Soluzioni compito 20 Febbraio 2018

1. Esercizio 1
   - $w = \frac{3}{5}\overrightarrow{i} + \frac{6}{5}\overrightarrow{j}$.
2. Esercizio 2
   - U è sottospazio; V non è sottospazio.
   - Una base di U è data da $\{(1,1,0),(-3,0,1)\}$.
   - $U + W = \{(1,1,0),(-3,0,1),(0,0,1)\}$.
   - $U + W$ non è somma diretta.
3. Esercizio 3
   - Per $k \neq -4$, $rg(A) = 3$; per $k = -4$, $rg(A) = 2$.
   - L'inversa della sottomatrice $\bar{A}$ data dalle prime tre colonne di A, per $k = 0$, è: $\bar{A}^{-1} \begin{pmatrix} -\frac{3}{2} & 0 & \frac{1}{2} \\ 4 & 0 & -1 \\ -\frac{3}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$.
4. Esercizio 4
   - Per $k \neq -1$ il sistema è incompatibile.
   - Per $k = 1$ il sistema è compatibile e ammette la soluzione $(1,1)$.
5. Esercizio 5
   - Per $\alpha = \frac{18}{7}$, $\ker f = \left\{(\frac{7}{3},2,1)\right\}$, Imf $= \{(-1,-4,3),(2,1,0)\}$; f non è iniettiva nè suriettiva.
   - Per $\alpha \neq \frac{18}{7}$, $\ker f = \{(0,0,0)\}$, Imf$= \mathbb{R}^3$ ed f è iniettiva e suriettiva.
   - Per $\alpha \neq \frac{18}{7}$ il vettore $(-1,\beta,1)^T$ appartiene ad Imf per ogni $\beta \in \mathbb{R}$. Viceversa, per $\alpha = \frac{18}{7}$ il vettore appartiene ad Imf se e solo se $\beta = \frac{4}{6}$.
6. Esercizio 6
   - $M_{\mathcal{B}}^{\mathcal{B}}(f) = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$.
   - dim Imf $= 2$ e $\mathcal{B}$ è un base di Imf; $\dim \ker f = 0$ e $\ker f = \{\underline{0}\}$.
7. Esercizio 7
   - Base ortonormale $\mathcal{B} = \{(1,0,0),(0,1,0),(0,0,-1)\}$.
   - Coefficienti di Fourier del generico elemento (x,y,z): $a_1 = x$, $a_2 = y$, $a_3 = -z$.

8. Esercizio 8

   - La matrice associata all'applicazione lineare f ammette gli autovalori $\lambda_1 = 1$, $\lambda_2 = 4$, $\lambda_3 = -1$, pertanto la matrice è diagonalizzabile.
   - La matrice diagonalizzante è

$$M = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

   - La matrice che rappresenta f rispetto a tale base è

$$D = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

9. Esercizio 9

   - $M = \begin{pmatrix} -8 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 0 \end{pmatrix}$.
   - La forma quadratica è indefinita.
   - Base ortonormale che diagonalizza la forma quadratica: $B = \left\{ \left(\frac{1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}} \right), (0, 1, 0), \left(-\frac{3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}} \right) \right\}$.

10. Esercizio 10

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$$s : \begin{cases} x = 1 - 4t \\ y = 2 + t \\ z = 3 - 2t \end{cases}$$

$$s' : \begin{cases} x = 1 + t \\ y = 2 - 2t \\ z = 3 + t \end{cases}$$

20-2-2018

1. $v_1 = \begin{pmatrix}1\\0\\1\end{pmatrix}$ $v_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $v_3 = \begin{pmatrix}1\\1\\2\end{pmatrix}$

Proiezione di ~~v_2~~ v_1 nel piano contenente v_2 e v_3

$$v_2 \times v_3 = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 2\vec{i} - \vec{k} = \begin{pmatrix}2\\0\\-1\end{pmatrix}$$

$$v_1' = v_1 - \left\langle v_1 , \frac{v_2 \times v_3}{\|v_2 \times v_3\|} \right\rangle \frac{v_2 \times v_3}{\|v_2 \times v_3\|} =$$

$$= \begin{pmatrix}1\\0\\1\end{pmatrix} - \frac{1}{5}\begin{pmatrix}2\\0\\-1\end{pmatrix} = \begin{pmatrix}+1-\frac{2}{5}\\0\\1+\frac{1}{5}\end{pmatrix} = \frac{3}{5}\vec{i} + \frac{6}{5}\vec{k}$$

2.

$$U = \{(x, y, z) \in \mathbb{R}^3 \; x - y + 3z = 0\} = \left\{\begin{pmatrix}y-3z\\y\\z\end{pmatrix}\right\}$$

Si dimostra che è un sottospazio di $\mathbb{R}^3$

Presi due elementi $\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$, $\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}$ t.c. $\begin{pmatrix}y_1-3z_1\\y_1\\z_1\end{pmatrix}$

e $\begin{pmatrix}y_2-3z_2\\y_2\\z_2\end{pmatrix}$ (ossia $x_1 = y_1 - 3z_1$ e $x_2 = y_2 - 3z_2$)

di U, e uno scalare $c \in \mathbb{R}$,

si consideri

$$c\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix} - \begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix} = \begin{pmatrix}cx_1 - x_2\\cy_1 - y_2\\cz_1 - z_2\end{pmatrix}$$

$$cx_1 - x_2 = c(y_1 - 3z_1) - (y_2 - 3z_2)$$
$$= (cy_1 - y_2) - 3(cz_1 - z_2)$$

Dunque $\begin{pmatrix} cx_1 - x_2 \\ cy_1 - y_2 \\ cz_1 - z_2 \end{pmatrix} \in U$ e U è sottospazio

$$V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \quad x^2 + y^2 - 4x = 0 \right\}$$

Dati $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ e $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ t.c. $x_1^2 + y_1^2 - 4x_1 = 0$, $x_2^2 + y_2^2 - 4x_2 = 0$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

$$(x_1 + x_2)^2 + (y_1 + y_2)^2 - 4(x_1 + x_2) =$$
$$= \underbrace{x_1^2 + y_1^2 - 4x_1}_{=0} + \underbrace{x_2^2 + y_2^2 - 4x_2}_{=0} + 2x_1x_2 + 2y_1y_2$$
$$= 2x_1x_2 + 2y_1y_2 \neq 0$$ per vettori di V non nulli

V non è sottospazio.

Base di $U = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$ Sono el. lin. indip.

$\begin{bmatrix} 1 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ ha rango 2

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \quad x = y \right\} = \left\{ \begin{pmatrix} x \\ x \\ z \end{pmatrix} \right\} = \left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$U+W = \left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$ $\dim U+W = 3$

(Sono una base

perchè $\begin{pmatrix} 1 & -3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ ha rango 3)

$U \cap W \neq \{0\}$ perchè $\dim U \cap W = -3+2+2 = 1$

$\dim U+W = \dim U + \dim W - \dim(U \cap W)$

$$U \cap W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{array}{l} x = y \\ x = y - 3z \end{array} \right\} = \left\{ \begin{pmatrix} x \\ x \\ 0 \end{pmatrix} \right\} = \left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right]$$

3.

$$A = \begin{pmatrix} 2 & 1 & 0 & 1 \\ -1 & 0 & 2 & 1 \\ 8 & 3 & k & 1 \end{pmatrix} \qquad \text{rango } A \geq 2$$

$$\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 8 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 8 & 3 & -2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 8-6 & 3 & -2 \end{vmatrix}$$

$$= - \begin{vmatrix} -1 & 1 \\ 2 & -2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 1 & 0 \\ -1 & 0 & 2 \\ 8 & 3 & k \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 3 & k \end{vmatrix} = - \begin{vmatrix} -1 & 2 \\ 2 & k \end{vmatrix}$$

$$= -(-k-4) = k+4$$

$k \neq -4$ rango $A = 3$

$k = -4$ rango $A = 2$

$$\bar{A} = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 2 \\ 8 & 3 & 0 \end{pmatrix} \qquad \det \bar{A} = -2(6-8) = 4$$

$$\bar{A}^{-1} = \frac{1}{4} (\text{adj } A)^T = \frac{1}{4} \begin{pmatrix} \begin{vmatrix} 0 & 2 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \\ -\begin{vmatrix} -1 & 2 \\ 8 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 8 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ -1 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 8 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 8 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} -6 & 0 & 2 \\ 16 & 0 & -4 \\ -3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -3/2 & 0 & \frac{1}{2} \\ 16/4 & 0 & -1 \\ -3/4 & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

4.
$$\begin{cases} x - y = 0 \\ 2x - y = 1 \\ kx + y = 0 \end{cases}$$

$$A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \\ k & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 + 2 \neq 0$$

rango $A = 2$

$$(A:b) = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ k & 1 & 0 \end{pmatrix} \qquad \begin{vmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ k & 1 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & -1 \\ k & 1 \end{vmatrix} = -(1+k)$$

$$-(1+k) = 0$$

$$\iff k = -1$$

$k \neq -1$ rango $(A:b) = 3$ sistema impossibile

$k = -1$ rango $(A:b) = 2$ sistema risolubile

$$\begin{cases} x - y = 0 \\ 2x - y = 1 \end{cases} \qquad \begin{matrix} x = y \\ y = 1 \end{matrix} \qquad Sol = (1, 1)$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1+2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{matrix} x - 1 = 0 \quad x = 1 \\ y = 1 \end{matrix}$$

5. $f: \mathbb{R}^3 \to \mathbb{R}^3$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} -y + 2z \\ 3x - 4y + z \\ \alpha x - 3y \end{pmatrix}$$

$$\ker f = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : \begin{array}{l} -y + 2z = 0 \\ 3x - 4y + z = 0 \\ \alpha x - 3y = 0 \end{array} \right\}$$

$$\left| \begin{pmatrix} 0 & -1 & 2 \\ 3 & -4 & 1 \\ \alpha & -3 & 0 \end{pmatrix} \right| = -\alpha + 2(-9 + 4\alpha) = -\alpha - 18 + 8\alpha$$
$$= 7\alpha - 18$$

Per $\alpha = \frac{18}{7}$ $\quad rg\, A = 2 \quad \ker f = \left\{ \begin{array}{l} 3x - 4y + z = 0 \\ \frac{18}{7} x - 3y = 0 \end{array} \right\} = \left[\begin{pmatrix} 7/3 \\ 2 \\ 1 \end{pmatrix} \right]$

$\alpha \neq \frac{18}{7}$ $\quad rg\, A = 3 \quad \ker f = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$ $\quad \dim \ker f = 0$

Per $\alpha = \frac{18}{7}$ $\quad$ Im f ha dimensione 2

base $= \left\{ \begin{pmatrix} -1 \\ -4 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$ $\quad$ nè iniettiva nè suriettiva

Per $\alpha \neq \frac{18}{7}$ $\quad$ Im $f = \mathbb{R}^3$ $\quad$ dim = 3

base $= \begin{pmatrix} 0 & -1 & 2 \\ 3 & -4 & 1 \\ \alpha & -3 & 0 \end{pmatrix}$ $\quad$ iniettiva e suriettiva

Per $\alpha = \frac{18}{7}$ $\quad \begin{pmatrix} -1 \\ \beta \\ 1 \end{pmatrix} \in \operatorname{Im} f \iff rg \begin{pmatrix} -1 & -1 & 2 \\ \beta & -4 & 1 \\ 1 & -3 & 0 \end{pmatrix} = 2$

$$\begin{vmatrix} -1 & -1 & 2 \\ \beta & -4 & 1 \\ 1 & -3 & 0 \end{vmatrix} = \cancel{4} - (3+1) + 2(-3\beta + 4) =$$

$$= -6\beta + 8 - 4 = 0 \iff \beta = \frac{4}{6}$$

per $\beta = \frac{4}{6}$ $\quad \begin{pmatrix} -1 \\ \beta \\ 1 \end{pmatrix} \in \operatorname{Im} f$

per $\beta \neq \frac{4}{6}$ il vettore non appartiene ad $\operatorname{Im} f$

Per $\alpha \neq \frac{18}{7}$ $\quad \begin{pmatrix} -1 \\ \beta \\ 1 \end{pmatrix} \in \operatorname{Im} f$ $\quad \forall \beta$ (per la suriettività di f)

6. $f : R^2 \to R^2$

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} = M_C^C(f)$$

$$B = \{(1,3) \ (3,5)\}$$

$$f\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 18 \end{pmatrix}_C$$

$$f\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 18 \\ 34 \end{pmatrix}_C$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = a\begin{pmatrix} 1 \\ 3 \end{pmatrix} + b\begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\begin{cases} x = a + 3b \\ y = 3a + 5b \end{cases} \Rightarrow \quad \begin{matrix} a = x - 3b \\ y = 3x - 9b + 5b \end{matrix} \quad \begin{cases} a = x - 3b \\ -4b = y - 3x \end{cases}$$

$$\begin{cases} a = x - 3\,\dfrac{3x - y}{4} = \dfrac{4x - 9x + 3y}{4} = \dfrac{-5x + 3y}{4} \\[2ex] b = \dfrac{3x - y}{4} = \dfrac{3x - y}{4} \end{cases}$$

$$f\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 18 \end{pmatrix}_C = \begin{pmatrix} \dfrac{-5 \cdot 10 + 3 \cdot 18}{4} \\[2ex] \dfrac{3 \cdot 10 - 18}{4} \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$f\begin{pmatrix} \cancel{18} & 3 \\ \cancel{34} & 5 \end{pmatrix} = \begin{pmatrix} 18 \\ 34 \end{pmatrix}_C = \begin{pmatrix} \dfrac{-5 \cdot 18 + 3 \cdot 34}{4} \\[2ex] \dfrac{3 \cdot 18 - 34}{4} \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \Rightarrow M_B^B(f) = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$$

Basta osservare che $M^{B}_{C}(\mathrm{id}_{\mathbb{R}^2}) = M = A$

$$M^{B}_{B}(f) = M^{C}_{B}(\mathrm{id}_{\mathbb{R}^2})\, M^{C}_{C}(f)\, M^{B}_{C}(\mathrm{id}_{\mathbb{R}^2}) = \underbrace{M^{-1}A}_{I}\, M = M = A$$

$$= \begin{pmatrix} -\frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$$

$\dim \mathrm{Im} f = 2$ $\qquad$ $\dim \mathrm{Ker} f = \{0\}$

B è base di $\mathrm{Im} f$.

7. $\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}2\\3\\0\end{pmatrix}\begin{pmatrix}3\\4\\-2\end{pmatrix}$

costruire una base ortonormale

$$v_1' = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$v_2' = \begin{pmatrix}2\\3\\0\end{pmatrix} - \frac{2}{1}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\3\\0\end{pmatrix}$$

$$v_3' = \begin{pmatrix}3\\4\\-2\end{pmatrix} - \frac{3}{1}\begin{pmatrix}1\\0\\0\end{pmatrix} - \frac{12}{9}\begin{pmatrix}0\\3\\0\end{pmatrix} = \begin{pmatrix}0\\4-\frac{12}{3}\\-2\end{pmatrix} = \begin{pmatrix}0\\0\\-2\end{pmatrix}$$

$$\frac{v_1'}{\|v_1'\|} = \begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \frac{v_2'}{\|v_2'\|} = \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \frac{v_3'}{\|v_3'\|} = \begin{pmatrix}0\\0\\-1\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = a_1\begin{pmatrix}1\\0\\0\end{pmatrix} + b_1\begin{pmatrix}0\\1\\0\end{pmatrix} + c_1\begin{pmatrix}0\\0\\-1\end{pmatrix}$$

$a_1 = x \qquad b_1 = y \qquad c_1 = -z$

8. $f(x,y,z) = (x+y+2z,\ x+2y+z,\ 2x+y+z)$

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda-1 & -1 & -2 \\ -1 & \lambda-2 & -1 \\ -2 & -1 & \lambda-1 \end{vmatrix}$$

$$(\lambda-1)\left[\lambda^2+2-3\lambda-1\right]+1(-\lambda+1-2)-2(\underbrace{1+2(\lambda-2)}_{1+2\lambda-4})$$

$$=(\lambda-1)(\lambda^2-3\lambda+1) \cancel{=} \lambda-1-4\lambda+6$$

$$=(\lambda-1)(\lambda^2-3\lambda+1) \underbrace{-5\lambda+5}_{-5(\lambda-1)}$$

$$=(\lambda-1)\left[\lambda^2-3\lambda+1-5\right]$$

$$=(\lambda-1)(\lambda^2-3\lambda-4)$$

$$\lambda = \frac{3 \pm \sqrt{9+16}}{2} < \begin{matrix} 4 \\ -1 \end{matrix}$$

$\lambda_1 = 1$ molt. algebrica 1

$\lambda_2 = 4$ " 1

$\lambda_3 = -1$ " 1

diagonalizzabile

$$V_{\lambda_1} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \begin{matrix} -y - 2z = 0 \\ -x \cancel{+} y - z = 0 \\ -2x - y = 0 \end{matrix} \right\} = \left\{ \begin{matrix} y = -2z \\ \cancel{y = -2x} \\ -2x + 2z = 0 \\ x = z \end{matrix} \right\}$$

$$= \left[\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right]$$

$$V_{\lambda_2} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \begin{matrix} 3x - y - 2z = 0 \\ -x + 2y - z = 0 \\ -2x - y + 3z = 0 \end{matrix} \right\} = \left\{ \begin{matrix} y = 3x \cancel{+} 2z \\ -x + 6x - 4z - z = 0 \\ \cancel{5}x = \cancel{5}z \\ y = x \end{matrix} \right.$$

$$= \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$V_{\lambda_3} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \begin{matrix} -2x - y - 2z = 0 \\ -x - 3y - z = 0 \\ \cancel{-2x - y - 2z = 0} \end{matrix} \right\} = \left\{ \begin{matrix} x = -3y - z \\ 6y + \cancel{2z} - y - \cancel{2z} = 0 \\ 5y = 0 \\ x = -z \end{matrix} \right.$$

$$= \left[\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$B = \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

9. $q(x,y,z) = -8x^2 + y^2 + 6xz$

$$A = \begin{pmatrix} -8 & & +3 \\ & 1 & \\ 3 & & 0 \end{pmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda+8 & 0 & -3 \\ 0 & \lambda-1 & 0 \\ -3 & 0 & \lambda \end{vmatrix} = (\lambda+8)(\lambda-1)\lambda + -3\left(3(\lambda-1)\right)$$

$$= (\lambda-1)\left[\lambda^2 + 8\lambda - 9\right]$$

$$\frac{-8 \pm \sqrt{64+36}}{2} \begin{matrix} \nearrow 1 \\ \searrow -9 \end{matrix}$$

$\lambda_1 = 1$

$\lambda_2 = 1$

$\lambda_3 = -9$

forma q. indefinita

$$V_1 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{matrix} 9x - 3z = 0 \\ -3x + z = 0 \end{matrix} \right\} = \left\{ \begin{pmatrix} x \\ y \\ 3x \end{pmatrix} \right\} = \left[\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right]$$

$\cancel{z} = 3x$

$$V_{-9} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{matrix} -x - 3z = 0 \\ y = 0 \\ -3x - 9z = 0 \end{matrix} \right\} = \left[\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right]$$

$x = -3z$

$$\begin{pmatrix} \frac{1}{\sqrt{10}} \\ 0 \\ \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -\frac{3}{\sqrt{10}} \\ 0 \\ \frac{1}{\sqrt{10}} \end{pmatrix}$$ è base ortonormale

10. $s.$ $P_0 = 1,2,3$

$$r: \begin{array}{l} x+2y-z+3=0 \\ 2x+2y-3z+5=0 \end{array}$$

$$v_1 = \begin{vmatrix} 2 & -1 \\ 2 & -3 \end{vmatrix} \quad v_2 = -\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} \quad v_3 = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix}$$

$= -6+2$ $\quad = +3-2$ $\quad \cancel{v_3} = 2-4$

$= -4$ $\quad = 1$ $\quad = -2$

(1) $$s. \begin{cases} x = 1 - 4t \\ y = 2 + t \\ z = \cancel{2}\, 3 - 2t \end{cases}$$

(2) $v_1\ v_2\ v_3$ sono ortogonali a $(3,4,5)$

$$\begin{cases} 3v_1 + 4v_2 + 5v_3 = 0 \\ v_1 + v_2 + v_3 = 0 \end{cases}$$

$$r \begin{cases} x = 4+t \\ y = -7+t \\ z = t \end{cases} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$v_1 = -v_2 - v_3 = 2v_3 - v_3 = v_3$ $\qquad \begin{matrix} v_1 \\ -2v_1 \\ v_1 \end{matrix}$

$-3v_2 - 3v_3 + 4v_2 + 5v_3 = 0$

$v_2 + 2v_3 = 0 \qquad v_2 = -2v_3$

Perfeito

$$s = \begin{cases} x = 1 + t \\ y = 2 - 2t \\ z = 3 + t \end{cases}$$