow \overline{A+B} = \overline{A} \cdot \overline{B}$

$X \cdot \overline{X} = 0$

$X = \overline{A} \cdot \overline{B}$

$\overline{X} = \overline{\overline{A+B}}$

$\overline{A} \cdot \overline{B} \cdot (\overline{\overline{A+B}}) = \overline{A}\,\overline{B}\,(A+B) = \underbrace{\overline{A}\,\overline{B}\,A}_{0} + \underbrace{\overline{A}\,\overline{B}\,B}_{0}$

$= 0 + 0 = 0$

PORTE LOGICHE

VAR BOOLEANE A, B, C, ... Algebrico 0/1 ⟹ CIRCUITALE

SEGNALI ELETTRICI

2 VALORI D.D.P

Volt

ELEMENTI CIRCUITALI ⟶

TTL → 5 V ⟹ VERO 1

TTL → 0 V ⟹ FALSO ∅

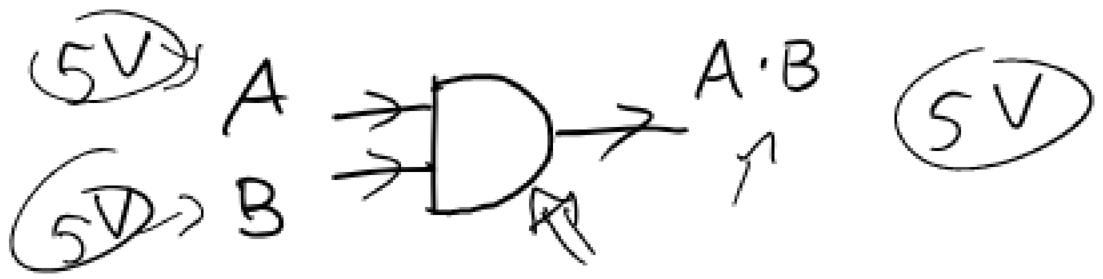
0, 12 V

NIM 0, -800 mV

0, 24 V

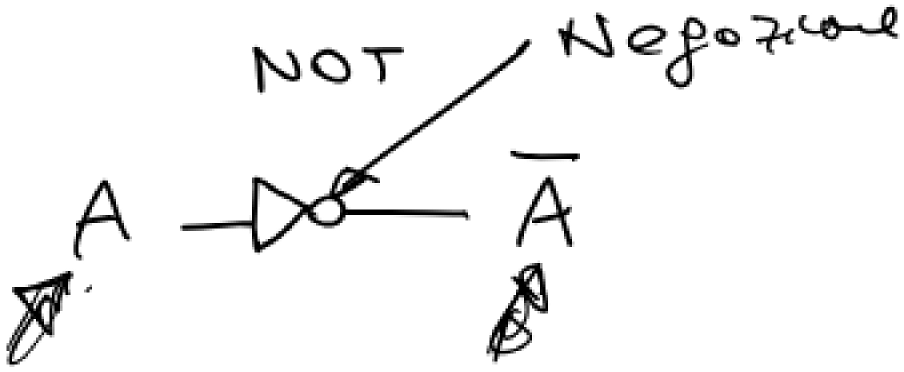
TTL → 0, 3.3 V

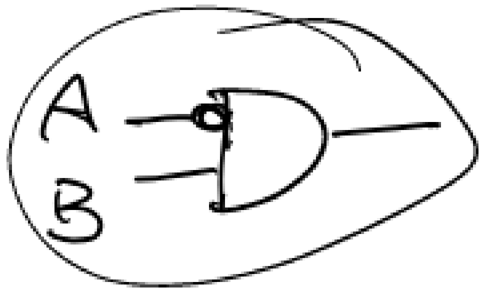
AND

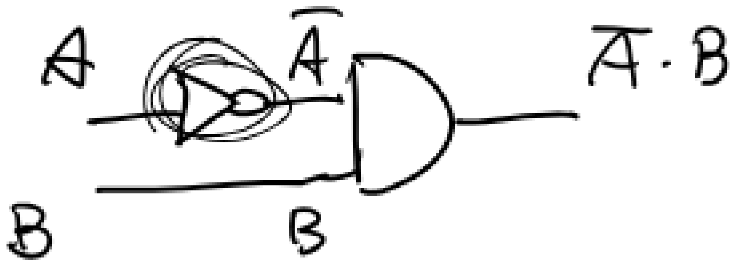


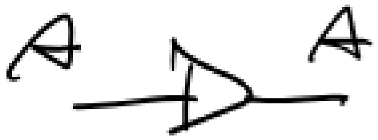
OR

NOT



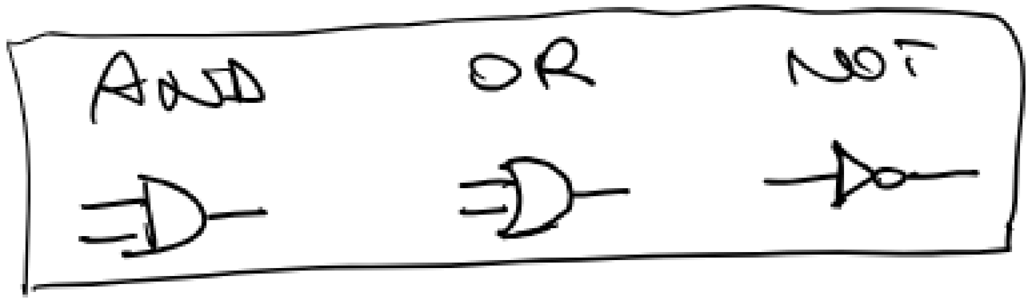






IDENTITA'

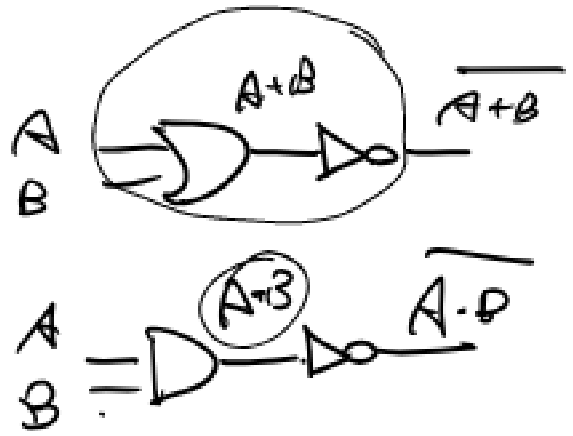
AND OR NOT

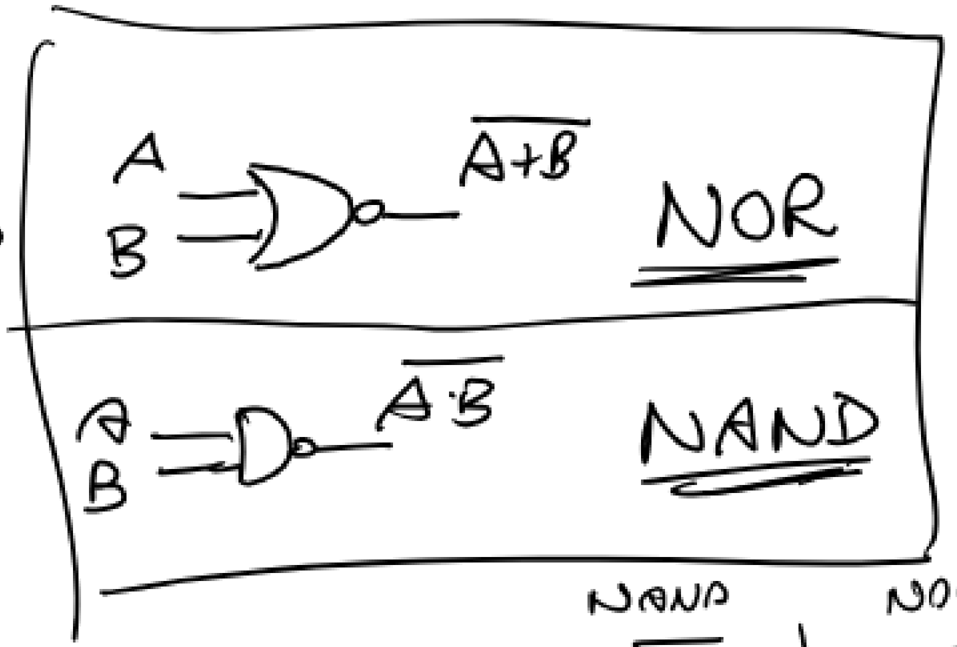


PORTE UNIVERSALI

$\overline{A+B} = \overline{A}\cdot\overline{B}$

$\overline{A\cdot B} = \overline{A}+\overline{B}$





$\overline{\overline{A+B}} = \overline{\overline{A}\cdot\overline{B}}$

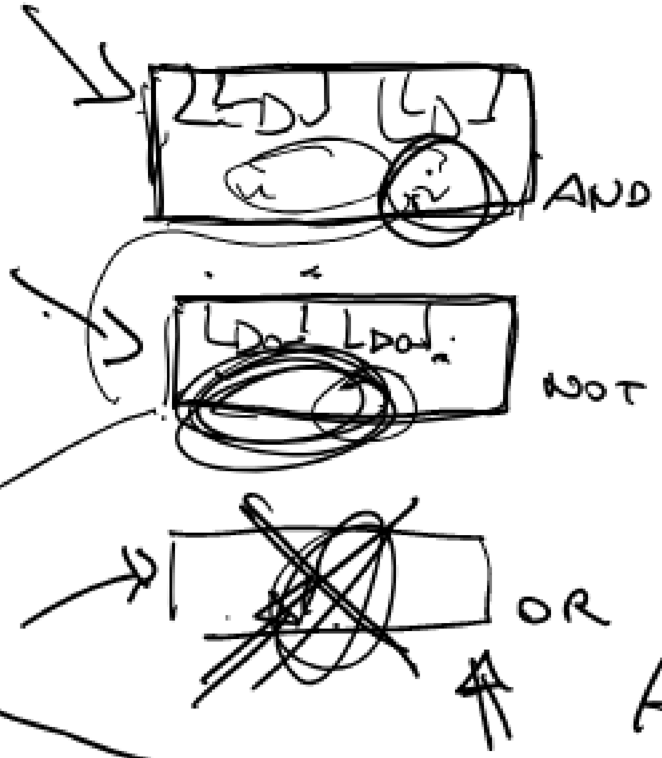
$\boxed{A+B = \overline{\overline{A}\cdot\overline{B}}}$

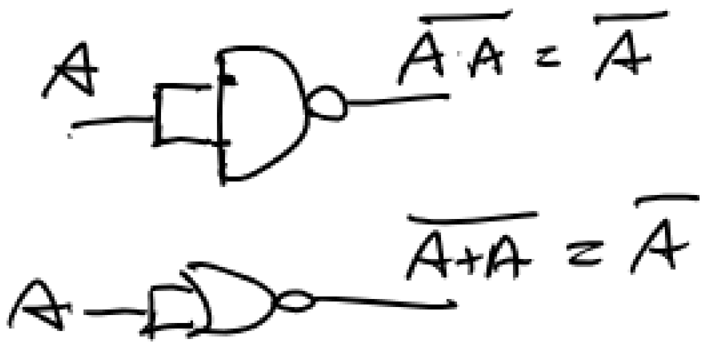
A	B	NAND $\overline{A\cdot B}$	NOR $\overline{A+B}$
0 =	0	1	1
0	1	1	0
1	0	1	0
1 =	1	0	0

?

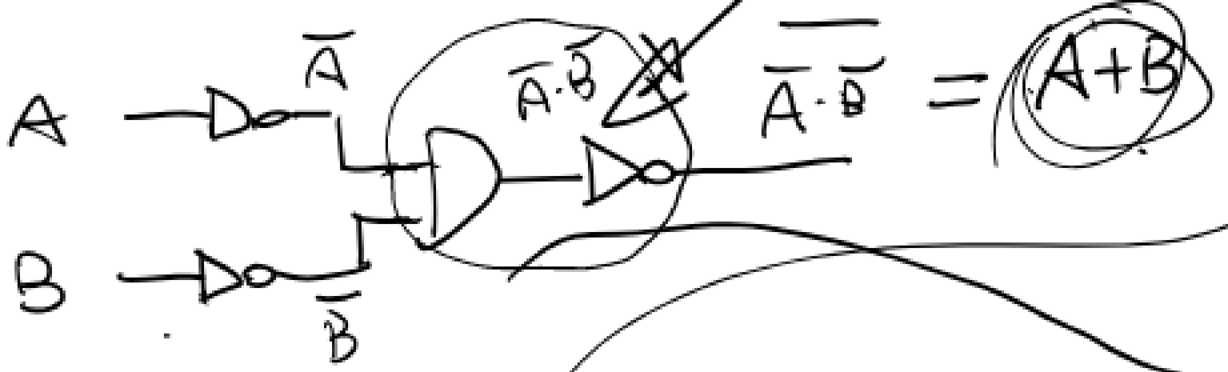
NOT

NOT



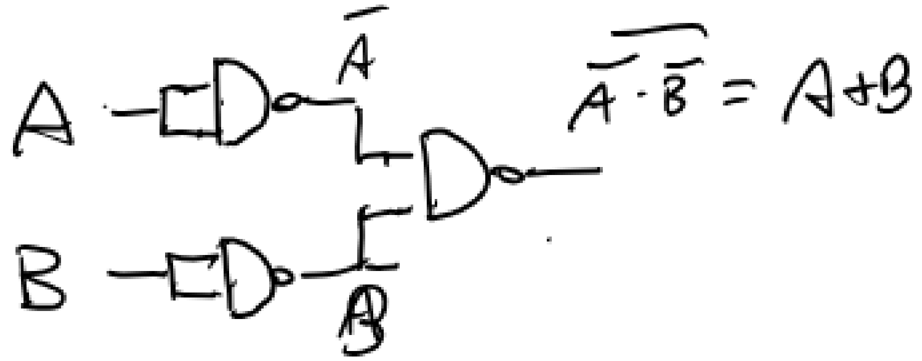


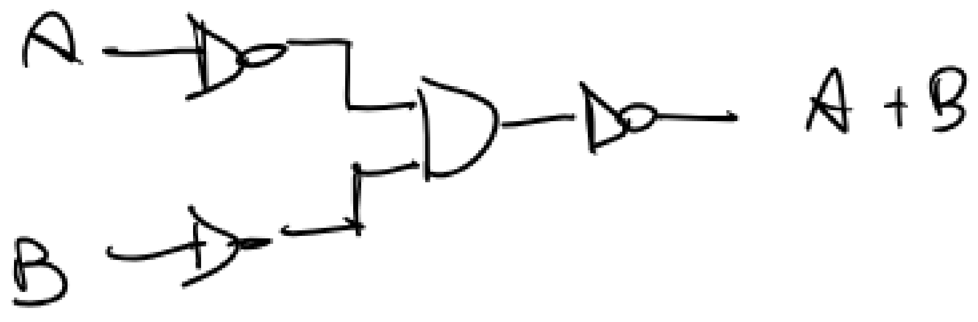
PORTE AND, NOT.



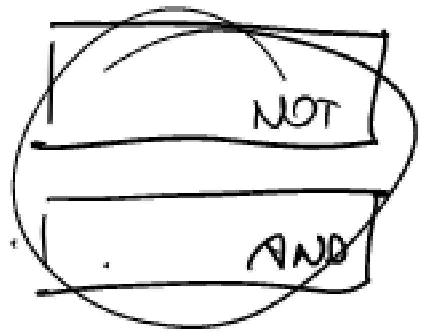
$\overline{\overline{A}\cdot\overline{B}} = A+B$

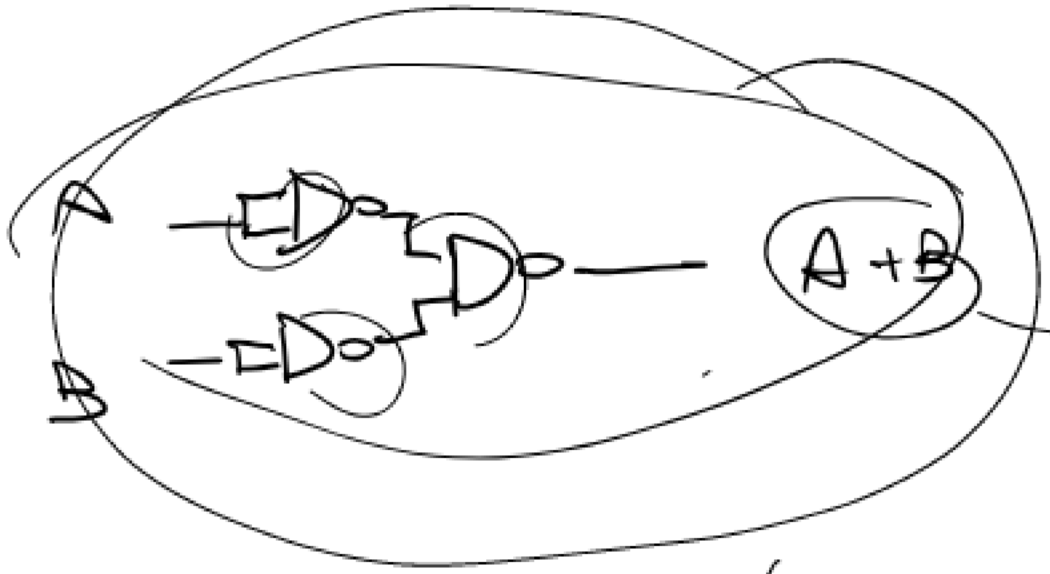
3 PORTE NOT
1 PORTA AND } 1 PORTA OR



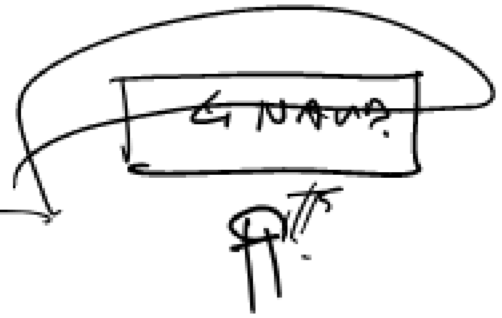


3 NOT
1 AND } → 2 C.I.





→ 3 NAND



FPGA . . . .

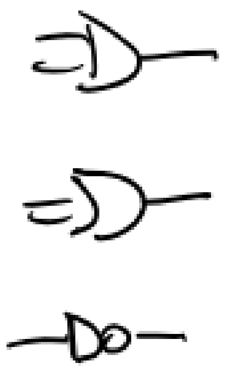
OP. LOG. BASE

T. De Morg. AND, NOT / OR, NOT

AND

OR

NOT



PORTE UNIVERSALI

NAND A B  $\overline{A \cdot B}$ ⇒ NOT $\boxed{A = B}$

NOR A B  $\overline{A + B}$

Vale 1 se solo una delle due vari. è vera

XOR (EXCLUSIVE OR)

$A \oplus B$

A B  $A \oplus B$

XNOR

$\overline{A \oplus B}$

A B  $\overline{A \oplus B}$

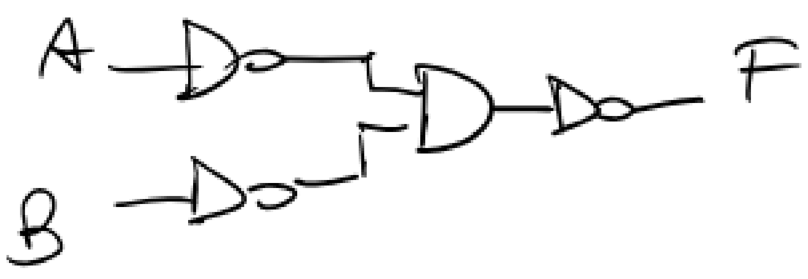
A	B	$A \oplus B$	$\overline{A \oplus B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

~~XOR~~ = AND, OR, NOT ?

SI | SI RIESCE

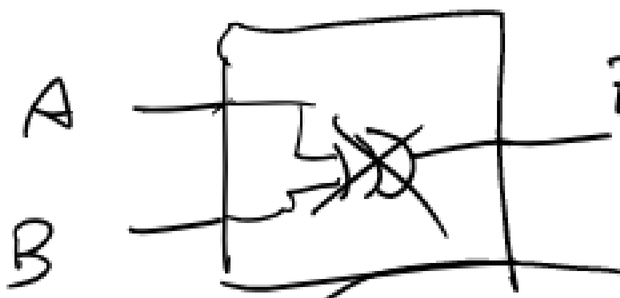
COME???

ES1) $A + B = \overline{\overline{A} \cdot \overline{B}}$ $\quad F = \overline{\overline{A} \cdot \overline{B}}$



ES2) XOR

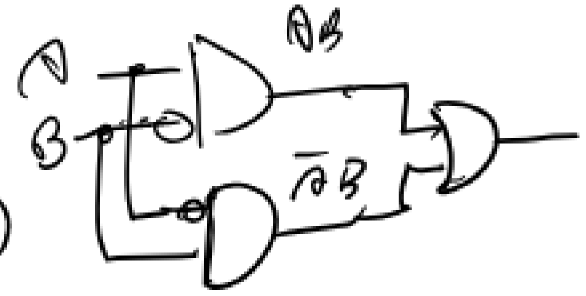
A	B	$A \oplus B = F$
0	0	0
0	1	1
1	0	1
1	1	0



$F = A \oplus B$

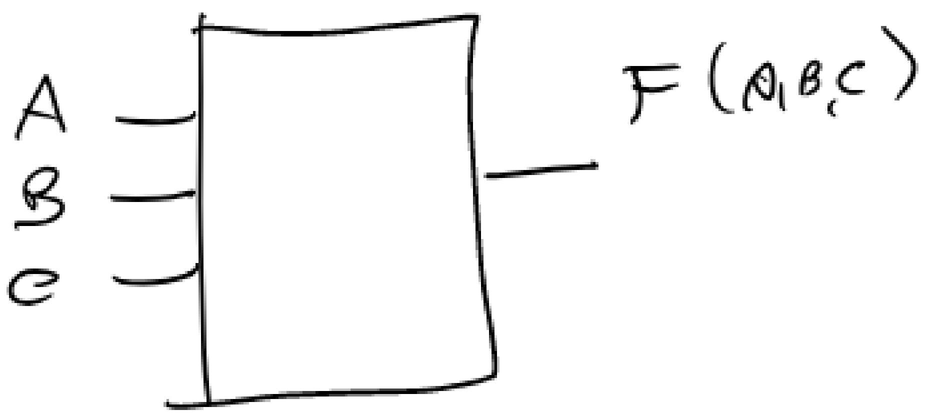
$F = A\overline{B} + \overline{A}B$

$= (A+B) \cdot (\overline{A+B})$



D,E.

A	B	C	F(A,B,C)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1



1) Rapp. algebrica $F(A,B,C,\dots)$ $\Longrightarrow$ Disegnare circuito SCHEMA $\longrightarrow$ SIM Ver. TAV. VER

2) F TAV VERITÀ $\longrightarrow$ Rapp. algebrica

SVILUPPO IN MINTERM o MAXTERM

A	B	minterm m	maxterm M	$m_1 = \bar{A}\cdot\bar{B}$	m_2	m_3	m_4	F	F
0	0	$\bar{A}\cdot\bar{B} = m_1$	$A+B = M_1$	1	0	0	0	0	F_1
0	1	$\bar{A}\cdot B = m_2$	$A+\bar{B} = M_2$	0	1	0	0	1	F_2
1	0	$A\cdot\bar{B} = m_3$	$\bar{A}+B = M_3$	0	0	1	0	1	F_3
1	1	$A\cdot B = m_4$	$\bar{A}+\bar{B} = M_4$	0	0	0	1	1	F_4

AND fra A e B aff. o neg. $= 1$

OR fra A e B aff. o neg. $= 0$

$$F = \sum_{i=1}^{4} m_i \cdot F_i = \prod_{i=1}^{4} (M_i + F_i)$$

SUCC. OR — TAV VER — AND — SUCC. AND — OR

SVILUPPO MINTERM / MAXTERM

A	B	m	M	F	F
0	0	$\bar{A}\bar{B} = m_1$	$A+B = M_1$	F_1	$\boxed{0}$
0	1	$\bar{A}B = m_2$	$A+\bar{B} = M_2$	F_2	1
1	0	$A\bar{B} = m_3$	$\bar{A}+B = M_3$	F_3	1
1	1	$AB = m_4$	$\bar{A}+\bar{B} = M_4$	F_4	$\boxed{0}$

$$F = \sum_{i=1}^{4} F_i \cdot m_i = \prod_{i=1}^{4} F_i + M_i$$

$$= \prod_{\substack{i=1 \\ F_i=0}}^{4} M_i$$

$$F = F_1 m_1 + F_2 m_2 + F_3 m_3 + F_4 \cdot m_4 = \sum_{\substack{i=1 \\ F_i=1}}^{4} m_i$$

$$= F_1 \bar{A}\bar{B} + \cancel{F_2 \bar{A} B} + F_3 A\bar{B} + F_4 A B$$

$0 \cdot \bar{A}B$

$F_2 = 0 \quad \overset{\shortparallel}{0}$

$F_1 = 1$

$1 \cdot \bar{A}\bar{B} = \bar{A}\bar{B}$ m_1

$F_i \neq 0$

$$F = \bar{A}B + A\bar{B}$$

$$= (A+B)(\bar{A}+\bar{B})$$

$$= \cancel{A\bar{A}} + A\bar{B} + B\bar{A} + \cancel{B\bar{B}}$$

$$= A\bar{B} + \bar{A}B$$

$$F = (F_1 + M_1) \cdot (F_2 + M_2) \cdot \ldots$$

$F_1 = 1$: $\cancel{1 + M_1} = 1$

$F_2 = 0$: $\cancel{0} + M_2$

$M_2 = A + \bar{B}$

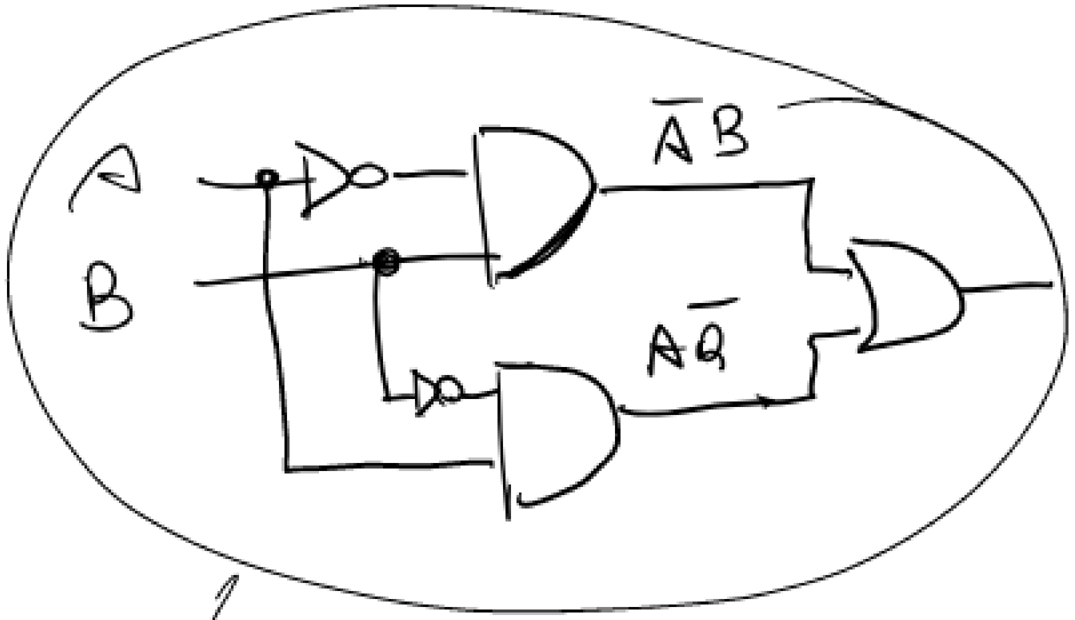
SVILUPPO MINTERM / MAXTERM

$$F = \sum_{\substack{i=1 \\ F_i=1}}^{2^{NVAR}} m_i = \prod_{\substack{i=1 \\ F_i=0}}^{2^{NVAR}} M_i$$

A	B	C	
0	0	0	$\rightarrow \bar{A}\bar{B}\bar{C}$
0	0	1	$\rightarrow \bar{A}\bar{B}C$
0	1	1	$\rightarrow \bar{A}BC$
⋮			⋮

A	B	F	m
0	0	0	$\bar{A}\bar{B}$
0	1	1	$\bar{A}B$
1	0	1	$A\bar{B}$
1	1	0	AB

$F = \bar{A}B + A\bar{B}$

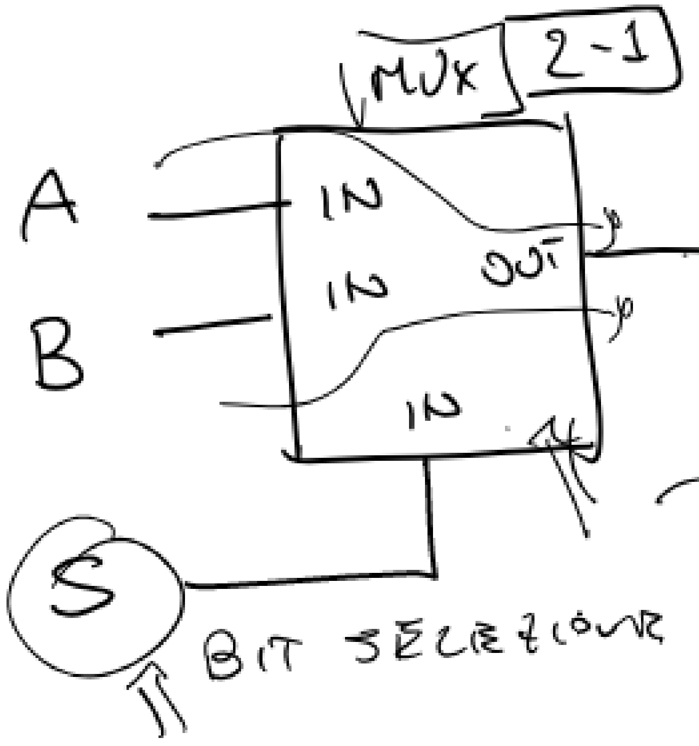


$F = A \oplus B$

NAND / NOR ? ???

AND ⇔ OR ?

MUX (MULTIPLEXER)



$F = \begin{cases} A & \text{quando } S=0 \\ B & \text{quando } S=1 \end{cases}$

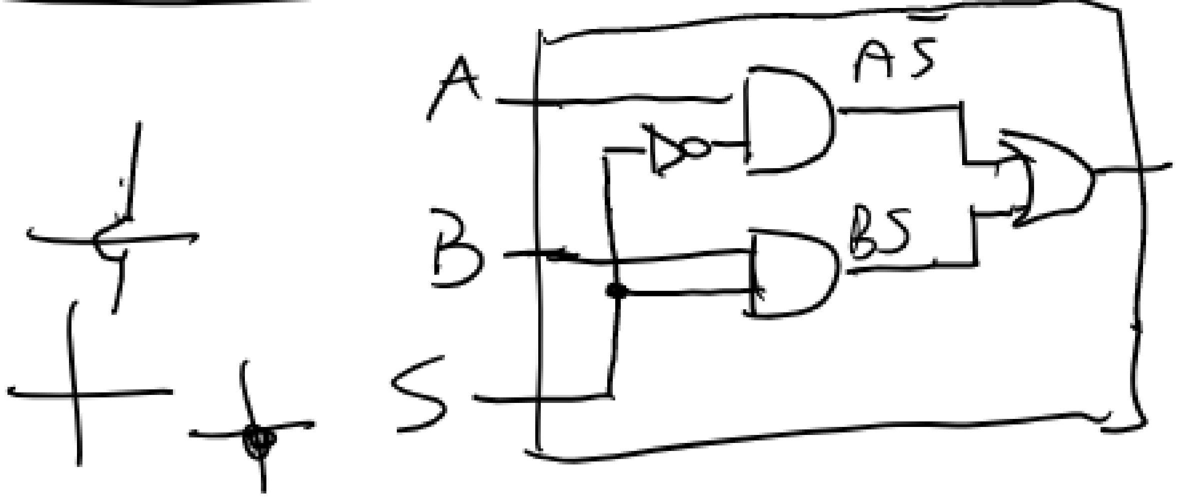
MIS.

S	A	B	F	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{S} A \bar{B}$
0	1	1	1	$\bar{S} A B$
1	0	0	0	
1	0	1	1	$S \bar{A} B$
1	1	0	0	
1	1	1	1	$S A B$

$$F = \bar{S} A \bar{B} + \bar{S} A B + S \bar{A} B + S A B$$

$$= \bar{S} A \underbrace{(\cancel{\bar{B} + B})}_{1} + S B \underbrace{(\cancel{\bar{A} + A})}_{1}$$

$$\boxed{F = \bar{S} A + S B}$$ MUX



$\bar{S} A + S B = F_{MUX}$

MUX 4-1

MUX 16-1